# سیگنال‌ها و سیستم‌ها

## درس ۲۴

# تبدیل لاپلاس (۱)

## The Laplace Transform (1)

کاظم فولادی قلعه
دانشکده مهندسی، دانشکدگان فارابی
دانشگاه تهران

http://courses.fouladi.ir/sigsys

# طرح درس

## COURSE OUTLINE

| |
|---|
| **بازنمایی‌های چندگانه‌ی سیستم‌های پیوسته–زمان** |
| Multiple Representations of CT Systems |
| **انگیزه و تعریف تبدیل لاپلاس (دوطرفه)** |
| Motivation and Definition of the (Bilateral) Laplace Transform |
| **مثال‌هایی از تبدیل‌های لاپلاس و نواحی همگرایی آنها (ROC)** |
| Examples of Laplace Transforms and Their Regions of Convergence (ROCs) |
| **خصوصیات نواحی همگرایی** |
| Properties of ROCs |

تبدیل لاپلاس (۱)

# ۱

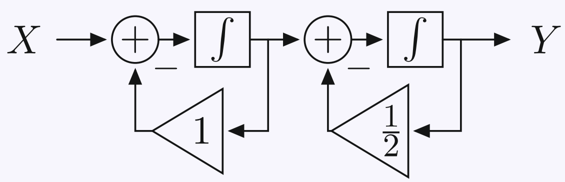
# بازنمایی‌های چندگانه‌ی سیستم‌های پیوسته–زمان

# نقشه‌ی مفهومی سیستم‌های پیوسته–زمان

## Concept Map of Continuous-Time Systems

سیستم‌های پیوسته–زمان را می‌توان با روش‌های مختلفی بازنمایی کرد.

### Block Diagram




### System Functional

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

### Impulse Response

$$h(t) = 2(e^{-t/2} - e^{-t})\,u(t)$$

### Differential Equation

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

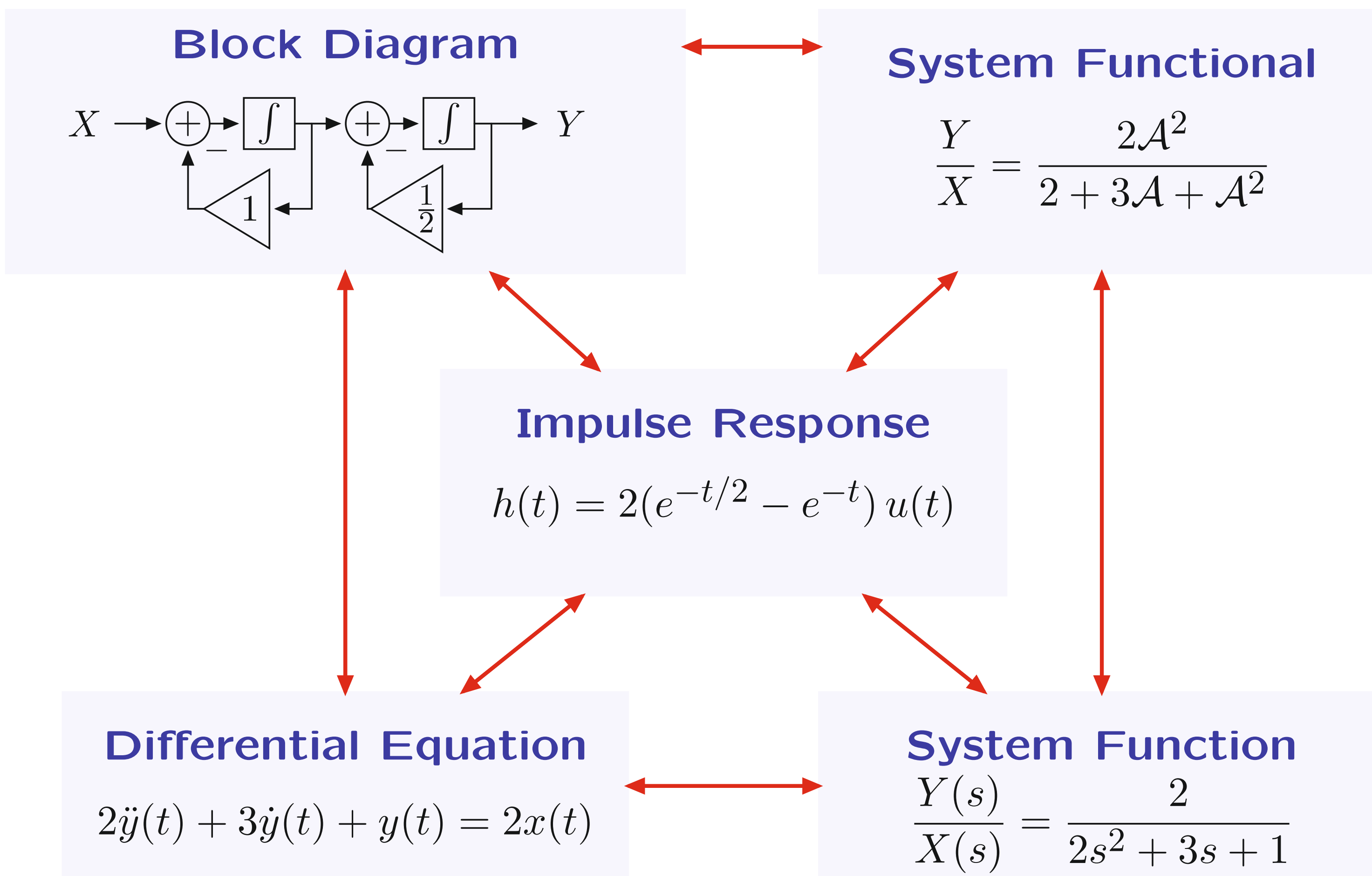
### System Function

$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته-زمان

## Concept Map of Continuous-Time Systems

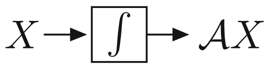
بین بازنمایی‌های مختلف رابطه‌هایی وجود دارد.



