سیگنال‌ها و سیستم‌ها

درس ۲۳

سیستم‌های مخابراتی (۲)

Communication Systems (2)

کاظم فولادی قلعه
دانشکده مهندسی، پردیس فارابی
دانشگاه تهران

http://courses.fouladi.ir/sigsys

طرح درس

Course Outline

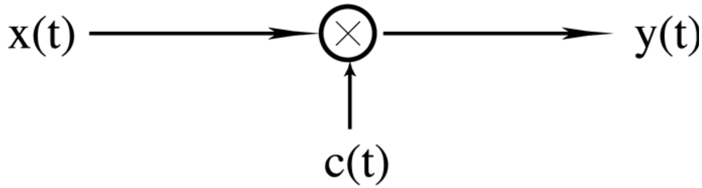
مدولاسیون دامنه‌ی با یک حامل متناوب دلخواه	AM with an Arbitrary Periodic Carrier
حامل قطار پالس و مالتی‌پلکس تقسیم زمانی	Pulse Train Carrier and Time-Division Multiplexing
مدولاسیون فرکانس سینوسی	Sinusoidal Frequency Modulation
مدولاسیون فاز	Phase Modulation
مدولاسیون دامنه‌ی سینوسی گسسته–زمان	DT Sinusoidal AM
نمونه‌برداری گسسته–زمان، دسیماسیون و درون‌یابی	DT Sampling, Decimation, and Interpolation

سیستم‌های مخابراتی (۲)

۱

مدولاسیون دامنه‌ی با یک حامل متناوب دلخواه

AM with an Arbitrary *Periodic* Carrier



$c(t)$ – periodic with period T, carrier frequency $\omega_c = \frac{2\pi}{T}$

Remember: periodic in $t \longleftrightarrow$ discrete in ω

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c) \quad \left(a_k = \frac{1}{T} \text{ for impulse train }\right)$$

$$\Downarrow$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega) = X(j\omega) * \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c)$$

$$= \sum_{k=-\infty}^{\infty} a_k X(j(\omega - k\omega_c))$$

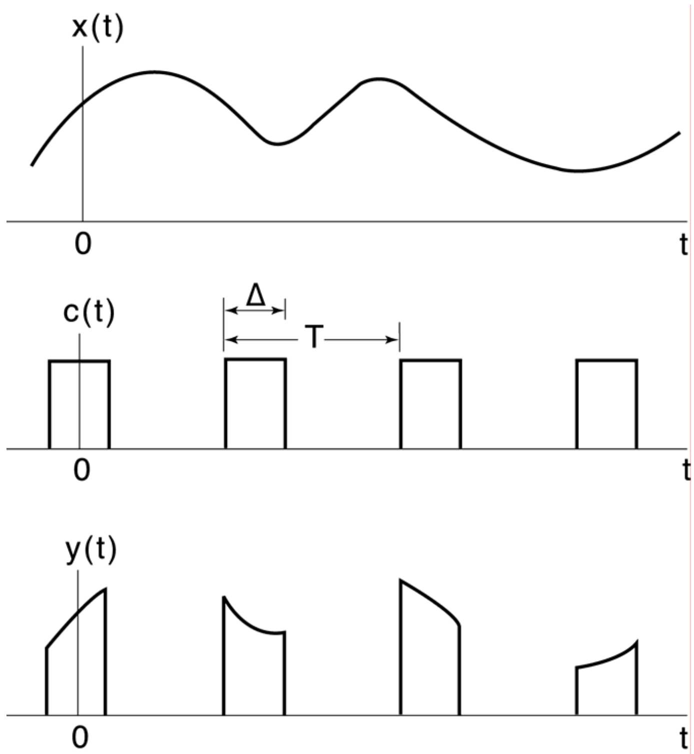
سیگنال‌ها و سیستم‌ها

سیستم‌های مخابراتی (۲)

۲

حامل قطار پالس و مالتی‌پلکس تقسیم زمانی

Modulating a (Periodic) Rectangular Pulse Train



$$y(t) = x(t) \cdot c(t)$$

Modulating a Rectangular Pulse Train Carrier, cont'd

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c)$$

and

$$a_0 = \frac{\Delta}{T}, \quad a_k = \frac{\sin(k\omega_c \Delta/2)}{\pi k}$$

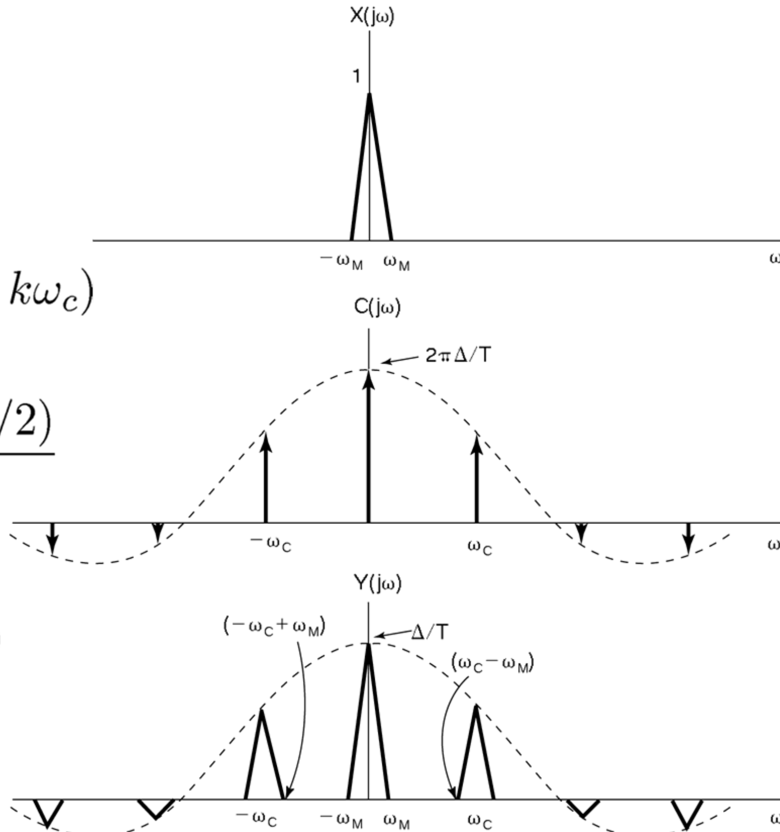
for rectangular pulse

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)$$

Drawn assuming:

$\omega_c > 2\omega_M$

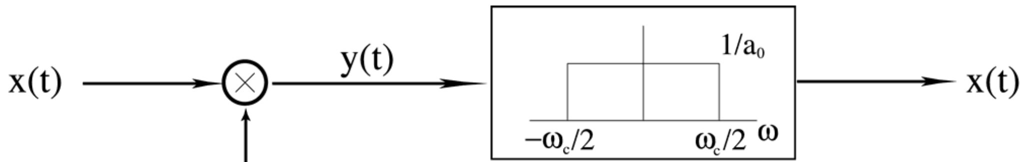
Nyquist rate is met



Observations

1) We get a similar picture with any $c(t)$ that is periodic with period T

2) As long as $\omega_c = 2\pi/T > 2\omega_M$, there is no overlap in the shifted and scaled replicas of $X(j\omega)$. Consequently, assuming $a_o \neq 0$:



$$c(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_c t}$$

$x(t)$ can be recovered by passing $y(t)$ through a LPF

3) Pulse Train Modulation is the basis for Time-Division Multiplexing
— Assign *time* slots instead of *frequency* slots to different channels, *e.g.* AT&T wireless phones

4) Really only need *samples* $\{x(nT)\}$ when $\omega_c > 2\omega_M$
$\Rightarrow$ Pulse Amplitude Modulation

سیستم‌های مخابراتی (۲)

۳

مدولاسیون فرکانس سینوسی

مدولاسیون فرکانس

FREQUENCY MODULATION (FM)

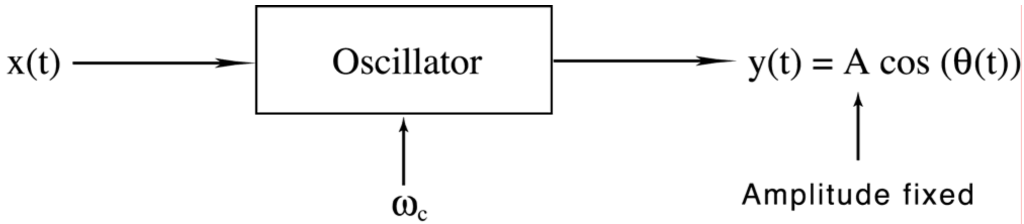
مدولاسیون فرکانس
Frequency Modulation (FM)

$$y_3(t) = \cos\left(\omega_c t + \underbrace{k\int_{-\infty}^{t} x(\tau)d\tau}_{\phi(t)}\right)$$

$$\omega_i(t) = \omega_c + \frac{d}{dt}\phi(t) = \omega_c + kx(t)$$

در FM سیگنال فرکانس لحظه‌ای حامل را مدوله می‌کند.

