بسم الله الرحمن الرحیم

دانشگاه تهران

سیگنال‌ها و سیستم‌ها

درس ۱۱

بازنمایی سری فوریه سیگنال‌های متناوب (۳)

Fourier Series Representation of Periodic Signals (3)

کاظم فولادی قلعه
دانشکده مهندسی، پردیس فارابی
دانشگاه تهران

http://courses.fouladi.ir/sigsys

طرح درس

COURSE OUTLINE

سری فوریه و سیستم‌های خطی تغییرناپذیر با زمان
Fourier Series and LTI Systems
پاسخ فرکانسی و فیلترینگ
Frequency Response and Filtering
مثال‌ها و نمونه‌های نمایشی
Examples and Demos

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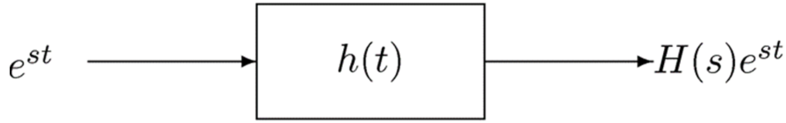
سری فوریه و سیستم‌های خطی تغییرناپذیر با زمان

The Eigenfunction Property of Complex Exponentials

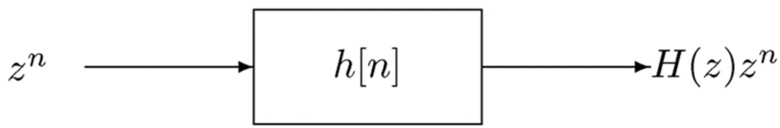
CT:



CT
"System Function"

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

DT:

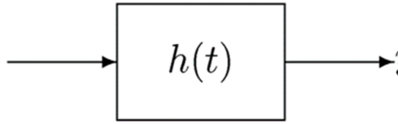


DT
"System Function"

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

Fourier Series: Periodic Signals and LTI Systems

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$



$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0) a_k e^{jk\omega_0 t}$$

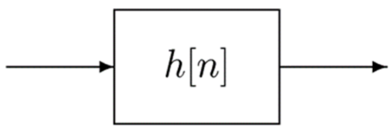
$$a_k \longrightarrow \underbrace{H(jk\omega_0)}_{\text{“gain”}} a_k$$

So $|a_k| \longrightarrow |H(jk\omega_0)||a_k|$
or powers of signals get
modified through filter/system

$$H(jk\omega_0) = |H(jk\omega_0)| e^{j\angle H(jk\omega_0)}$$

includes both amplitude & phase

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$$



$$y[n] = \sum_{k=<N>} H(e^{jk\omega_0}) a_k e^{jk\omega_0 n}$$

$$a_k \longrightarrow \underbrace{H(e^{jk\omega_0})}_{\text{“gain”}} a_k$$

$$H(e^{jk\omega_0}) = |H(e^{jk\omega_0})| e^{j\angle H(e^{jk\omega_0})}$$

includes both amplitude & phase

بازنمایی سری فوریه سیگنال‌های متناوب (۳)

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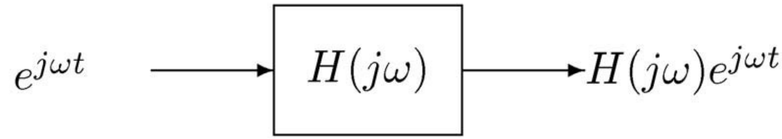
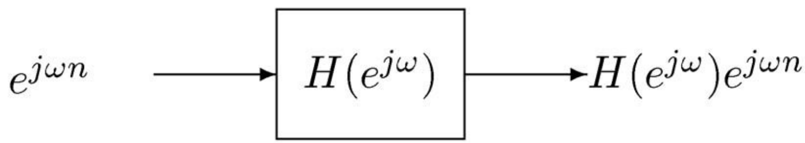
پاسخ فرکانسی و فیلترینگ

The Frequency Response of an LTI System



$$(s = j\omega)$$

CT Frequency response: $$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$



$$(z = e^{j\omega})$$

DT Frequency response: $$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

Frequency Shaping and Filtering

- By choice of $H(j\omega)$ (or $H(e^{j\omega})$) as a function of ω, we can *shape* the frequency composition of the output
  - Preferential amplification
  - Selective filtering of some frequencies

بازنمایی سری فوریه سیگنال‌های متناوب (۳)

۳

مثال‌ها و نمونه‌های نمایشی

فیلترینگ

FILTERING

سیستم‌های LTI:

ـ نمی‌توانند فرکانس جدیدی ایجاد کنند.

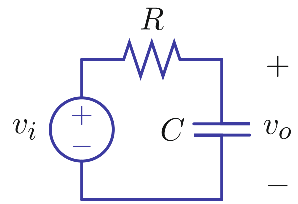
ـ می‌توانند مؤلفه‌های فرکانسی موجود را تغییر اندازه یا شیفت فاز بدهند.