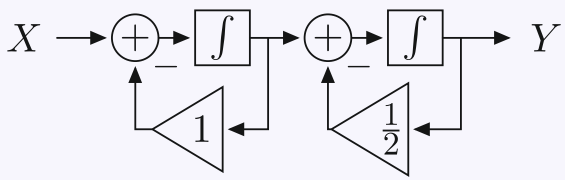
# نقشه‌ی مفهومی سیستم‌های پیوسته–زمان

## Concept Map of Continuous-Time Systems

دو تعبیر برای انتگرال:

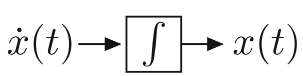


### Block Diagram



### System Functional

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$



### Impulse Response

$$h(t) = 2(e^{-t/2} - e^{-t})\, u(t)$$

### Differential Equation

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

### System Function

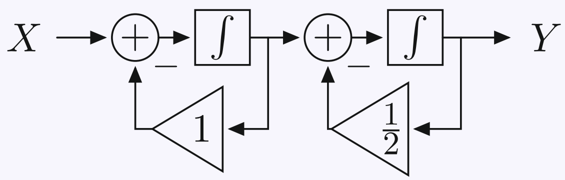
$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته-زمان

## Concept Map of Continuous-Time Systems

رابطه‌ی میان «معادله‌ی تابعی سیستم» و «تابع سیستم»:

**Block Diagram**




**System Functional**

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

$\mathcal{A} \to \frac{1}{s}$

**Impulse Response**

$$h(t) = 2(e^{-t/2} - e^{-t})\,u(t)$$

**Differential Equation**

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

**System Function**

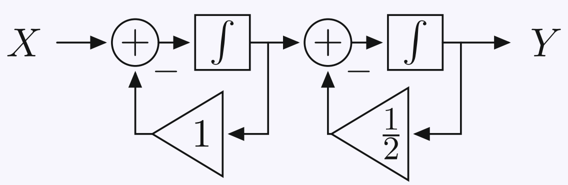
$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته‌-زمان

## CONCEPT MAP OF CONTINUOUS-TIME SYSTEMS

رابطه‌ی میان «معادله‌ی تابعی سیستم» و «پاسخ ضربه»:

### Block Diagram




### System Functional

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

### Impulse Response

$$h(t) = 2(e^{-t/2} - e^{-t})\, u(t)$$

### Differential Equation

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

### System Function

$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته–زمان

رابطه‌ی میان «معادله‌ی تابعی سیستم» و «پاسخ ضربه»

## Concept Map of Continuous-Time Systems

چگونه می‌توان پاسخ ضربه را از روی معادله‌ی تابعی سیستم به‌دست آورد؟

معادله‌ی تابعی را با استفاده از **کسرهای جزئی** بسط می‌دهیم:

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2+3\mathcal{A}+\mathcal{A}^2} = \frac{\mathcal{A}^2}{(1+\frac{1}{2}\mathcal{A})(1+\mathcal{A})} = \frac{2\mathcal{A}}{1+\frac{1}{2}\mathcal{A}} - \frac{2\mathcal{A}}{1+\mathcal{A}}$$

فرم‌های هر جمله را **بازشناسی** می‌کنیم: هر یک متناظر با یک نمایی هستند؛
یا اینکه هر جمله را در قالب یک **سری** بسط می‌دهیم:

$$\frac{Y}{X} = 2\mathcal{A}\Big(1-\frac{1}{2}\mathcal{A}+\frac{1}{4}\mathcal{A}^2-\frac{1}{8}\mathcal{A}^3+-\cdots\Big) - 2\mathcal{A}\Big(1-\mathcal{A}+\mathcal{A}^2-\mathcal{A}^3+-\cdots\Big)$$

Let $X = \delta(t)$. Then

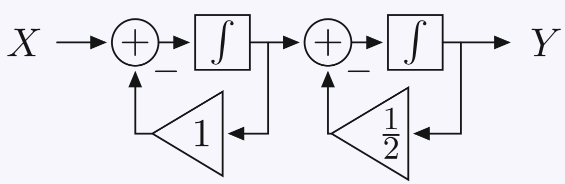
$$Y = 2\Big(1-\frac{1}{2}t+\frac{1}{8}t^2-\frac{1}{48}t^3+-\cdots\Big)u(t) - 2\Big(1-t+\frac{1}{2}t^2-\frac{1}{3!}t^3+-\cdots\Big)u(t)$$
$$= 2\Big(e^{-t/2}-e^{-t}\Big)u(t)$$

# نقشه‌ی مفهومی سیستم‌های پیوسته-زمان

## Concept Map of Continuous-Time Systems

رابطه‌ی میان «معادله‌ی تابعی سیستم» و «پاسخ ضربه»:

### Block Diagram




### System Functional

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

series

partial fractions

### Impulse Response

$$h(t) = 2(e^{-t/2} - e^{-t})\, u(t)$$

### Differential Equation

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

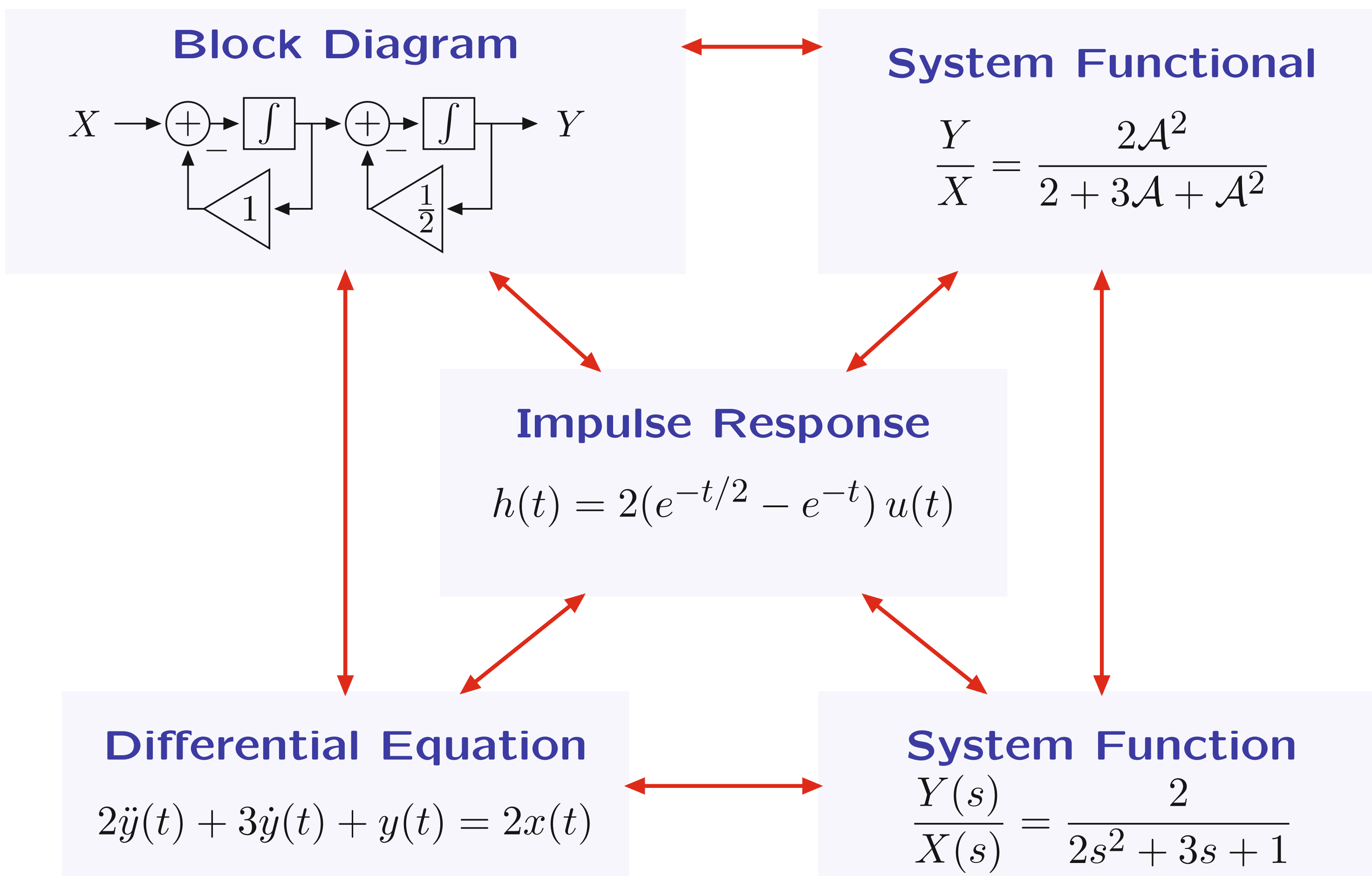
### System Function

$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته–زمان

## Concept Map of Continuous-Time Systems

با **تبدیل لاپلاس** می‌توان رابطه‌های جدیدی را تعیین کرد.

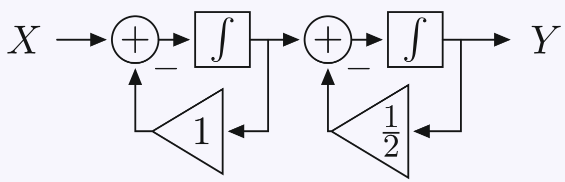


# نقشه‌ی مفهومی سیستم‌های پیوسته-زمان

## Concept Map of Continuous-Time Systems

رابطه‌ی میان معادله‌ی دیفرانسیل با تابع سیستم:

**Block Diagram**




**System Functional**

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

**Impulse Response**

$$h(t) = 2(e^{-t/2} - e^{-t})\, u(t)$$

**Differential Equation**

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

$\mathcal{L}$ →

**System Function**

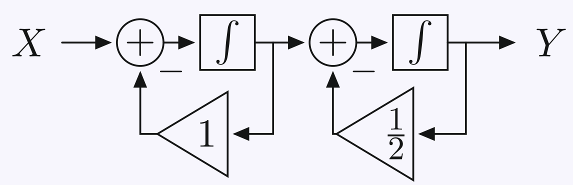
$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

# نقشه‌ی مفهومی سیستم‌های پیوسته-زمان

## Concept Map of Continuous-Time Systems

رابطه‌ی میان پاسخ ضربه و تابع سیستم:

### Block Diagram




### System Functional

$$\frac{Y}{X} = \frac{2\mathcal{A}^2}{2 + 3\mathcal{A} + \mathcal{A}^2}$$

### Impulse Response

$$h(t) = 2(e^{-t/2} - e^{-t})\, u(t)$$

$\mathcal{L}$

### Differential Equation

$$2\ddot{y}(t) + 3\dot{y}(t) + y(t) = 2x(t)$$

### System Function

$$\frac{Y(s)}{X(s)} = \frac{2}{2s^2 + 3s + 1}$$

سیگنال‌ها و سیستم‌ها

تبدیل لاپلاس (۱)