Sinusoidal *F*requency *M*odulation (FM)

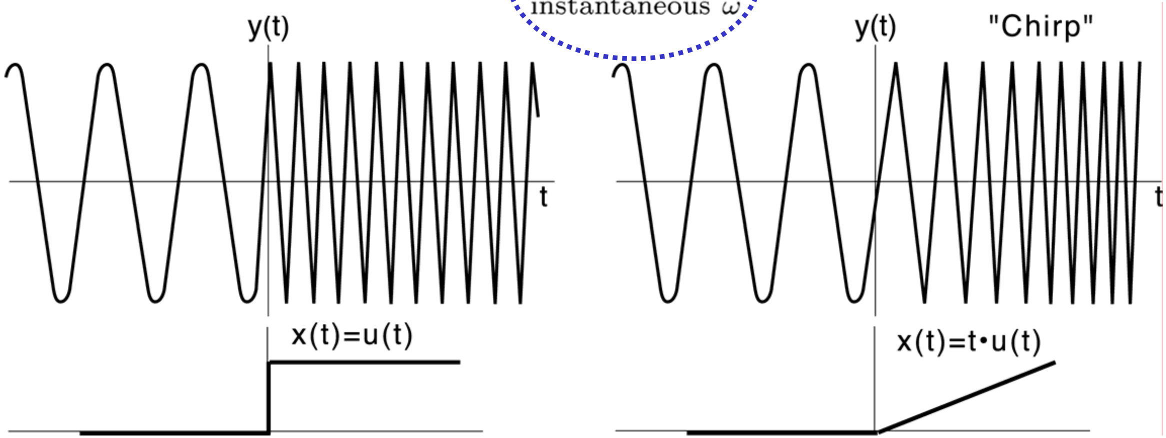


Phase modulation: $\theta(t) = \omega_c t + \theta_0 + k_p x(t)$

Frequency modulation: $\frac{d\theta}{dt} = \underbrace{\omega_c + k_f x(t)}_{\text{instantaneous } \omega}$

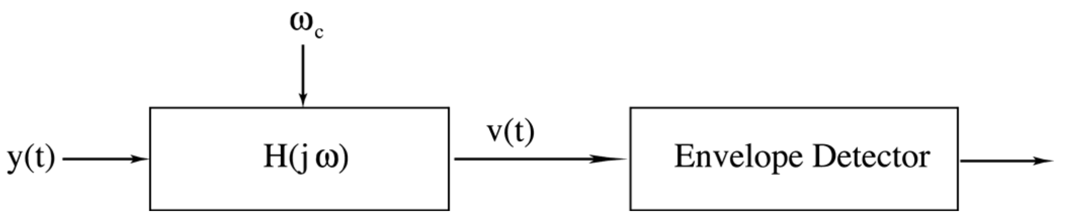
x(*t*) is signal to be transmitted

FM



Sinusoidal FM (continued)

- Transmitted power does not depend on $x(t)$: average power = $A^2/2$
- Bandwidth of $y(t)$ *can* depend on *amplitude* of $x(t)$
- Demodulation
  a) Direct tracking of the phase $\theta(t)$ (by using phase-locked loop)
  b) Use of an LTI system that acts like a differentiator



$H(j\omega)$— Tunable band-limited differentiator, over the bandwidth of $y(t)$

$$H(j\omega) \cong j\omega$$

$$\Downarrow$$

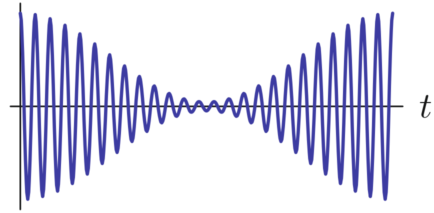
$$v(t) \cong \frac{dy(t)}{dt} = -\underbrace{(\omega_c + k_f x(t))}_{d\theta/dt} A \sin\theta(t)$$

…looks like AM envelope detection

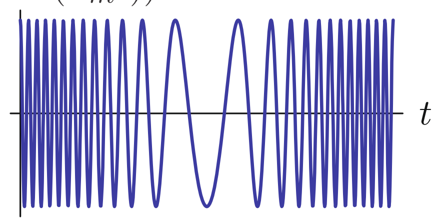
مقایسه‌ی مدولاسیون دامنه با مدولاسیون فرکانس

Compare AM to FM for $x(t) = \cos(\omega_m t)$.

AM: $y_1(t) = (\cos(\omega_m t) + 1.1)\cos(\omega_c t)$



FM: $y_3(t) = \cos(\omega_c t + m\sin(\omega_m t))$



Advantages of FM:

- constant power
- no need to transmit carrier (unless DC important)
- bandwidth?

مقایسه‌ی مدولاسیون دامنه با مدولاسیون فرکانس

Early investigators thought that narrowband FM could have arbitrarily narrow bandwidth, allowing more channels than AM. Wrong!

$$y_3(t) = \cos\left(\omega_c t + k\int_{-\infty}^{t} x(\tau)d\tau\right)$$

$$= \cos(\omega_c t) \times \cos\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) - \sin(\omega_c t) \times \sin\left(k\int_{-\infty}^{t} x(\tau)d\tau\right)$$

If $k \to 0$ then

$$\cos\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) \to 1$$

$$\sin\left(k\int_{-\infty}^{t} x(\tau)d\tau\right) \to k\int_{-\infty}^{t} x(\tau)d\tau$$

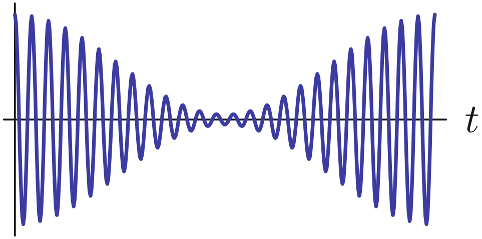
$$y_3(t) \approx \cos(\omega_c t) - \sin(\omega_c t) \times \left(k\int_{-\infty}^{t} x(\tau)d\tau\right)$$

Bandwidth of narrowband FM is the same as that of AM!
(integration does not change bandwidth)

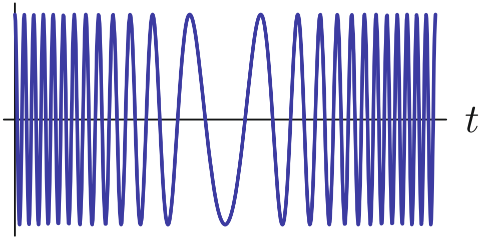
مقایسه‌ی مدولاسیون دامنه با مدولاسیون فرکانس

Wideband FM is useful because it is robust to noise.

AM: $y_1(t) = (\cos(\omega_m t) + 1.1)\cos(\omega_c t)$



FM: $y_3(t) = \cos(\omega_c t + m\sin(\omega_m t))$



FM generates a very redundant signal, which is resilient to additive noise.

سیستم‌های مخابراتی (۲)

۴

مدولاسیون فاز

مدولاسیون فاز/فرکانس

Phase/Frequency Modulation

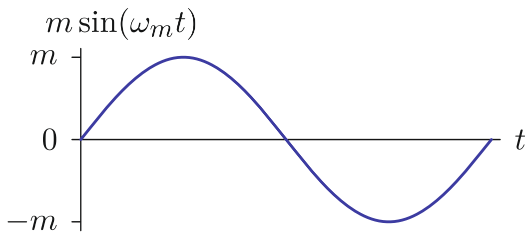
Find the Fourier transform of a PM signal.

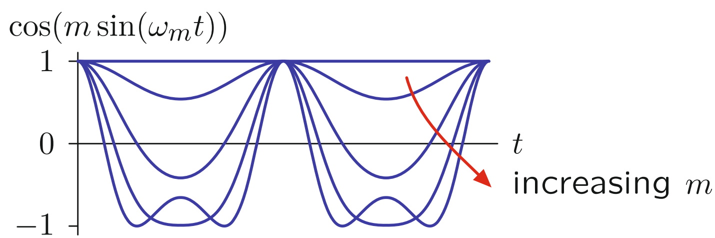
$$x(t) = \sin(\omega_m t)$$

$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m \sin(\omega_m t))$$

$$= \cos(\omega_c t)\cos(m \sin(\omega_m t)) - \sin(\omega_c t)\sin(m \sin(\omega_m t))$$

$x(t)$ is periodic in $T = \frac{2\pi}{\omega_m}$, therefore $\cos(m \sin(\omega_m t))$ is periodic in T.





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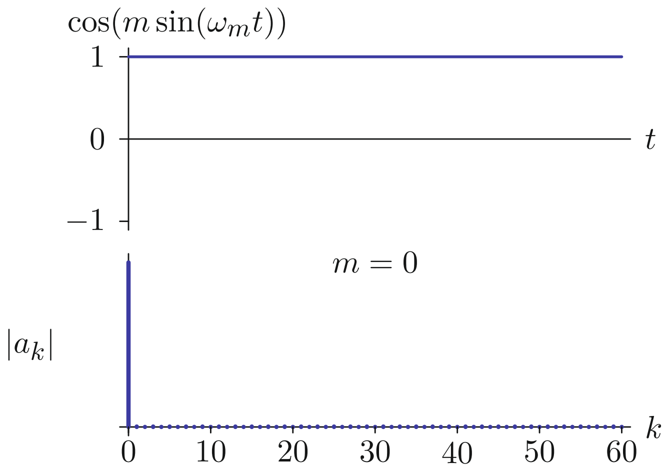

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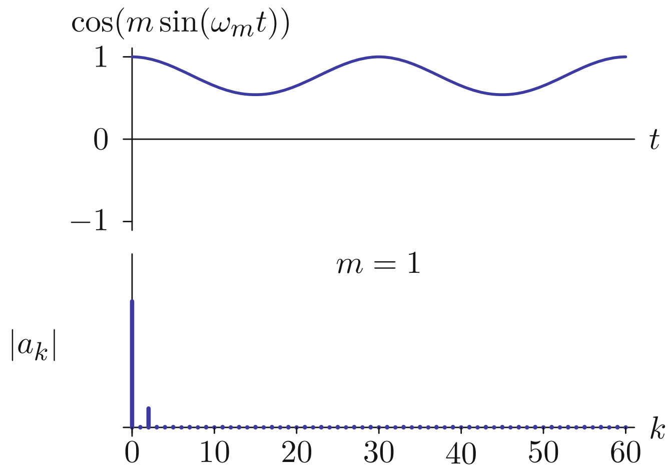