فیلترینگ

مثال

FILTERING

Example: Low-Pass Filtering with an RC circuit

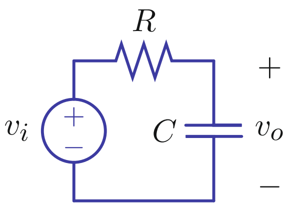


فیلترینگ

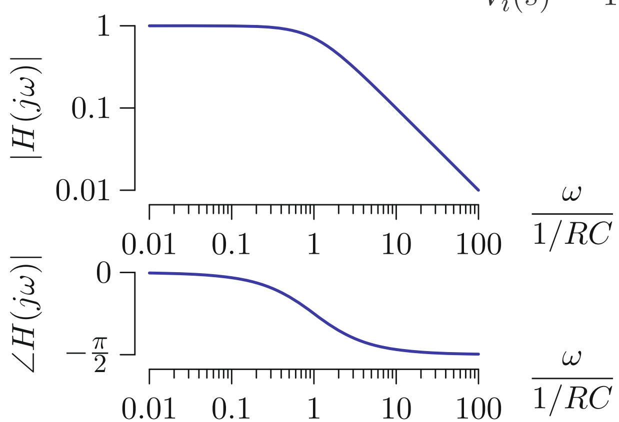
مثال

FILTERING

Calculate the frequency response of an RC circuit.



$$\begin{aligned}
\text{KVL:} \quad & v_i(t) &&= Ri(t) + v_o(t) \\
\text{C:} \quad & i(t) &&= C\dot{v}_o(t) \\
\text{Solving:} \quad & v_i(t) &&= RC\dot{v}_o(t) + v_o(t) \\
& V_i(s) &&= (1 + sRC)V_o(s) \\
& H(s) &&= \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC}
\end{aligned}$$

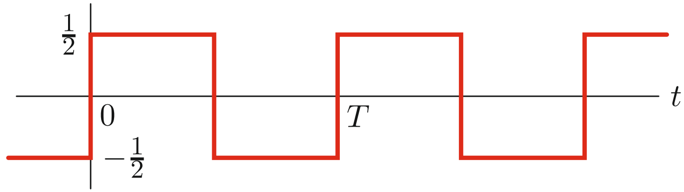


فیلترینگ

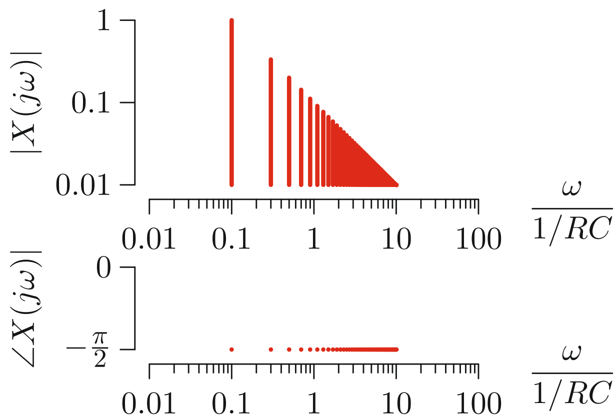
مثال

FILTERING

Let the input be a square wave.



$$x(t) = \sum_{k \text{ odd}} \frac{1}{j\pi k} e^{j\omega_0 kt} \quad ; \qquad \omega_0 = \frac{2\pi}{T}$$

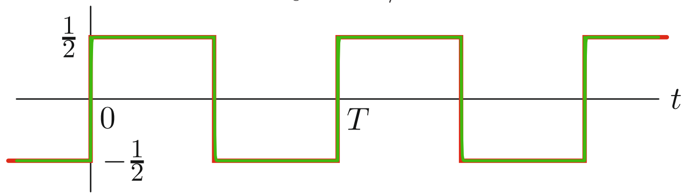


فیلترینگ

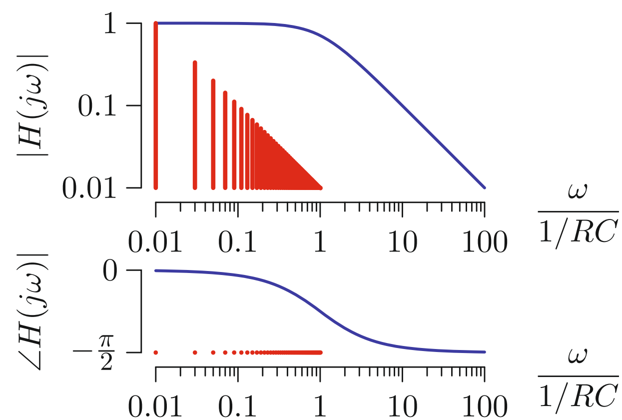
مثال

FILTERING

Low frequency square wave: $\omega_0 << 1/RC$.



$$x(t) = \sum_{k \text{ odd}} \frac{1}{j\pi k} e^{j\omega_0 k t} \; ; \qquad \omega_0 = \frac{2\pi}{T}$$