# ۲

# تعریف تبدیل لاپلاس

# تبدیل لاپلاس

## THE LAPLACE TRANSFORM

**تبدیل لاپلاس** تابعی از زمان $t$ را به تابعی از $s$ نگاشت می‌دهد.

$$X(s) = \int x(t)e^{-st}dt$$

| **دوطرفه**<br>*Bilateral* | **یک‌طرفه**<br>*Unilateral* |
|---|---|
| $$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$ | $$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$ |

هر دو خواص مهم مشترکی دارند + تفاوت‌ها

# Motivation for the Laplace Transform

- CT Fourier transform enables us to do a lot of things, e.g.
  - Analyze frequency response of LTI systems
  - Sampling
  - Modulation
  
  ⋮

- Why do we need yet another transform?

- One view of Laplace Transform is as an *extension* of the Fourier transform to allow analysis of broader class of signals and systems

- In particular, Fourier transform *cannot* handle large (and important) classes of signals and *unstable* systems, i.e. when

$$\int_{-\infty}^{\infty} |x(t)| dt = \infty$$

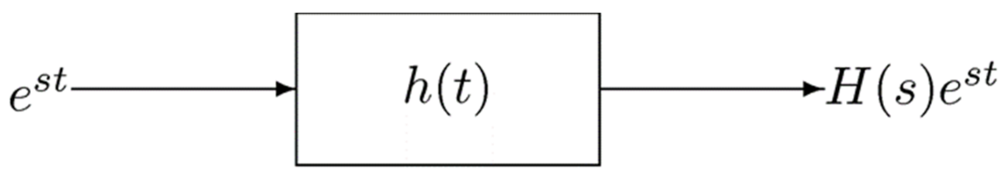
# Motivation for the Laplace Transform (continued)

- In many applications, we do need to deal with *unstable* systems, e.g.
  - Stabilizing an inverted pendulum
  - Stabilizing an airplane or space shuttle

    ⋮

  - Instability is *desired* in some applications, e.g. oscillators and lasers

- How do we analyze such signals/systems?
  Recall from Chapter #3, eigenfunction property of LTI systems:




$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt \qquad \text{(assuming this converges)}$$

  - $e^{st}$ is an eigenfunction of *any* LTI system
  - $s = \sigma + j\omega$ can be complex in general

# The (Bilateral) Laplace Transform

$$x(t) \longleftrightarrow X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \mathcal{L}\{x(t)\}$$

$s = \sigma + j\omega$ is a *complex* variable – Now we explore the full range of $s$

Basic ideas:

absolute integrability needed

(1) $$X(s) = X(\sigma + j\omega) = \int_{-\infty}^{\infty} [x(t)e^{-\sigma t}] e^{-j\omega t} dt = \mathcal{F}\{x(t)e^{-\sigma t}\}$$

(2) A critical issue in dealing with Laplace transform is convergence:

— *X*(*s*) generally exists only for *some* values of *s*,
located in what is called the *region of convergence* (ROC)

$$\text{ROC} = \{s = \sigma + j\omega \text{ so that } \int_{-\infty}^{\infty} \underbrace{\left|x(t)e^{-\sigma t}\right|}_{\substack{\text{Depends}\\ \text{only on } \sigma\\ \text{not on } \omega}} dt < \infty\}$$

absolute integrability condition

(3) If $s = j\omega$ is in the ROC (i.e. $\sigma = 0$), then

$$X(s)\big|_{s=j\omega} = \mathcal{F}\{x(t)\}$$

# ۳

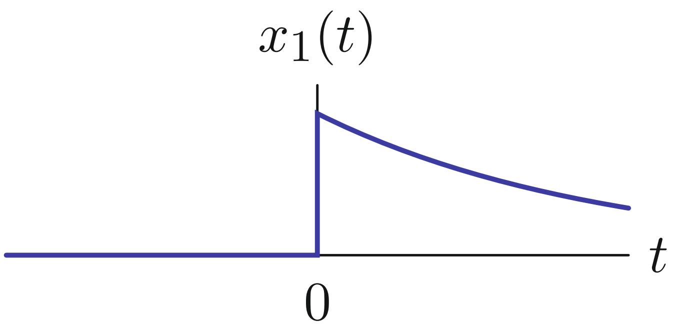
# مثال‌هایی از تبدیل‌های لاپلاس و نواحی همگرایی آنها (ROC)

# تبدیل لاپلاس

مثال

## LAPLACE TRANSFORM

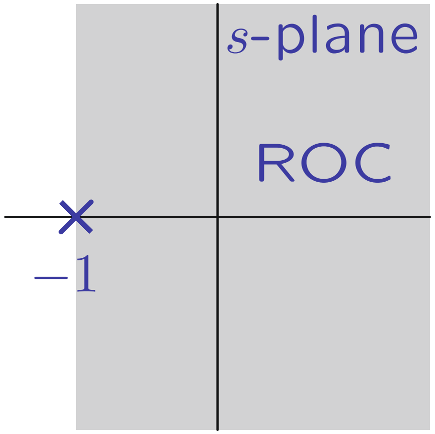
$$x_1(t) = \begin{cases} e^{-t} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$




$$X_1(s) = \int_{-\infty}^{\infty} x_1(t) e^{-st} dt = \int_{0}^{\infty} e^{-t} e^{-st} dt = \left. \frac{e^{-(s+1)t}}{-(s+1)} \right|_0^{\infty} = \frac{1}{s+1}$$