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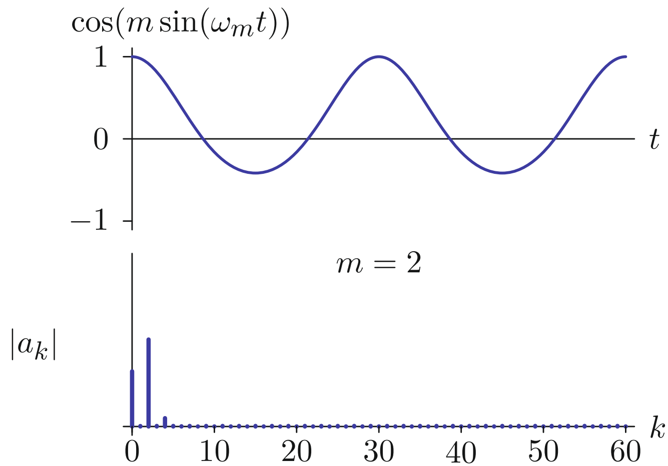

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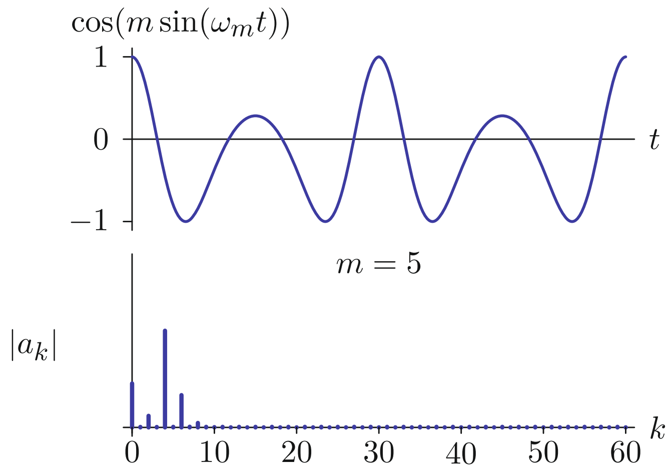

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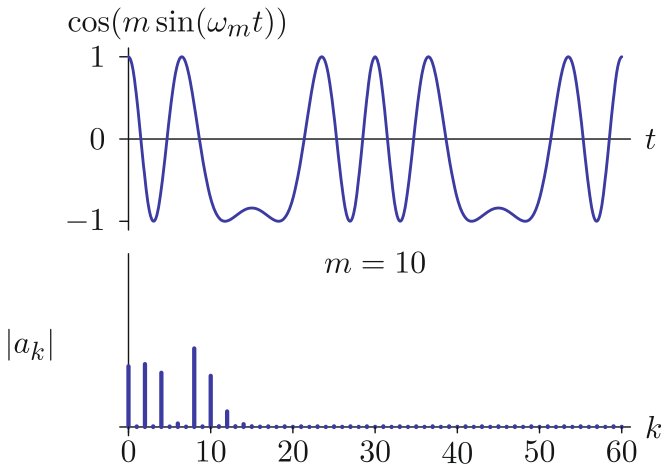

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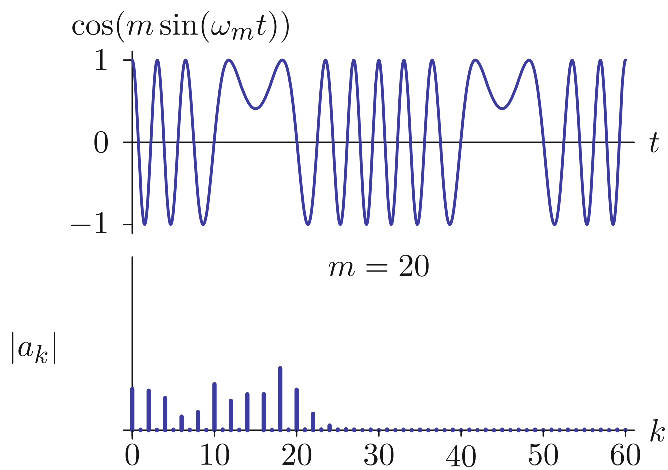

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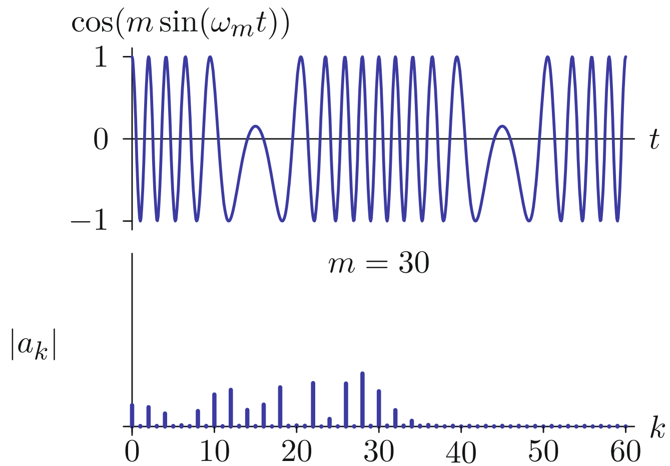

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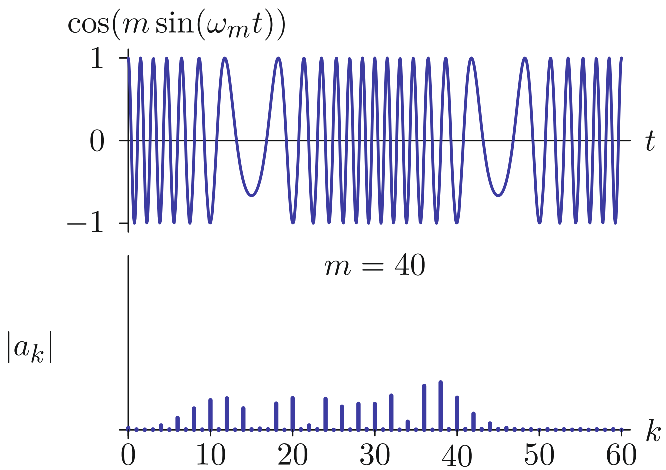

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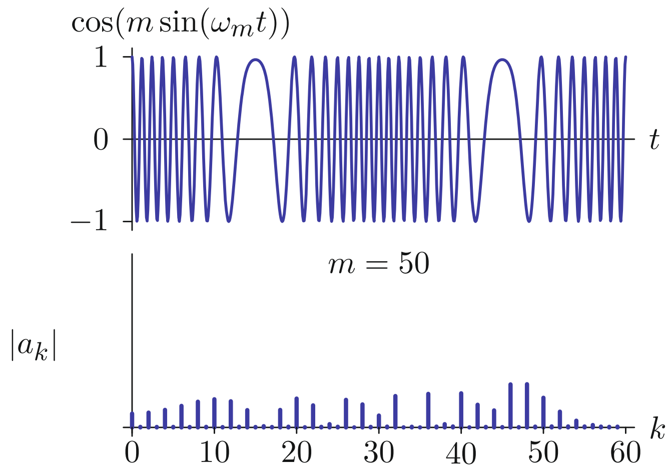

مدولاسیون فاز/فرکانس

PHASE/FREQUENCY MODULATION

Fourier transform of first part.

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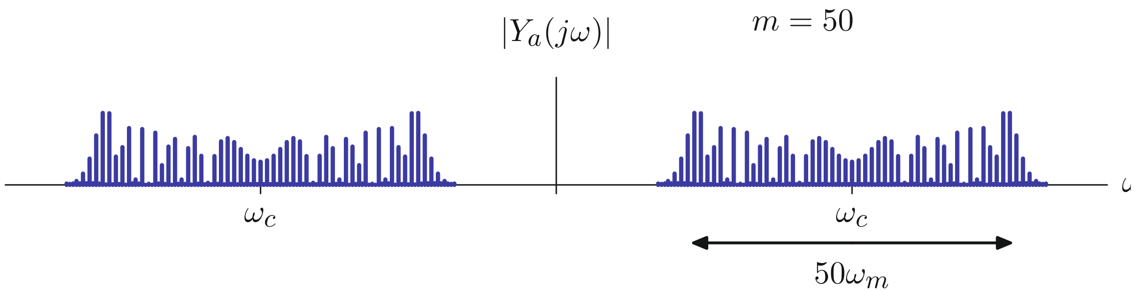

مدولاسیون فاز/فرکانس

Phase/Frequency Modulation

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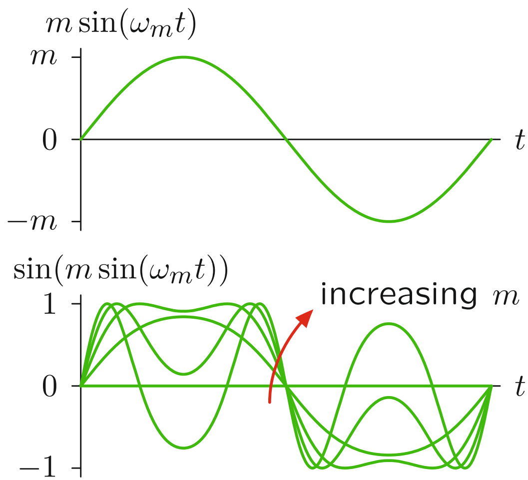