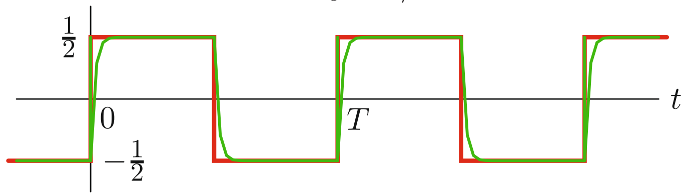


فیلترینگ

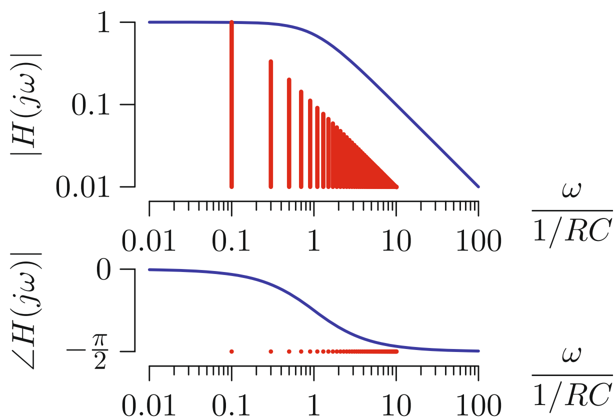
مثال

FILTERING

Higher frequency square wave: $\omega_0 < 1/RC$.



$$x(t) = \sum_{k \text{ odd}} \frac{1}{j\pi k} e^{j\omega_0 k t} \; ; \qquad \omega_0 = \frac{2\pi}{T}$$

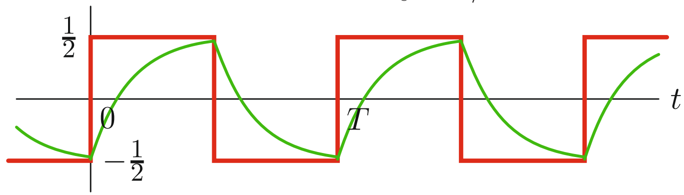


فیلترینگ

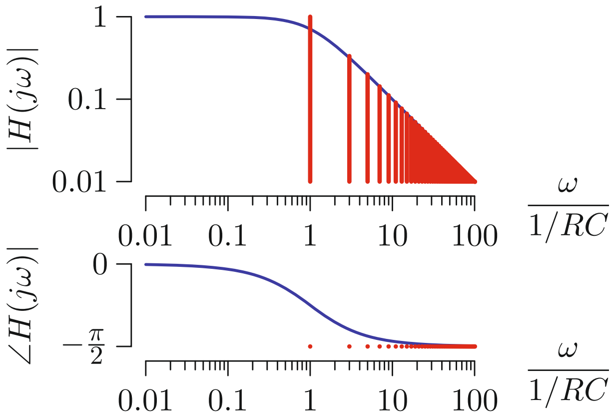
مثال

FILTERING

Still higher frequency square wave: $\omega_0 = 1/RC$.



$$x(t) = \sum_{k \text{ odd}} \frac{1}{j\pi k} e^{j\omega_0 kt} \; ; \quad \omega_0 = \frac{2\pi}{T}$$

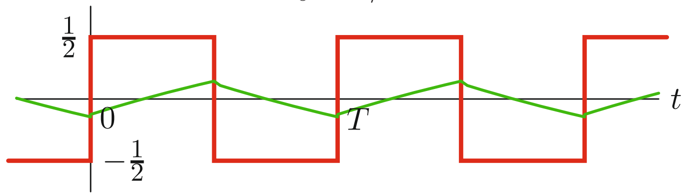


فیلترینگ

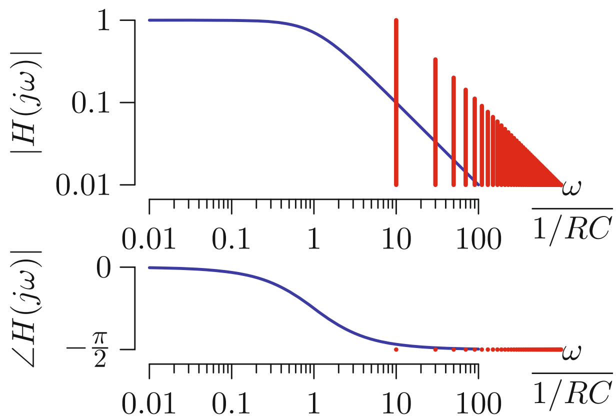
مثال

Filtering

High frequency square wave: $\omega_0 > 1/RC$.

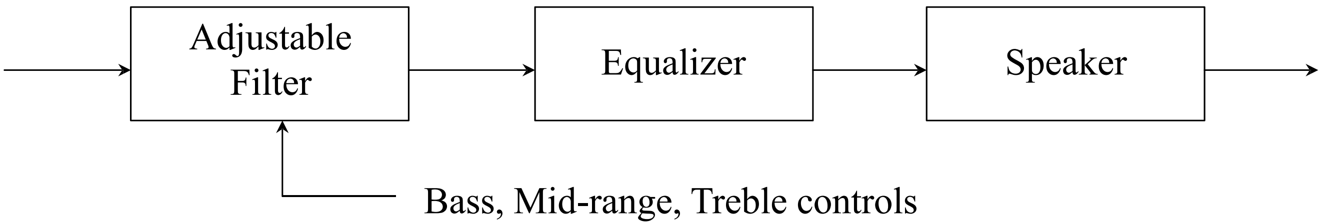


$$x(t) = \sum_{k \text{ odd}} \frac{1}{j\pi k} e^{j\omega_0 kt} \; ; \quad \omega_0 = \frac{2\pi}{T}$$