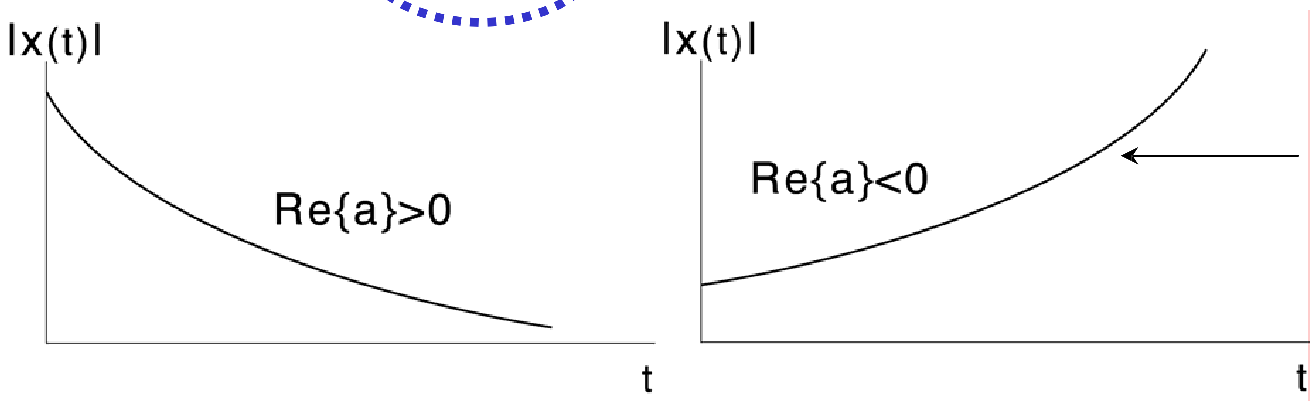
provided $\text{Re}(s+1) > 0$ which implies that $\text{Re}(s) > -1$.

$$\frac{1}{s+1} \; ; \quad \text{Re}(s) > -1$$



**Example #1:**

$$x_1(t) = e^{-at}u(t) \qquad (a \text{ – an arbitrary real or complex number})$$




*Unstable*:

- no *Fourier Transform*
- but *Laplace Transform* exists

$$X_1(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt = \int_0^{\infty} e^{-(s+a)t}dt$$

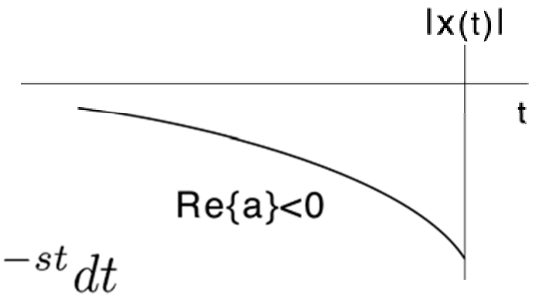
$$= -\frac{1}{s+a}e^{-(s+a)t}\Big|_0^{\infty} = -\frac{1}{s+a}[e^{-(s+a)\infty} - 1]$$

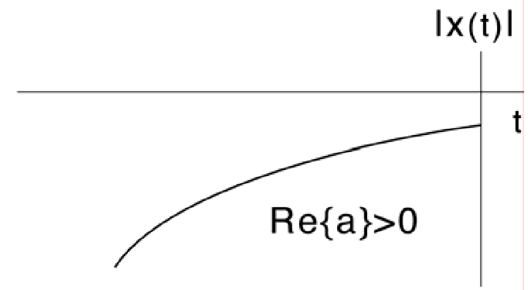
This converges only if $Re(s + a) > 0$, i.e. $Re(s) > -Re(a)$

$$\Downarrow$$

$$X_1(s) = \frac{1}{s+a}, \quad \underbrace{\Re e\{s\} > -\Re e\{a\}}_{\text{ROC}}$$

**Example #2:**




$$\begin{aligned} x_2(t) &= -e^{-at}u(-t) \\ X_2(s) &= -\int_{-\infty}^{\infty} e^{-at}u(-t)e^{-st}dt \\ &= -\int_{-\infty}^{0} e^{-(s+a)t}dt \\ &= +\frac{1}{s+a}e^{-(s+a)t}\Big|_{-\infty}^{0} = \frac{1}{s+a}[1-e^{(s+a)\infty}] \end{aligned}$$

This converges only if $Re(s+a) < 0$, i.e. $Re(s) < -Re(a)$

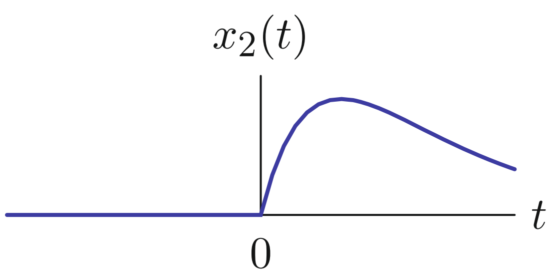
$$X_2(s) = \frac{1}{s+a}, \quad \underbrace{\Re e\{s\} < -\Re e\{a\}}_{\text{ROC}}$$ - Same as $X_1(s)$, but different ROC

**Key Point** (and key difference from *FT*): Need *both* $X(s)$ *and* ROC to uniquely determine $x(t)$. No such an issue for *FT*.