مدولاسیون فاز/فرکانس

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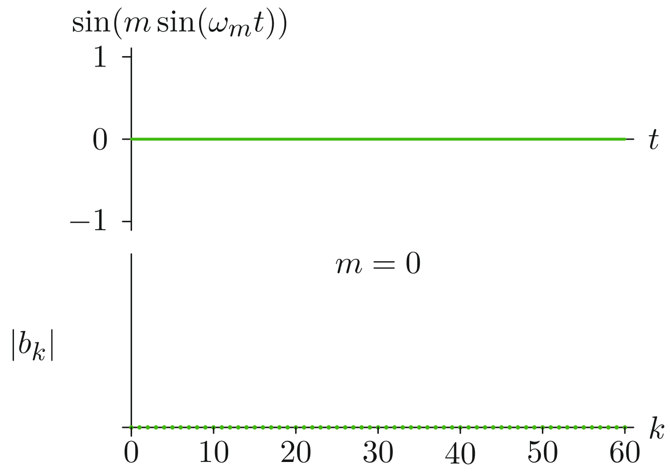

مدولاسیون فاز/فرکانس

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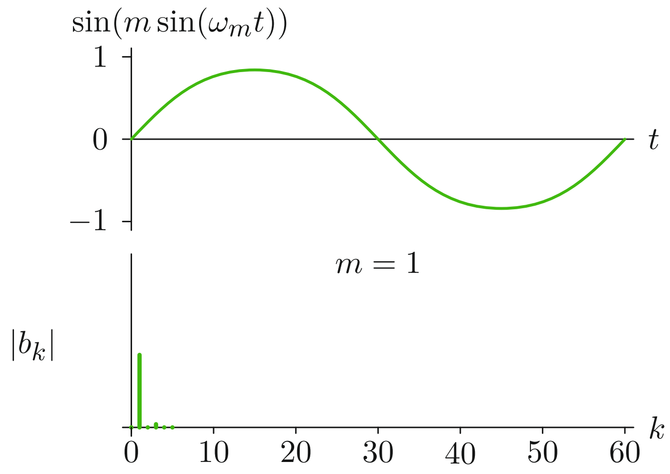

مدولاسیون فاز/فرکانس

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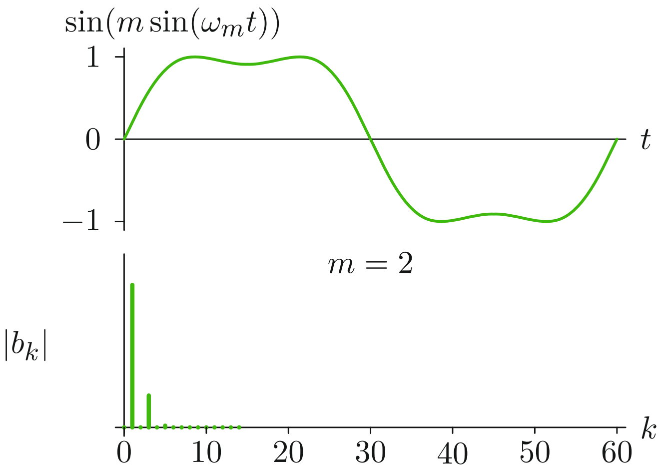

مدولاسیون فاز/فرکانس

Phase/Frequency Modulation

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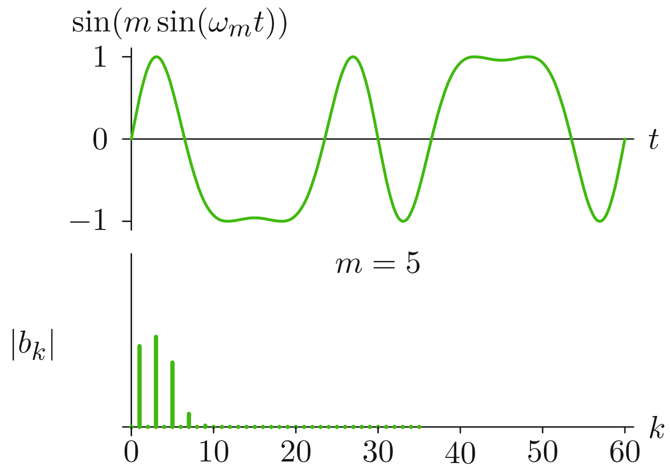

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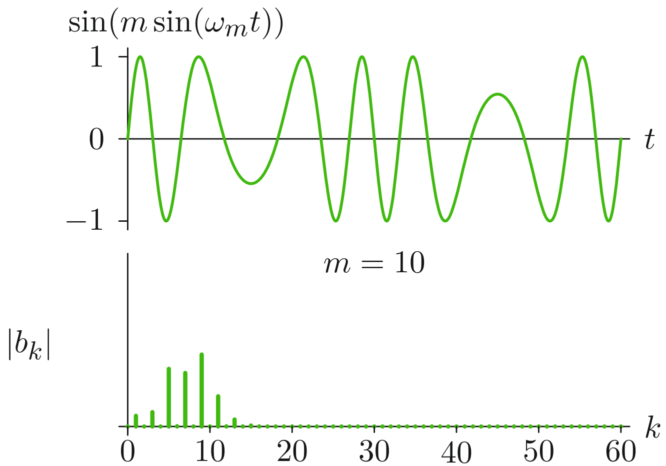

مدولاسیون فاز/فرکانس

PHASE/FREQUENCY MODULATION

Find the Fourier transform of a PM signal.

$$x(t) = \sin(\omega_m t)$$

$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m \sin(\omega_m t))$$

$$= \cos(\omega_c t)\cos(m\sin(\omega_m t)) - \sin(\omega_c t)\sin(m\sin(\omega_m t))$$

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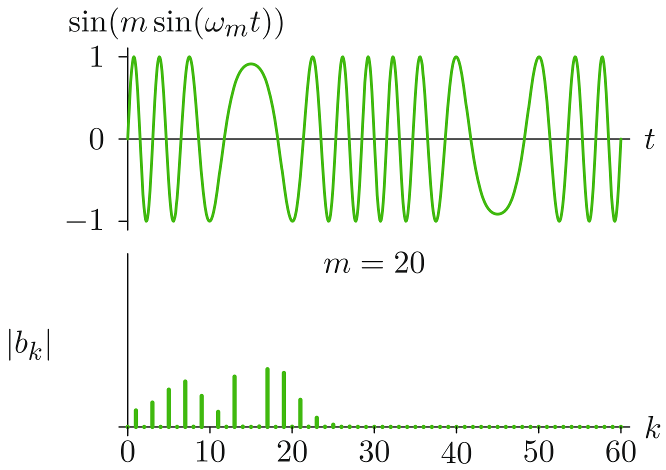

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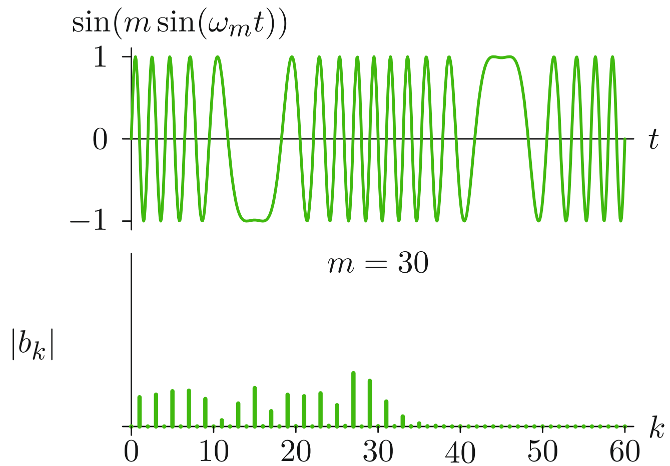

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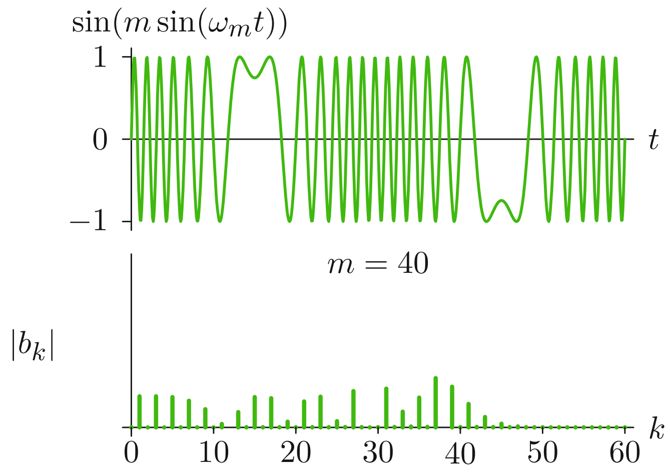

مدولاسیون فاز/فرکانس

Phase/Frequency Modulation

Find the Fourier transform of a PM signal.