Frequency Shaping and Filtering

Example #1: Audio System

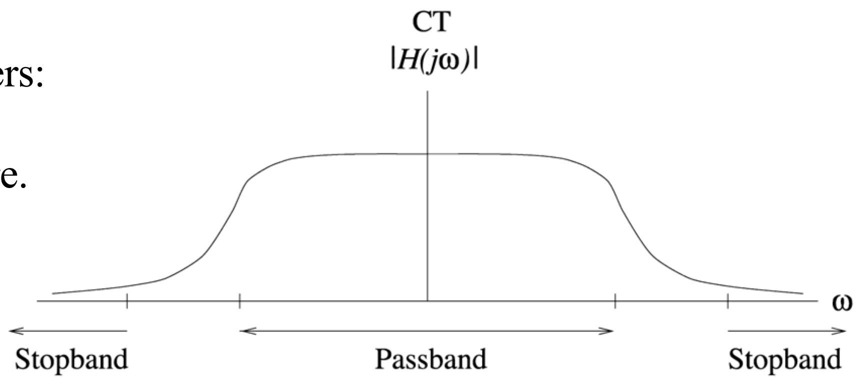


For audio signals, the amplitude is much more important than the phase.

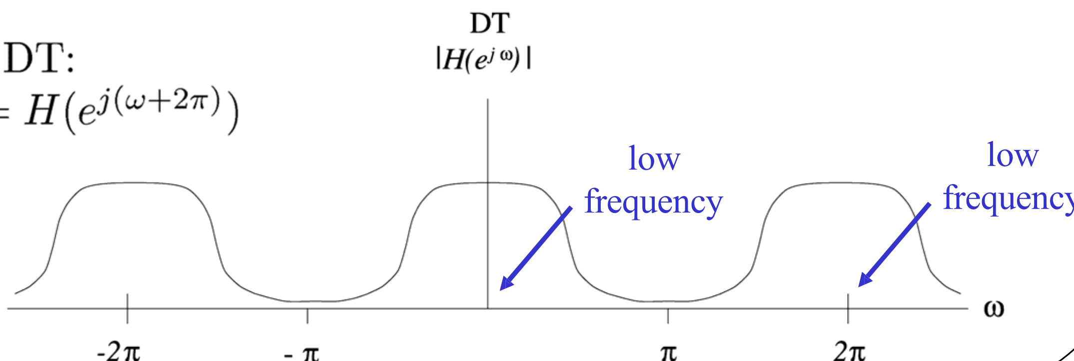
Example #2: Frequency Selective Filters

— Filter out signals outside of the frequency range of interest

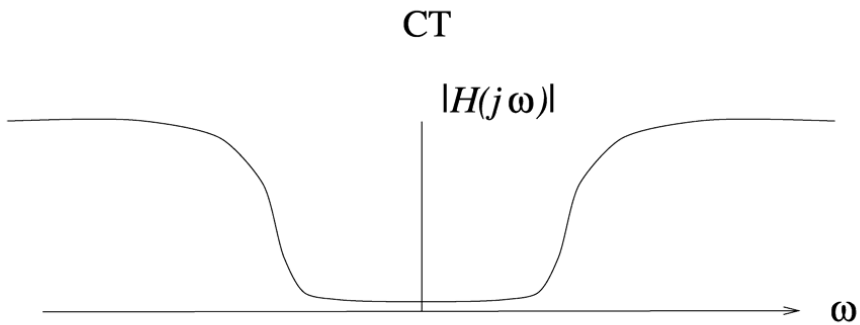
Lowpass Filters:
Only show
amplitude here.