# تبدیل لاپلاس

## مثال

## Laplace Transform




$$x_2(t) = \begin{cases} e^{-t} - e^{-2t} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$X_2(s) = \int_0^{\infty} (e^{-t} - e^{-2t}) e^{-st} dt$$

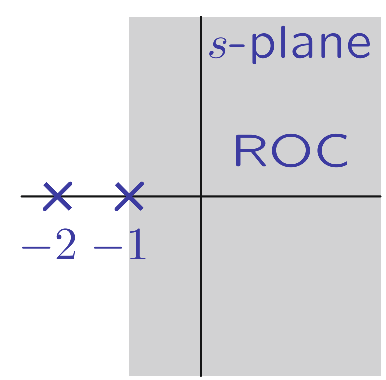
$$= \int_0^{\infty} e^{-t} e^{-st} dt - \int_0^{\infty} e^{-2t} e^{-st} dt$$

$$= \frac{1}{s+1} - \frac{1}{s+2} = \frac{(s+2)-(s+1)}{(s+1)(s+2)} = \frac{1}{(s+1)(s+2)}$$

These equations converge if

$\mathrm{Re}(s+1) > 0$ and $\mathrm{Re}(s+2) > 0$, thus $\mathrm{Re}(s) > -1$.

$$\frac{1}{(s+1)(s+2)} \; ; \quad \mathrm{Re}(s) > -1$$



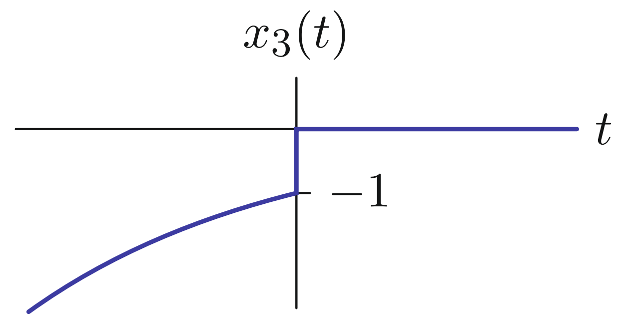
# تبدیل لاپلاس

مثال

## Laplace Transform

سیگنال‌های سمت چپی، تبدیل‌های لاپلاس سمت چپی دارند (فقط در تبدیل لاپلاس دوطرفه)

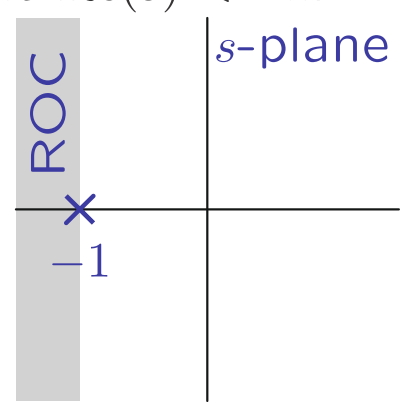
$$x_3(t) = \begin{cases} -e^{-t} & \text{if } t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$




$$X_3(s) = \int_{-\infty}^{\infty} x_3(t)e^{-st}dt = \int_{-\infty}^{0} -e^{-t}e^{-st}dt = \left.\frac{-e^{-(s+1)t}}{-(s+1)}\right|_{-\infty}^{0} = \frac{1}{s+1}$$

provided $\mathrm{Re}(s+1) < 0$ which implies that $\mathrm{Re}(s) < -1$.