$$x(t) = \sin(\omega_m t)$$
$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m \sin(\omega_m t))$$
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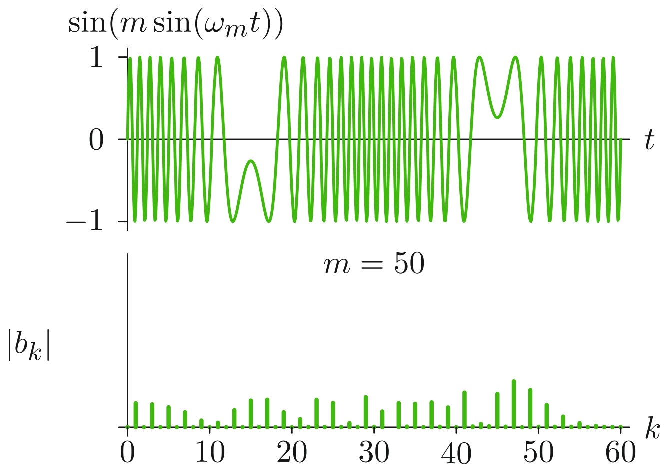
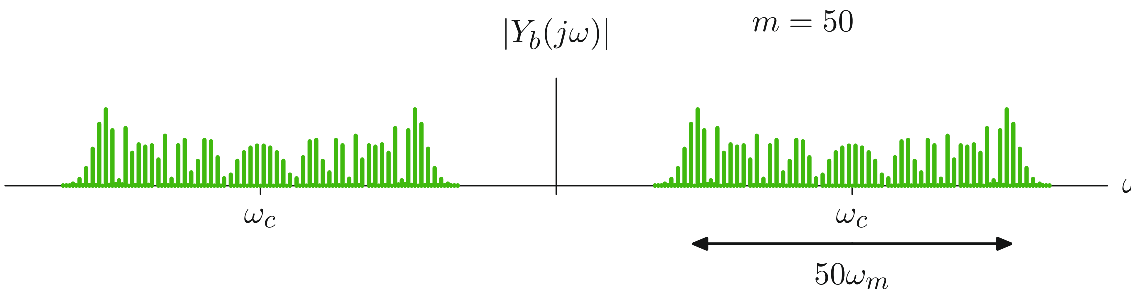


مدولاسیون فاز/فرکانس

Phase/Frequency Modulation

Fourier transform of second part.

$$
\begin{aligned}
x(t) &= \sin(\omega_m t) \\
y(t) &= \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \\
&= \cos(\omega_c t)\cos(m\sin(\omega_m t)) - \underbrace{\sin(\omega_c t)\sin(m\sin(\omega_m t))}_{y_b(t)}
\end{aligned}
$$



مدولاسیون فاز/فرکانس

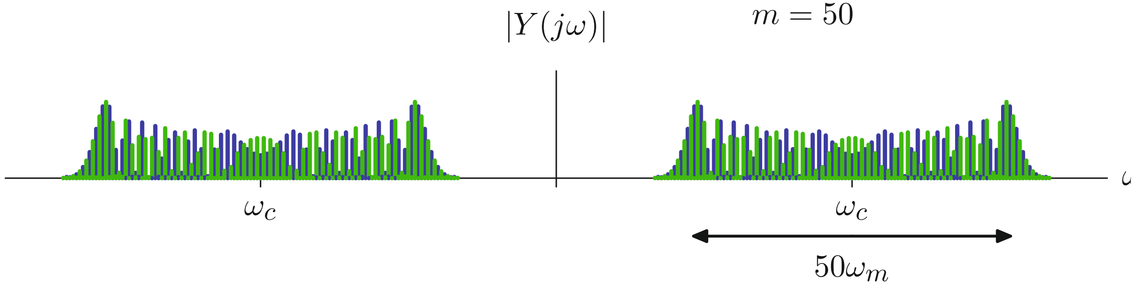
Phase/Frequency Modulation

Fourier transform.

$$x(t) = \sin(\omega_m t)$$

$$y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m\sin(\omega_m t))$$

$$= \underbrace{\cos(\omega_c t)\cos(m\sin(\omega_m t))}_{y_a(t)} - \underbrace{\sin(\omega_c t)\sin(m\sin(\omega_m t))}_{y_b(t)}$$



سیستم‌های مخابراتی (۲)

۵

مدولاسیون دامنه‌ی سینوسی گسسته-زمان

DT Sinusoidal AM

Multiplication $\leftrightarrow$ Periodic convolution

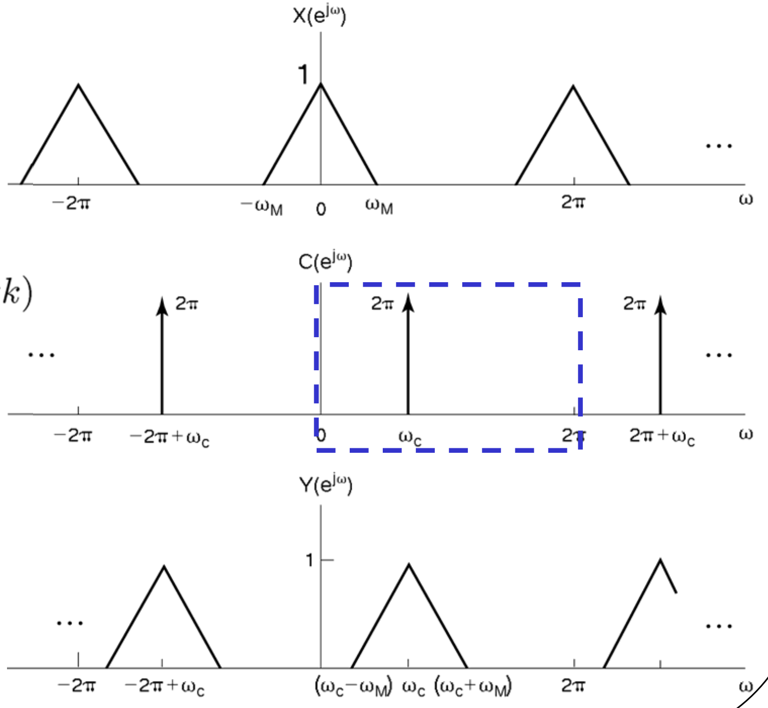
$$y[n] = x[n] \cdot c[n] \leftrightarrow Y(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) C(e^{j(\omega-\theta)}) d\theta$$

Example #1:

$$c[n] = e^{j\omega_c n}$$

$$\Rightarrow C(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_c + 2\pi k)$$

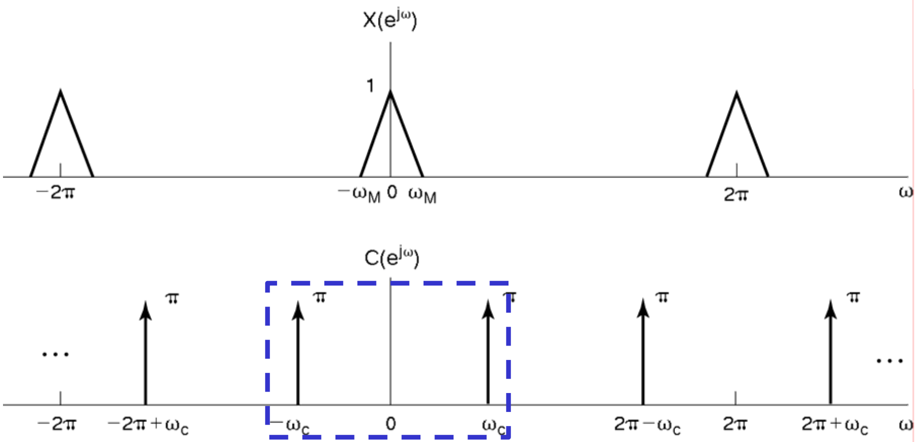
$$Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \otimes C(e^{j\omega})$$



Example #2: Sinusoidal AM

$$c[n] = \cos \omega_c n$$

$$C(e^{j\omega}) = \pi \left\{ \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_c + 2\pi k) + \delta(\omega + \omega_c + 2\pi k) \right\}$$



Drawn assuming:

$\omega_c - \omega_M > 0$ and
$2\pi - \omega_c - \omega_M > \omega_c + \omega_M$

i.e.,
$\omega_M < \omega_c < \pi - \omega_M$
$\Rightarrow \omega_M < \pi/2$

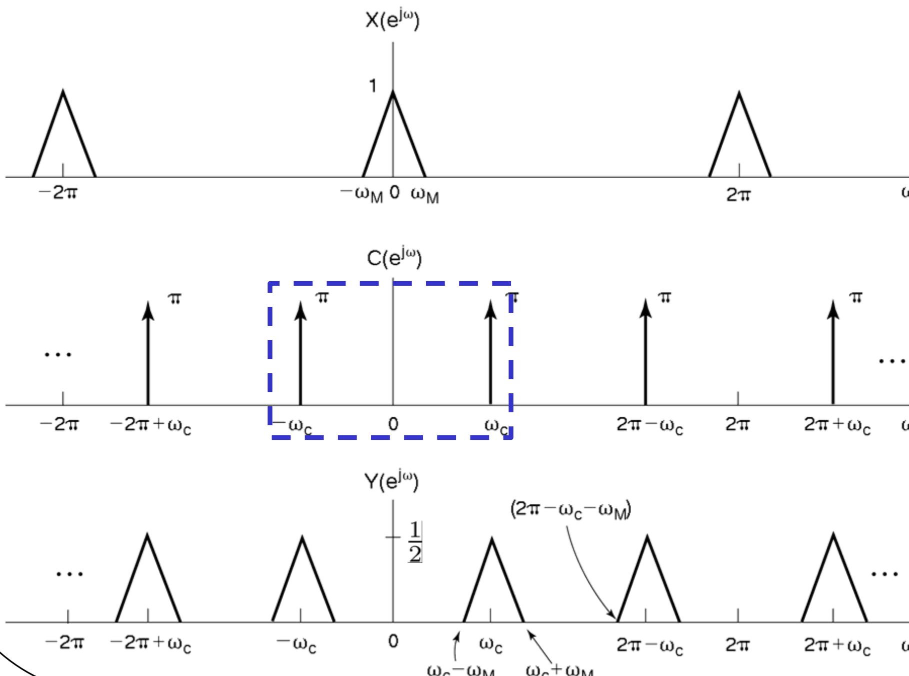
$$Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \otimes C(e^{j\omega})$$