Note for DT:
$H(e^{j\omega}) = H(e^{j(\omega+2\pi)})$



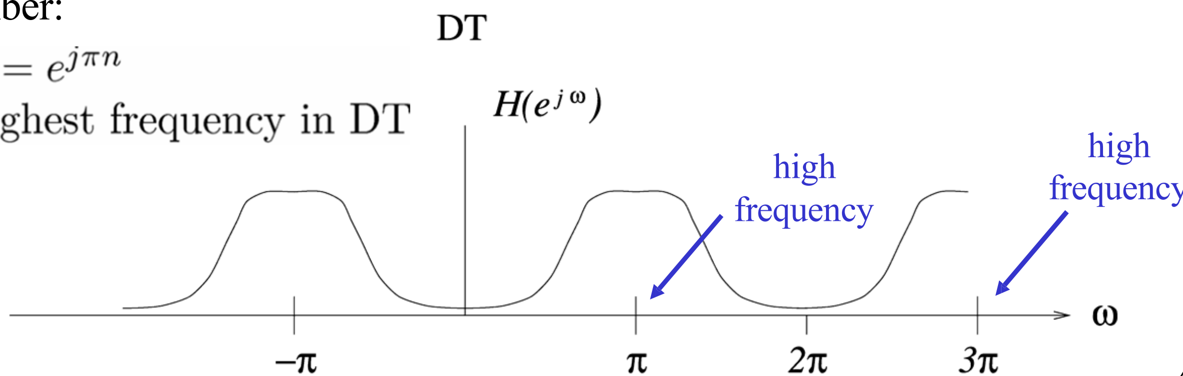
Highpass Filters



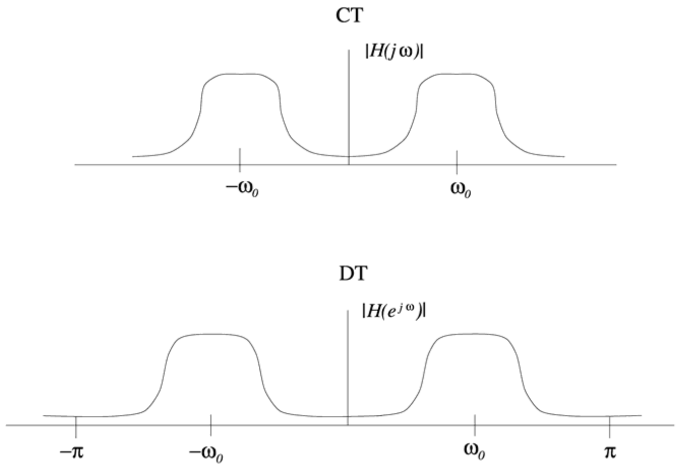
Remember:

$(-1)^n = e^{j\pi n}$

π = highest frequency in DT



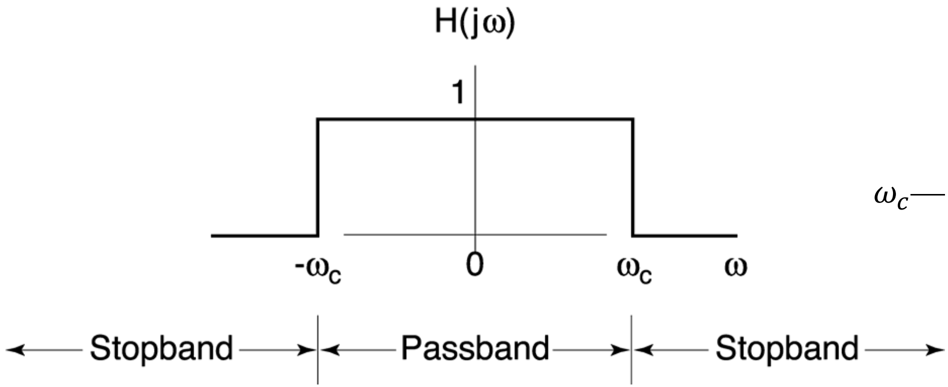
Bandpass Filters



Demo: Filtering effects on audio signals

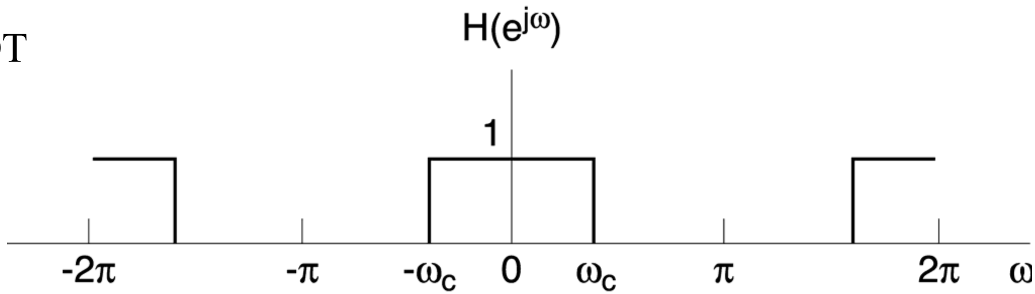
Ideal*ized* Filters

CT



ω_c — cutoff frequency

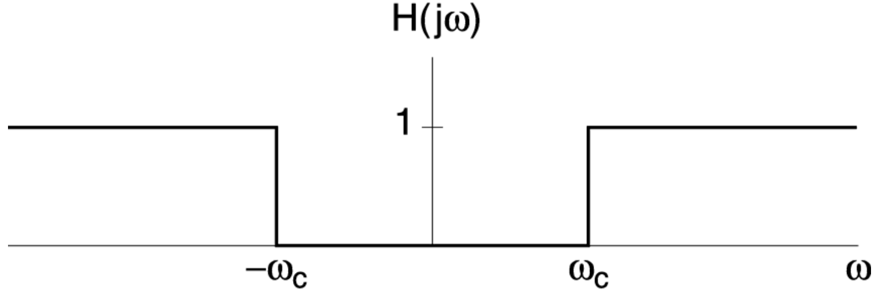
DT



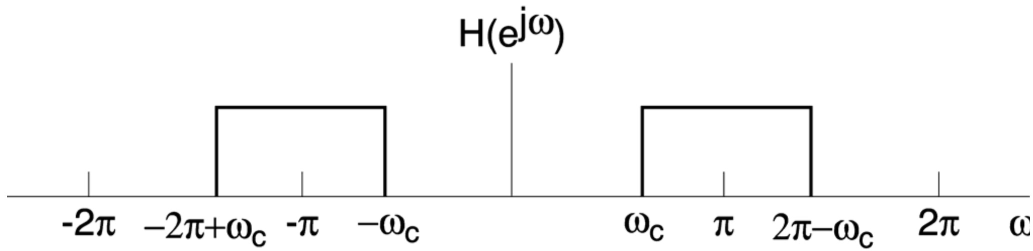
Note: $|H| = 1$ and $\angle H = 0$ for the ideal filters in the passbands, no need for the phase plot.