$$\frac{1}{s+1} \;;\quad \mathrm{Re}(s) < -1$$

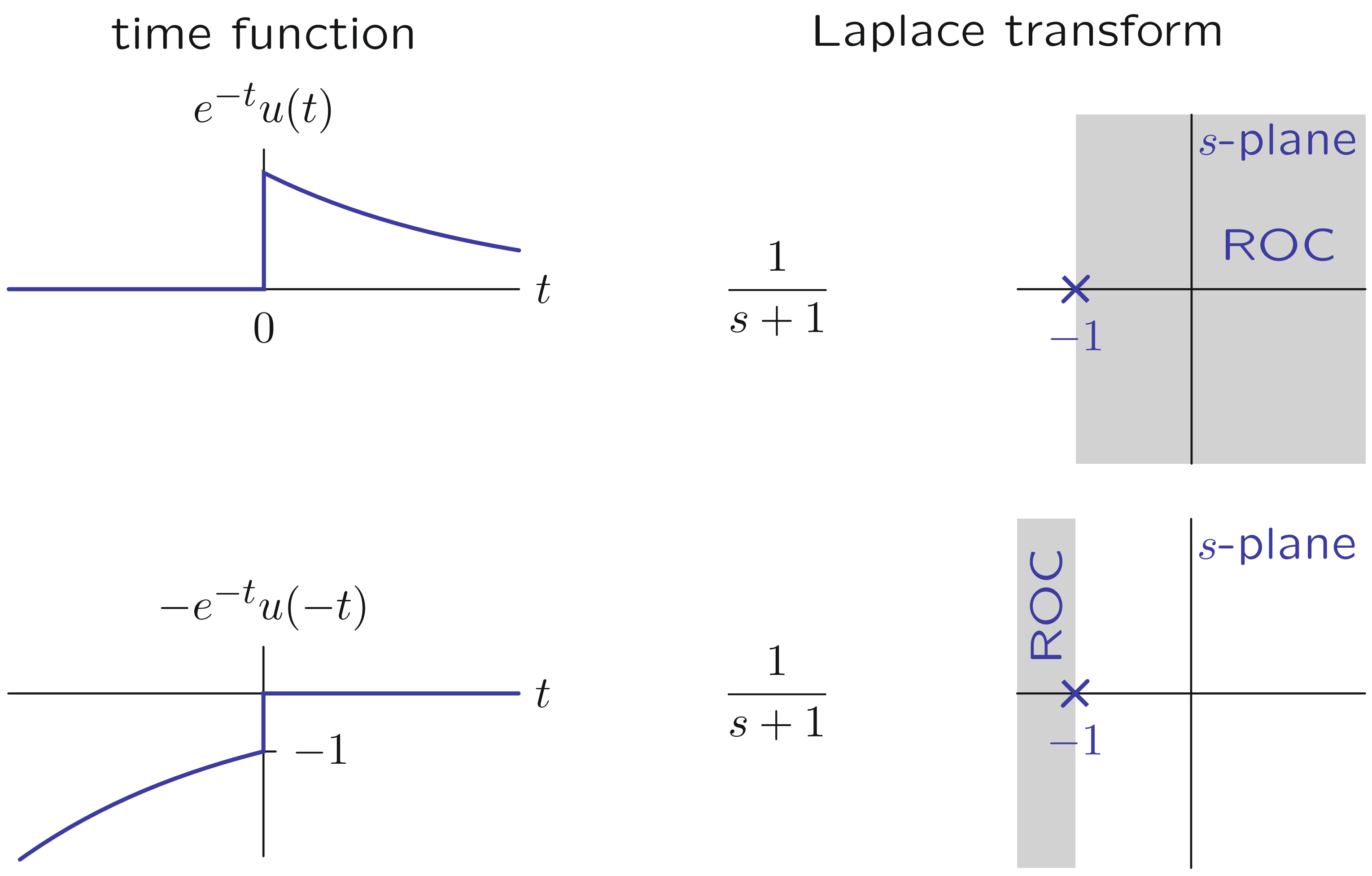


# تبدیل لاپلاس

## نواحی همگرایی سمت چپی و سمت راستی

## LEFT- AND RIGHT-SIDED ROCS

تبدیل‌های لاپلاس نمایی‌های سمت چپی و سمت راستی، شکل‌های مشابهی دارند،
اما نواحی همگرایی آنها به‌ترتیب سمت چپی و سمت راستی است.

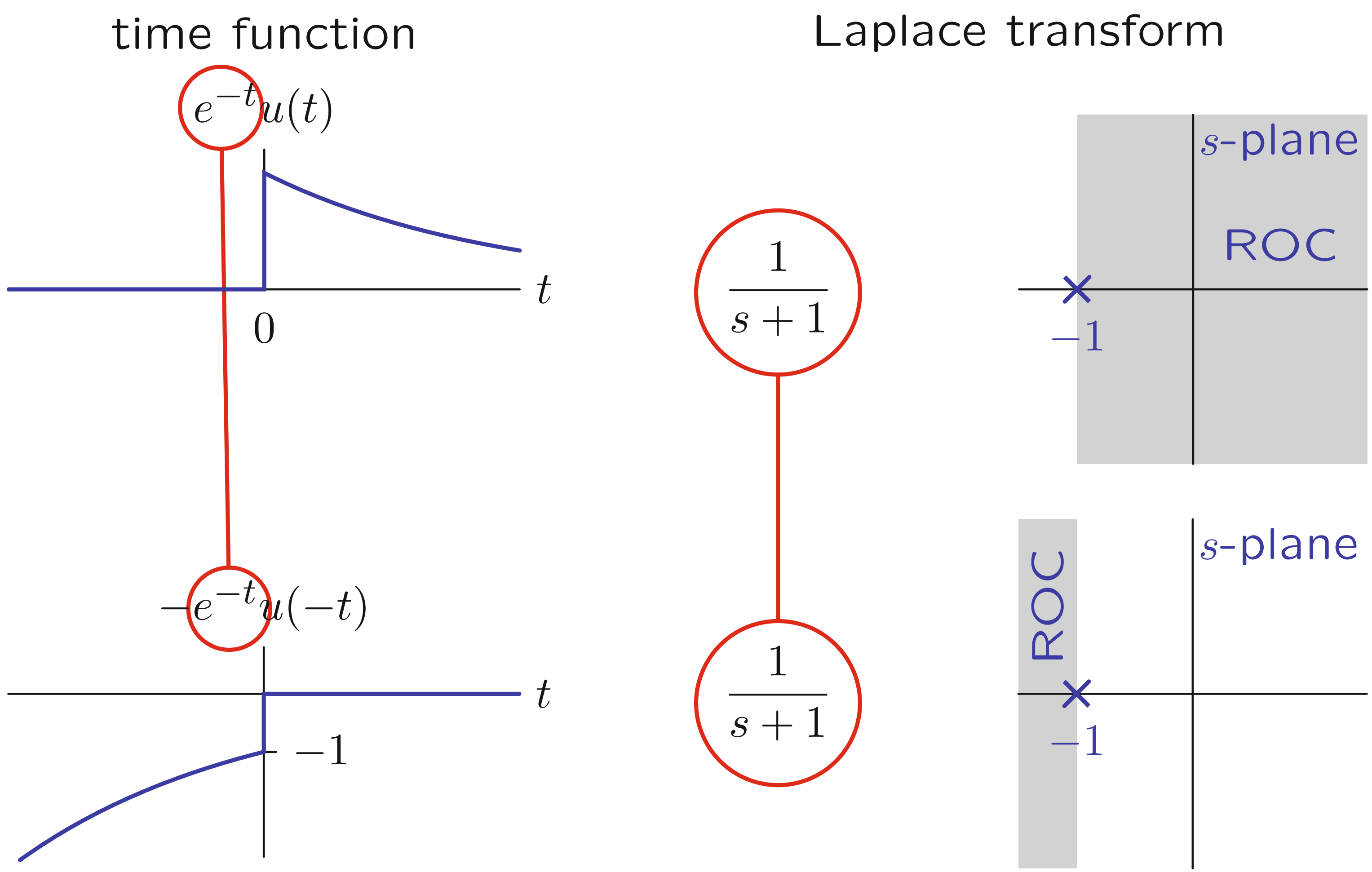


# تبدیل لاپلاس

## نواحی همگرایی دست چپی و دست راستی

## LEFT- AND RIGHT-SIDED ROCS

تبدیل‌های لاپلاس نمایی‌های سمت چپی و سمت راستی، شکل‌های مشابهی دارند،
اما نواحی همگرایی آنها به‌ترتیب سمت چپی و سمت راستی است.