No overlap of shifted spectra

Example #2: Sinusoidal AM

$$c[n] = \cos \omega_c n$$

$$C(e^{j\omega}) = \pi \left\{ \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_c + 2\pi k) + \delta(\omega + \omega_c + 2\pi k) \right\}$$



Drawn assuming:

$$\omega_c - \omega_M > 0 \quad \text{and}$$
$$2\pi - \omega_c - \omega_M > \omega_c + \omega_M$$

i.e.,

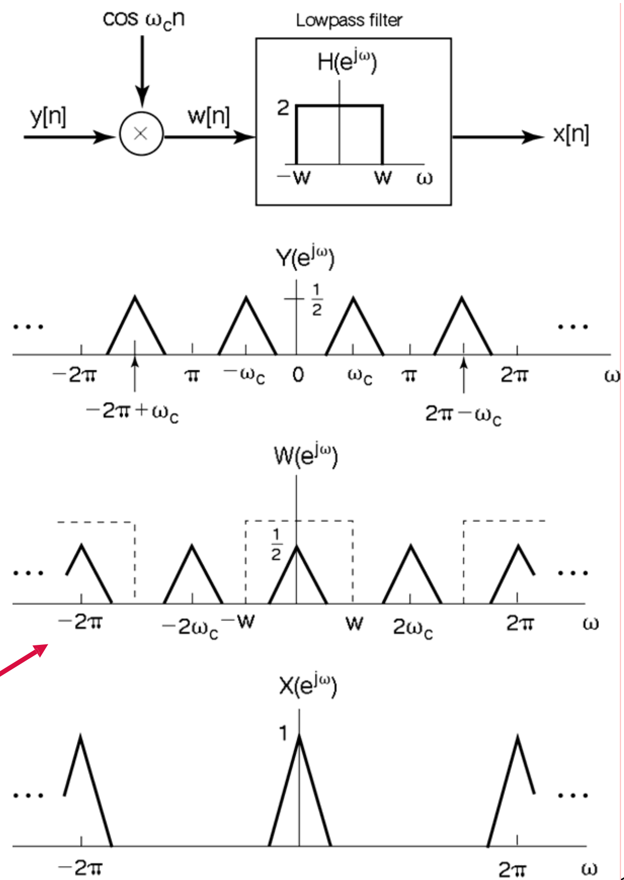
$$\omega_M < \omega_c < \pi - \omega_M$$
$$\Rightarrow \omega_M < \pi/2$$

$$Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \otimes C(e^{j\omega})$$

No overlap of shifted spectra

Example #2 (continued): Demodulation

Possible as long as there is no overlap of shifted replicas of $X(e^{j\omega})$



i.e., $\omega_c - \omega_M > 0$

and $\omega_c + \omega_M < 2\pi - \omega_c - \omega_M$

$\Rightarrow \omega_M < \omega_c < \pi - \omega_M$

$$W(e^{j\omega}) = \frac{1}{2\pi} Y(e^{j\omega}) \otimes C(e^{j\omega})$$

Misleading drawing – shown for a very special case of $\omega_c = \pi/2$

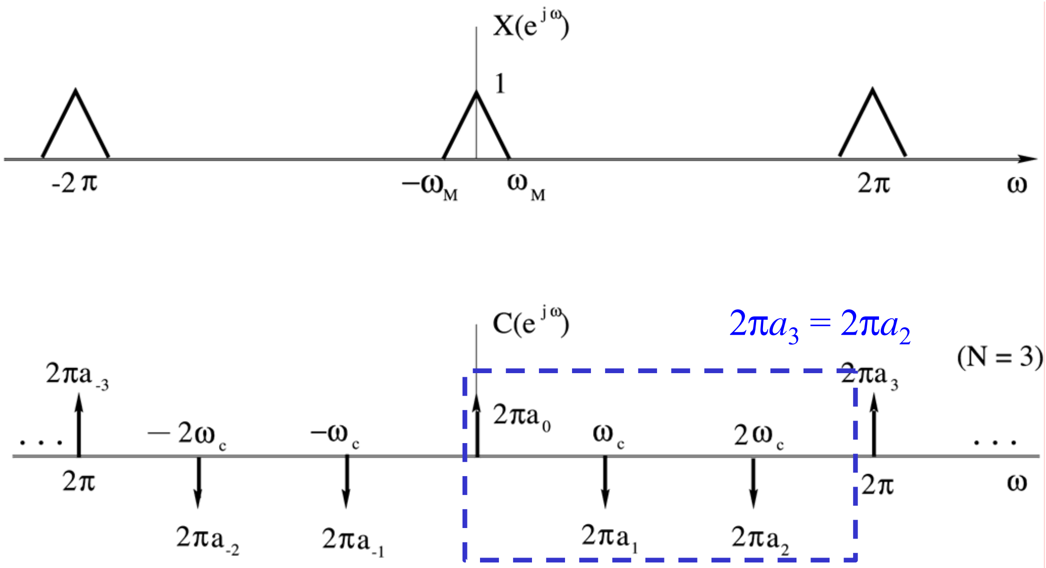
Example #3: An arbitrary periodic DT carrier

$$c[n] = \sum_{k=<N>} a_k e^{j2\pi kn/N} = c[n+N], \quad \omega_c = \frac{2\pi}{N}$$

$$C(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$$

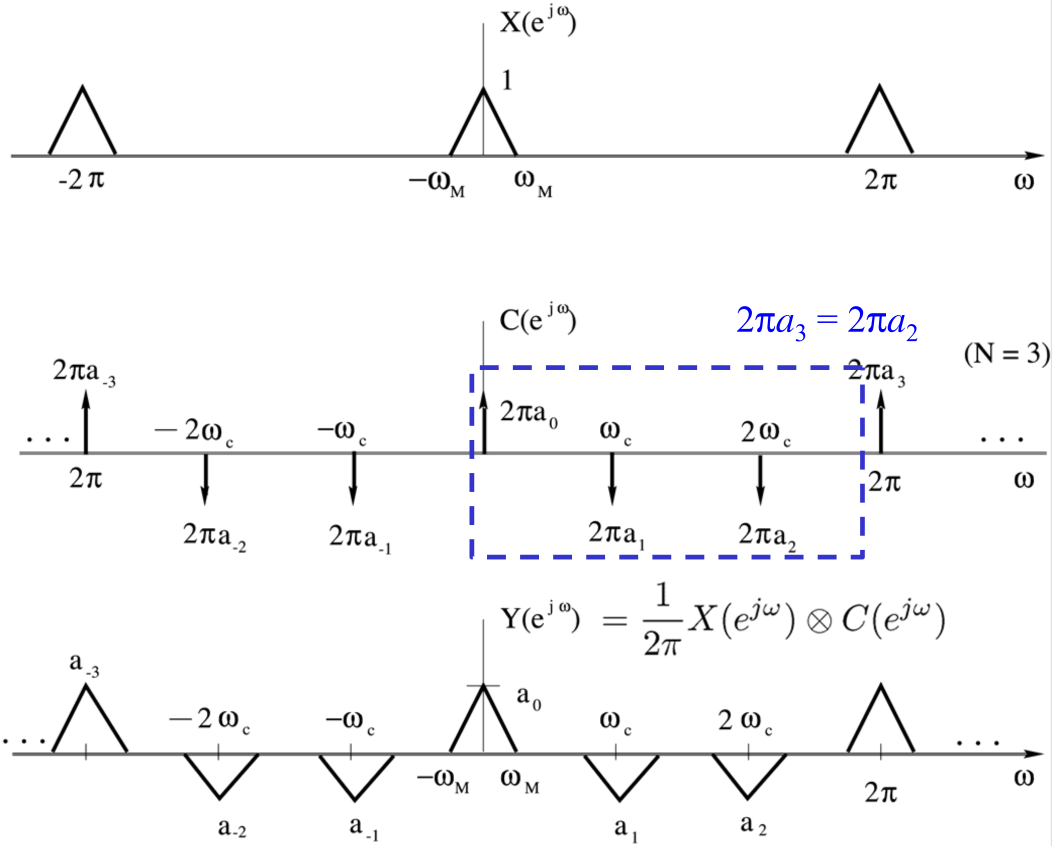
$$\begin{aligned} Y(e^{j\omega}) &= \frac{1}{2\pi} X(e^{j\omega}) \otimes C(e^{j\omega}) - \text{ periodic convolution} \\ &= \frac{1}{2\pi} X(e^{j\omega}) * \sum_{k=0}^{N-1} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right) - \text{regular convolution} \\ &= \sum_{k=0}^{N-1} a_k X(e^{j(\omega - 2\pi k/N)}) \end{aligned}$$

Example #3 (continued):



No overlap when: $\omega_c > 2\omega_M$ (Nyquist rate) $\Rightarrow$ $\omega_M < \pi/N$

Example #3 (continued):



No overlap when: $\omega_c > 2\omega_M$ (Nyquist rate) $\Rightarrow$ $\omega_M < \pi/N$