Highpass

CT

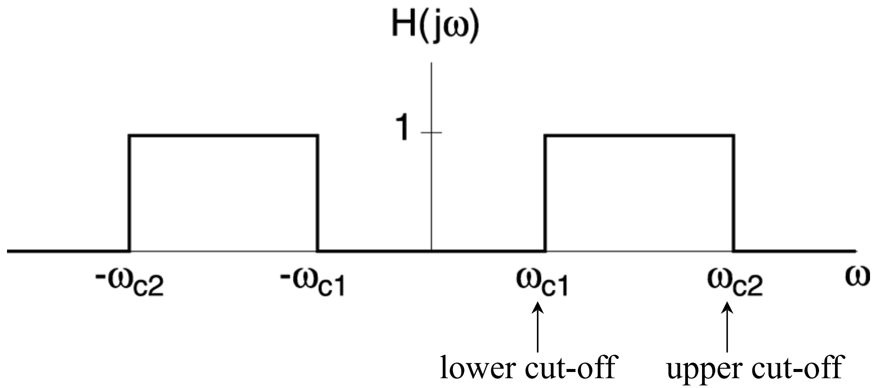


DT

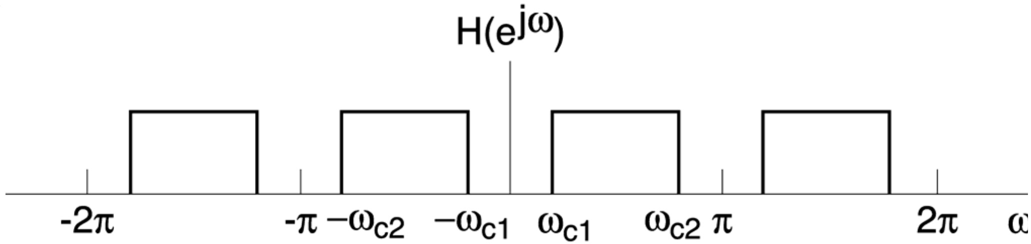


Bandpass

CT



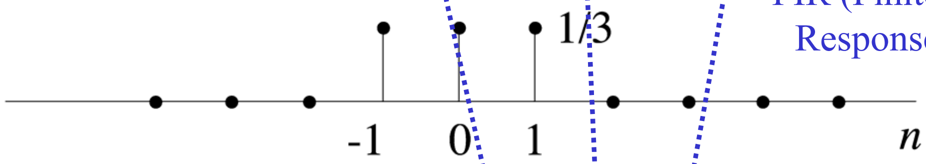
DT



Example #3: DT Averager/Smoother

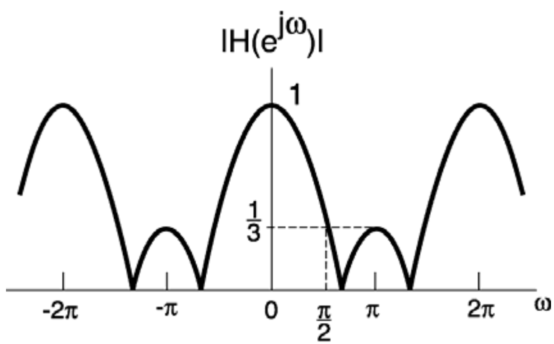
$$y[n] = \frac{1}{3}\{x[n-1] + x[n] + x[n+1]\}$$

$$h[n] = \frac{1}{3}\{\delta[n-1] + \delta[n] + \delta[n+1]\}$$



FIR (Finite Impulse Response) filters

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \frac{1}{3}\left[e^{-j\omega} + 1 + e^{j\omega}\right] = \frac{1}{3} + \frac{2}{3}\cos\omega$$

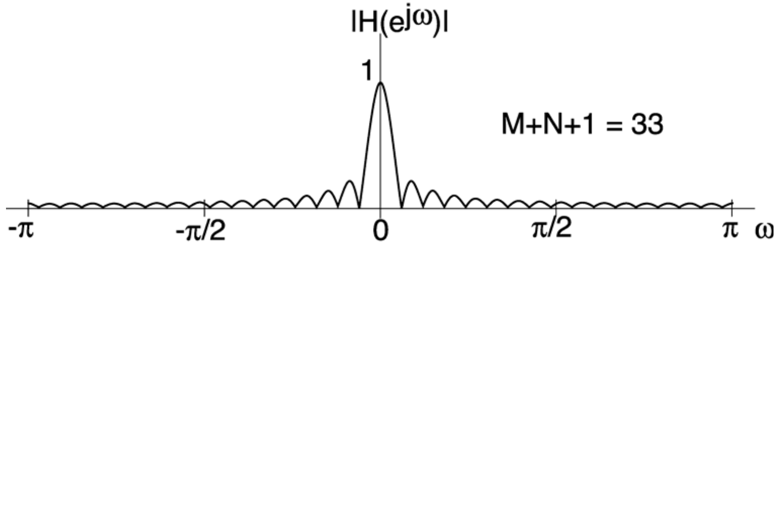


LPF

Example #4: Nonrecursive DT (*FIR*) filters

$$y[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} x[n-k] \longrightarrow h[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} \delta[n-k]$$

$$H(e^{j\omega}) = \frac{1}{N+M+1} \sum_{k=-N}^{M} e^{-jk\omega} = \frac{1}{N+M+1} e^{j\omega[(N-M)/2]} \frac{\sin[\omega(M+N+1)/2]}{\sin(\omega/2)}$$