سیگنال‌ها و سیستم‌ها

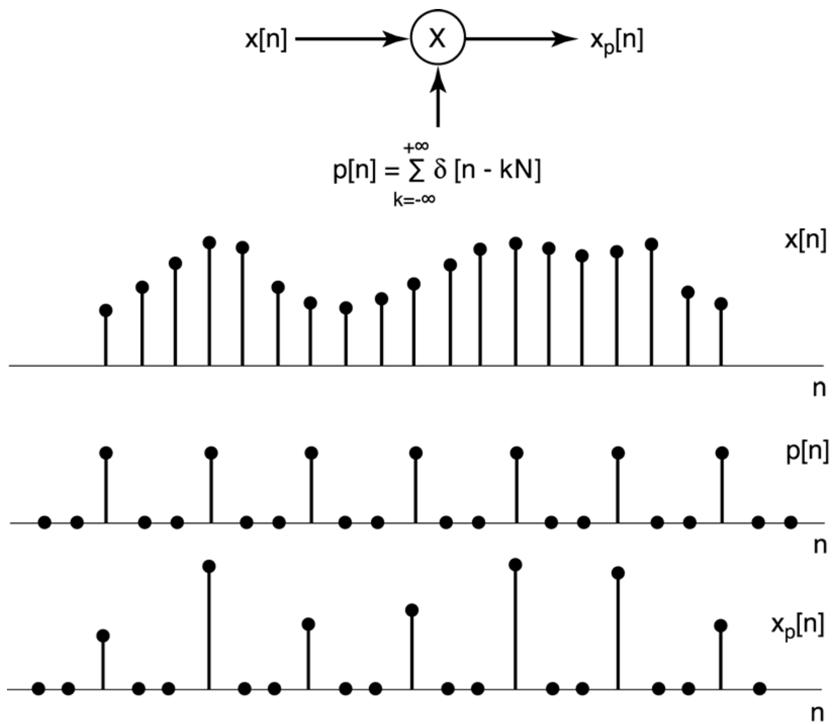
سیستم‌های مخابراتی (۲)

۶

نمونه‌برداری گسسته-زمان، دسیماسیون و درون‌یابی

DT Sampling

Motivation: Reducing the number of data points to be stored or transmitted, *e.g.* in CD music recording.



DT Sampling (continued)

$$p[n] \quad = \quad \sum_{k=-\infty}^{\infty} \delta[n-kN] \longleftrightarrow P(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s), \quad \omega_s = \frac{2\pi}{N}$$

$$\text{Note: } x_p[n] = x[n] \cdot p[n] = \sum_{k=-\infty}^{\infty} x[kN]\delta[n-kN]$$

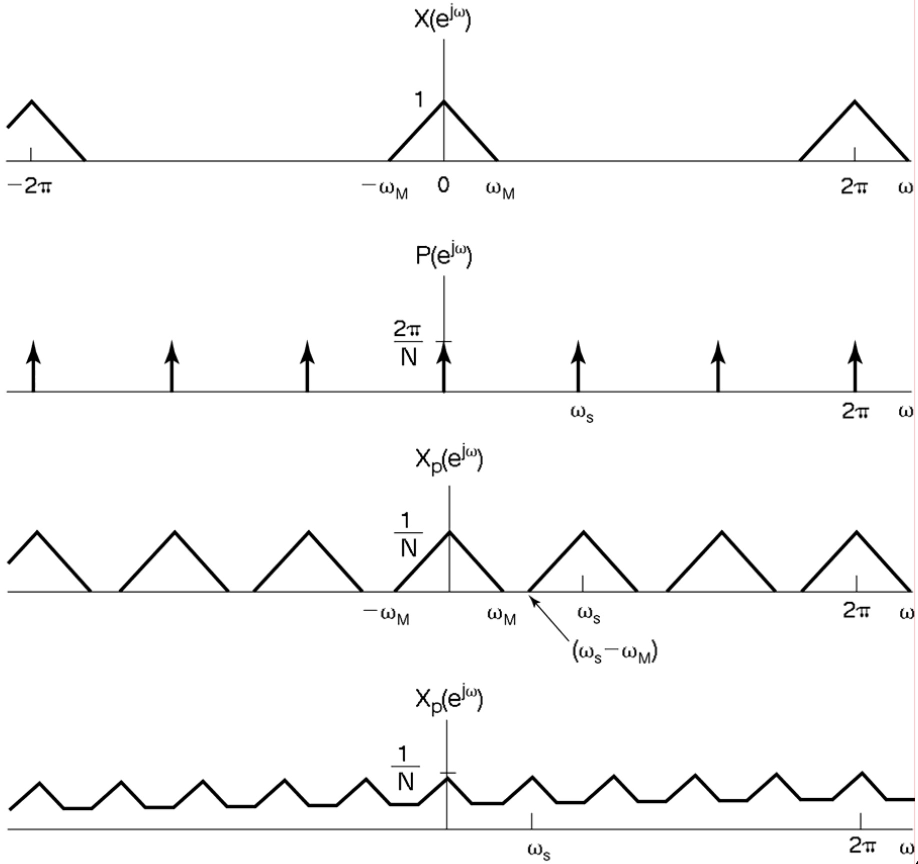
$$x_p[n] \quad = \quad \begin{cases} x[n], & \text{if } n \text{ is integer multiple of } N \\ 0, & \text{otherwise} \end{cases} \Rightarrow \text{ Pick one out of } N$$

$$X_p(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \otimes P(e^{j\omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\omega - k\omega_s)})$$

$$- \text{ periodic with period } \omega_s = \frac{2\pi}{N}$$

DT Sampling Theorem

We can reconstruct $x[n]$ if $\omega_s = 2\pi/N > 2\omega_M$

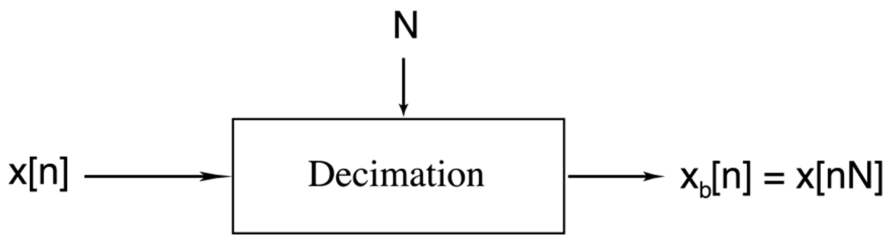


Drawn assuming
$\omega_s > 2\omega_M$
Nyquist rate is met
$\Rightarrow \quad \omega_M < \pi/N$

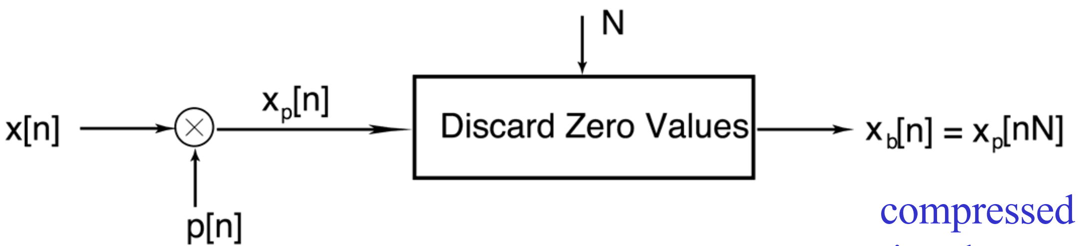
Drawn assuming
$\omega_s < 2\omega_M$
Aliasing!

Decimation — *Downsampling*

$x_p[n]$ has $(n - 1)$ zero values between nonzero values:
Why keep them around?

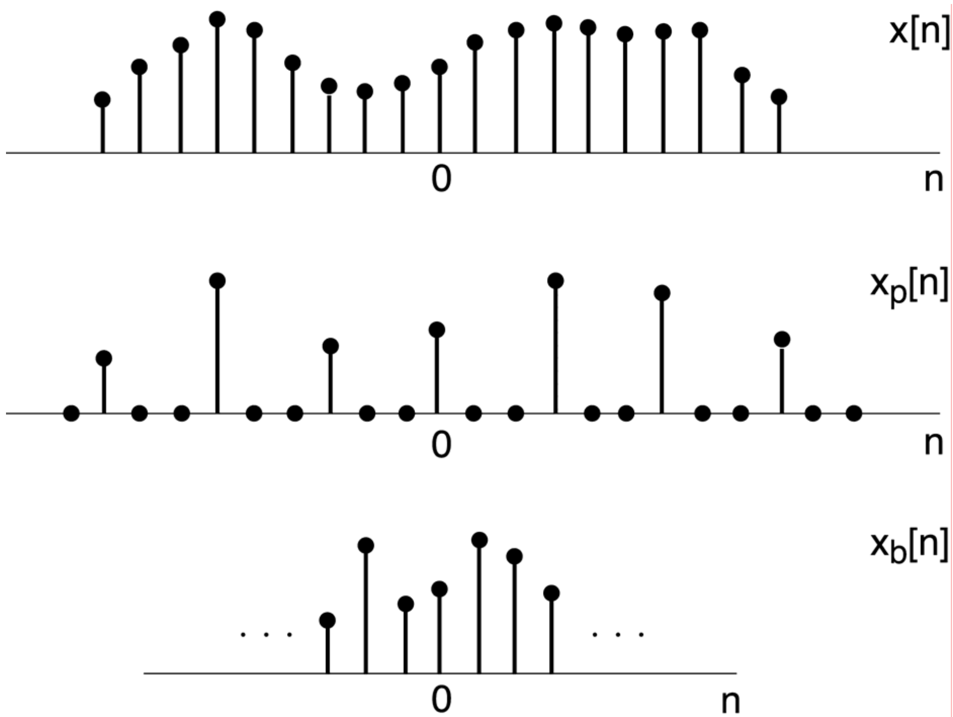


Useful to think of this as sampling followed by discarding the zero values



compressed in
time by N

Illustration of Decimation in the Time-Domain (for $N = 3$)



Decimation in the Frequency Domain

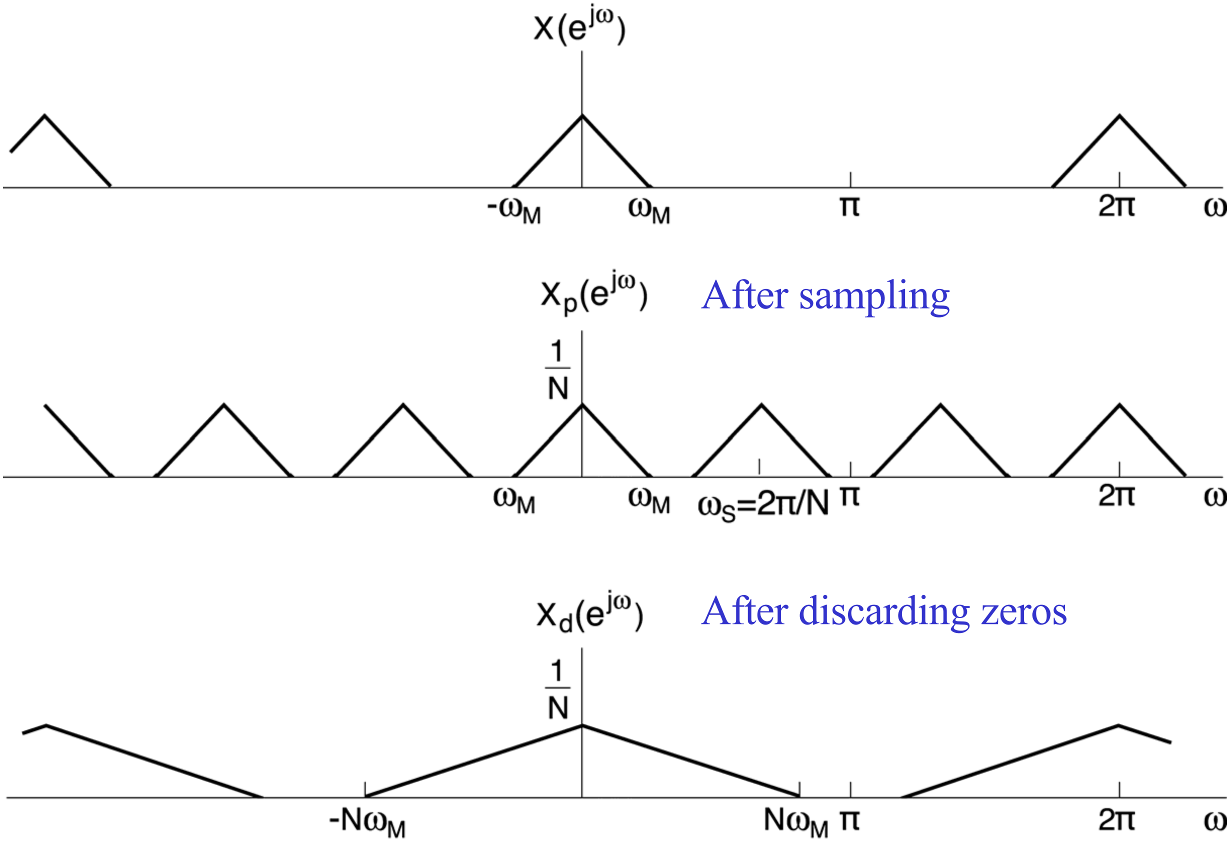
$$
\begin{aligned}
X_b(e^{j\omega}) &= \sum_{k=-\infty}^{\infty} x_b[k]e^{-j\omega k} \quad (x_b[k] = x_p[kN]) \\
&= \sum_{k=-\infty}^{\infty} x_p[kN]e^{-j\omega k} \quad \text{Let } n = kN \text{ or } k = n/N \\
&= \sum_{\substack{n=\text{ an integer} \\ \text{multiple of } N}}^{\infty} x_p[n]e^{-j\omega(n/N)} \\
&= \sum_{n=-\infty}^{\infty} x_p[n]e^{-j(\omega/N)n} \quad (\text{Since } x_p[n \neq kN] = 0) \\
&= X_p(e^{j(\omega/N)})
\end{aligned}
$$

Squeeze in time
Expand in frequency

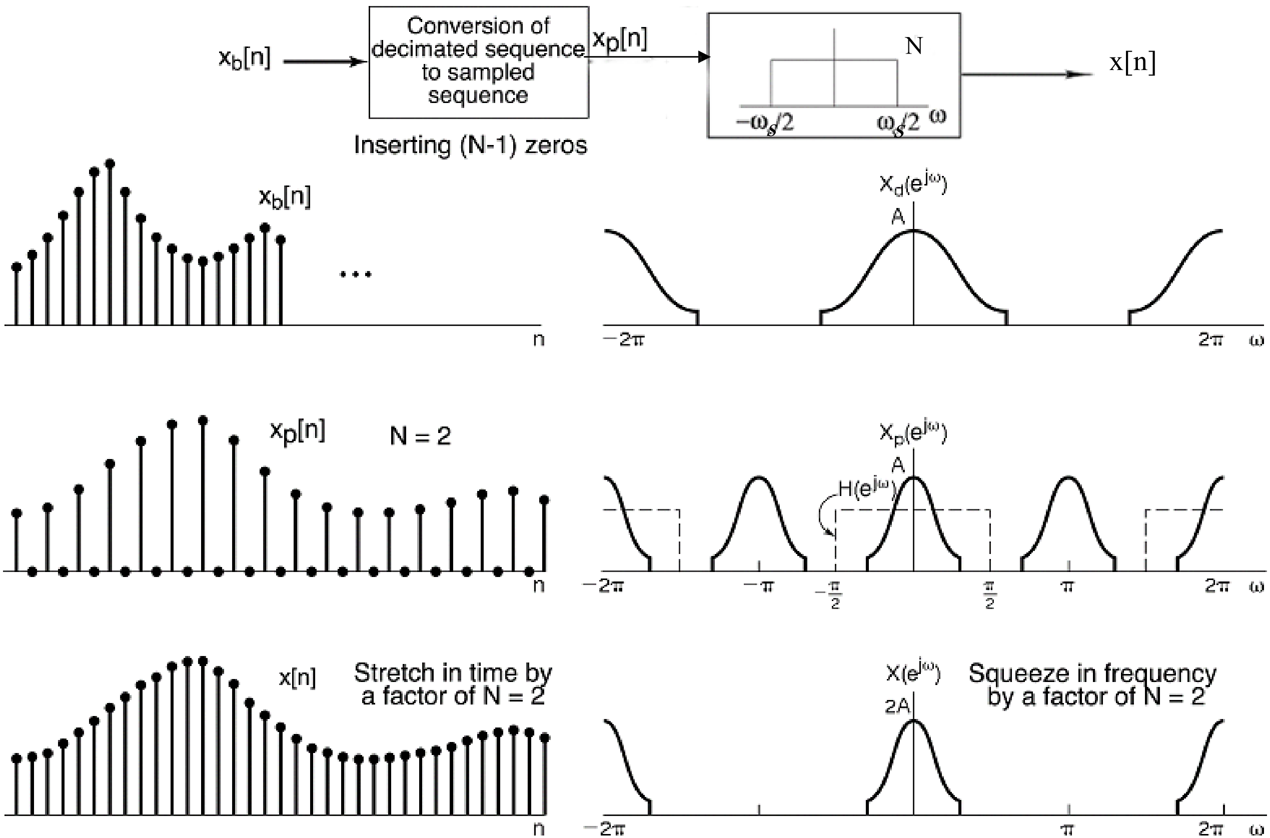
- still periodic with period 2π

since $X_p(e^{j\omega})$ is periodic with period $2\pi/N$

Illustration of Decimation in the Frequency Domain



The Reverse Operation: Upsampling (*e.g.* CD playback)

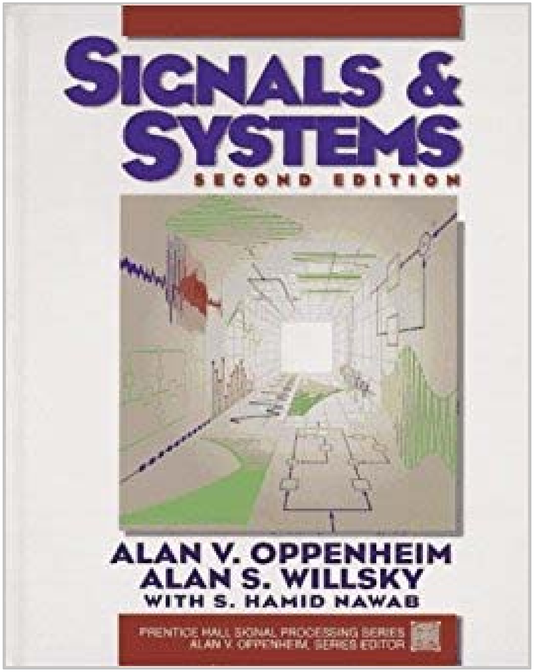


سیستم‌های مخابراتی (۲)

۷

منابع

منبع اصلی



A.V. Oppenheim, A.S. Willsky, S.H. Nawab,
Signals and Systems,
Second Edition, Prentice Hall, 1997.

Chapter 8