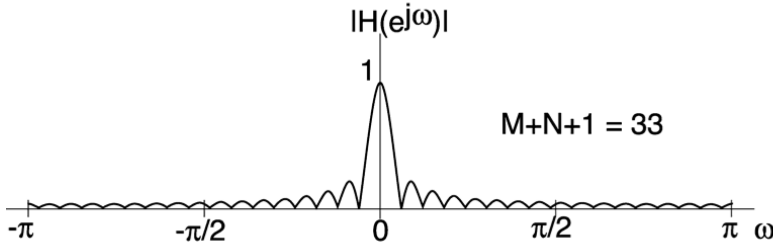


Rolls off at lower ω as M+N+1 increases

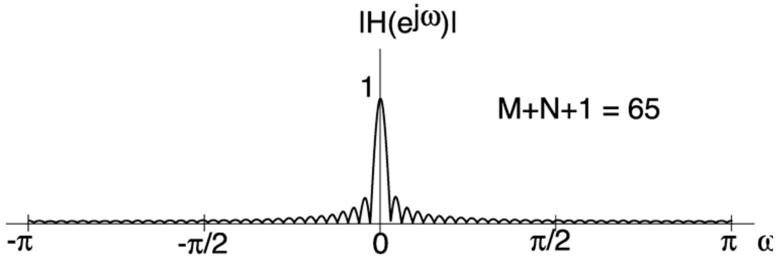
Example #4: Nonrecursive DT (*FIR*) filters

$$y[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k] \longrightarrow h[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} \delta[n-k]$$

$$H(e^{j\omega}) = \frac{1}{N+M+1}\sum_{k=-N}^{M} e^{-jk\omega} = \frac{1}{N+M+1} e^{j\omega[(N-M)/2]}\frac{\sin\left[\omega(M+N+1)/2\right]}{\sin(\omega/2)}$$



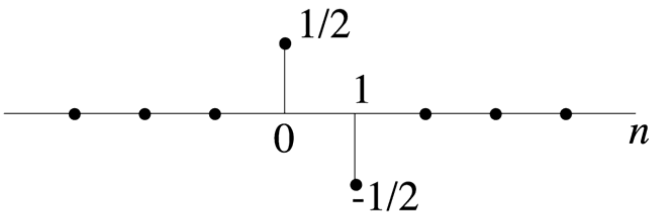
Rolls off at lower ω as M+N+1 increases



Example #5: Simple DT "Edge" Detector

— DT 2-point "differentiator"

$$y[n] = \frac{1}{2}\{x[n] - x[n-1]\}$$
$$h[n] = \frac{1}{2}\{\delta[n] - \delta[n-1]\}$$



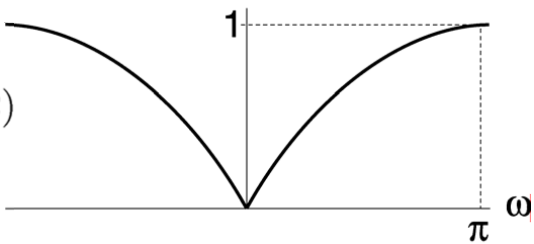
$$\frac{j}{2j}e^{-j\omega/2}(e^{j\omega/2} - e^{-j\omega/2}) = je^{-j\omega/2}\sin(\omega/2)$$

$$\Downarrow$$

$$H(e^{j\omega}) = \frac{1}{2}(1 - e^{-j\omega}) = je^{-j\omega/2}\sin(\omega/2)$$

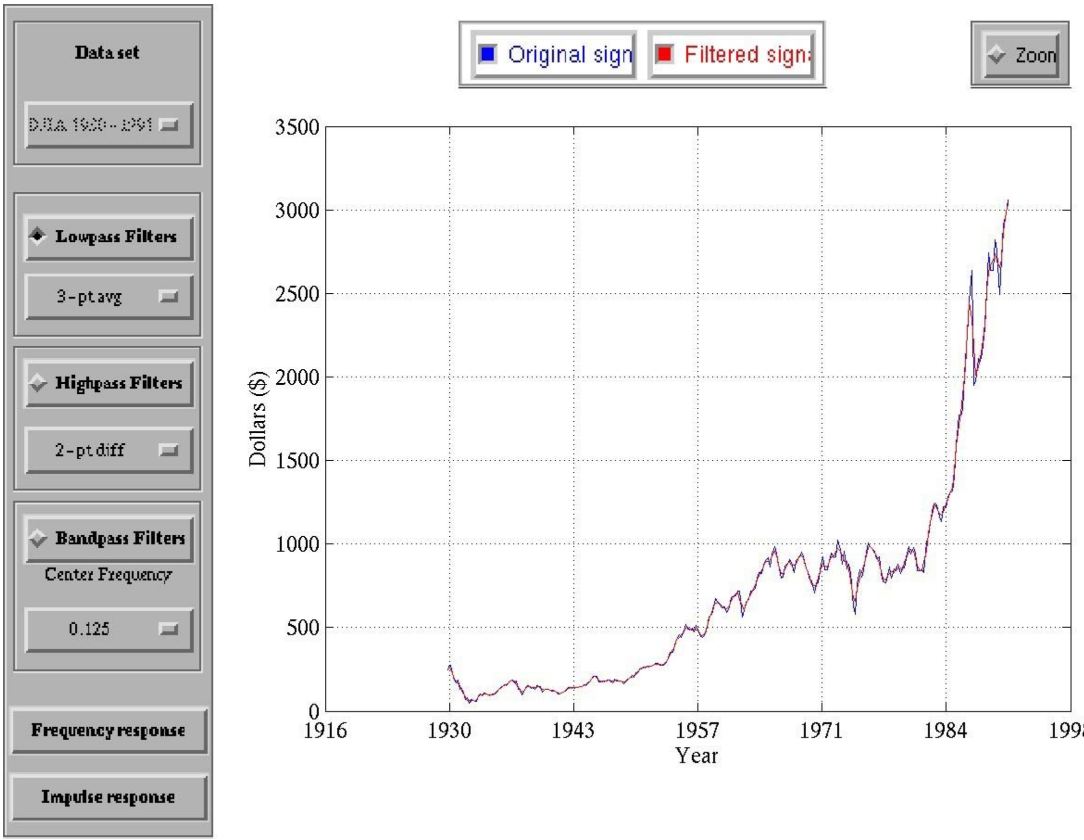
$$|H(e^{j\omega})| = |\sin(\omega/2)|$$

| H(ejω) |



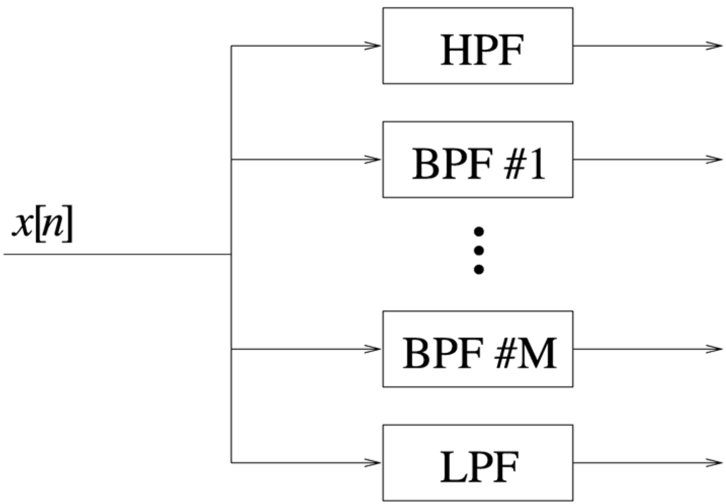
Passes high-frequency components

Demo: DT filters, LP, HP, and BP applied to DJ Industrial average



Example #6: Edge enhancement using DT differentiator

Example #7: A Filter Bank



Demo: Apply different filters to two-dimensional image signals.

Face of a monkey.

Image removed do to copyright considerations

HP

LP

LP

HP

BP

BP

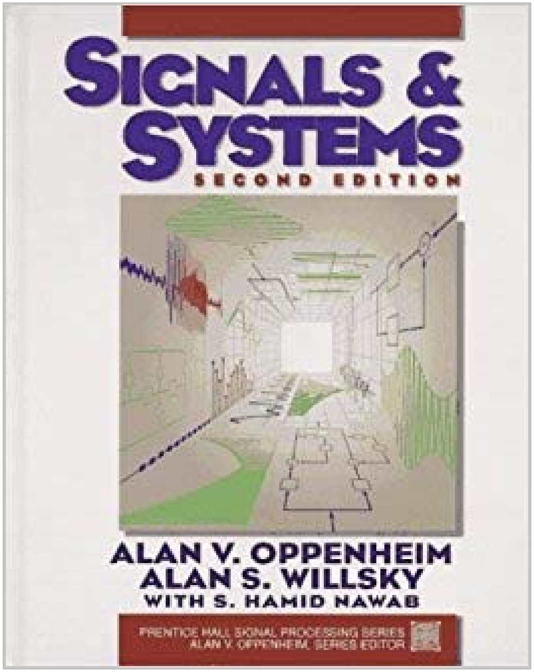
Note: To really understand these examples, we need to understand frequency contents of aperiodic signals ⇒ the Fourier Transform

بازنمایی سری فوریه سیگنال‌های متناوب (۳)

۴

منابع

منبع اصلی



A.V. Oppenheim, A.S. Willsky, S.H. Nawab,
Signals and Systems,
Second Edition, Prentice Hall, 1997.

Chapter 3