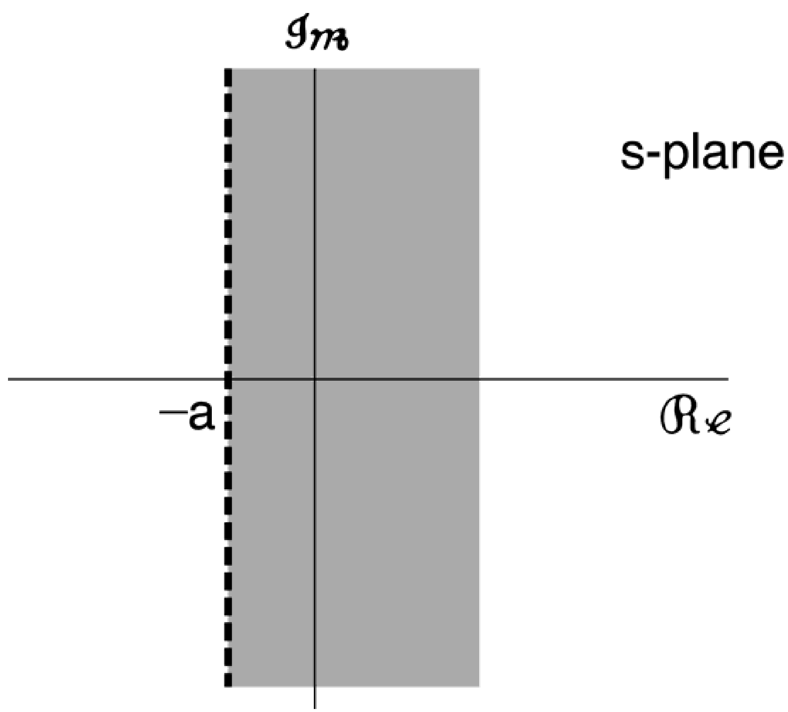


# Graphical Visualization of the ROC

**Example #1**

$$X_1(s) = \frac{1}{s+a}, \quad \Re e\{s\} > -\Re e\{a\}$$

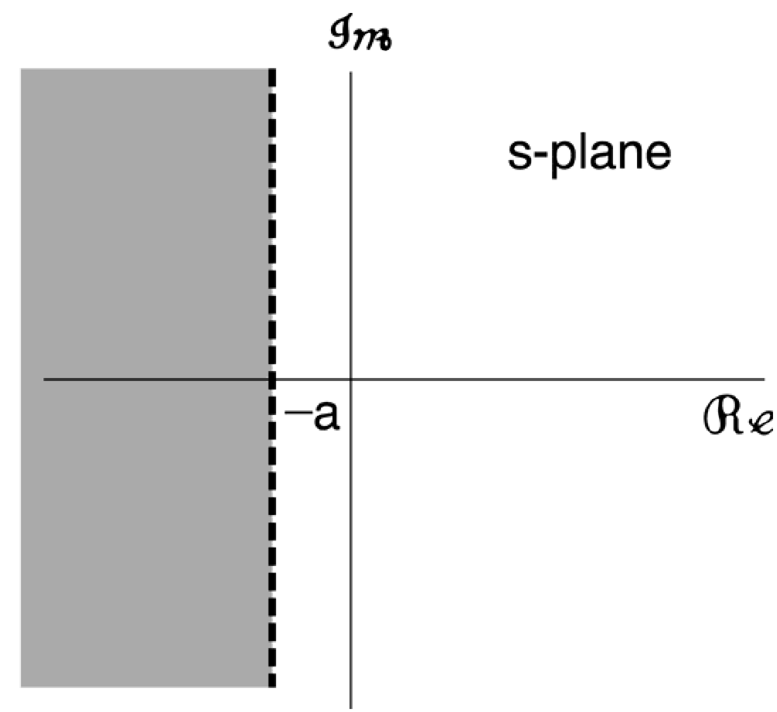
$x_1(t) = e^{-at}u(t)$ - right-sided signal




**Example #2**

$$X_2(s) = \frac{1}{s+a}, \quad \Re e\{s\} < -\Re e\{a\}$$

$x_2(t) = -e^{-at}u(-t)$ - left-sided signal



# Rational Transforms

- Many (but by no means all) Laplace transforms of interest to us are rational functions of $s$ (e.g., Examples #1 and #2; in general, impulse responses of LTI systems described by LCCDEs), where

$$X(s) = \frac{N(s)}{D(s)}, \quad N(s), D(s) \text{ – polynomials in } s$$

- Roots of $N(s)$ = zeros of $X(s)$

- Roots of $D(s)$ = poles of $X(s)$

- Any $x(t)$ consisting of a linear combination of complex exponentials for $t > 0$ and for $t < 0$ (e.g., as in Example #1 and #2) has a rational Laplace transform.

**Example #3**

$$x(t) = 3e^{2t}u(t) - 2e^{-t}u(t)$$

$$X(s) = \int_0^{\infty} [3e^{2t} - 2e^{-t}]e^{-st}dt$$

$$= 3\underbrace{\int_0^{\infty} e^{-(s-2)t}dt}_{\text{Requires } \Re e\{s\} > 2} - 2\underbrace{\int_0^{\infty} e^{-(s+1)t}dt}_{\text{Requires } \Re e\{s\} > -1}$$

**BOTH required → ROC intersection**

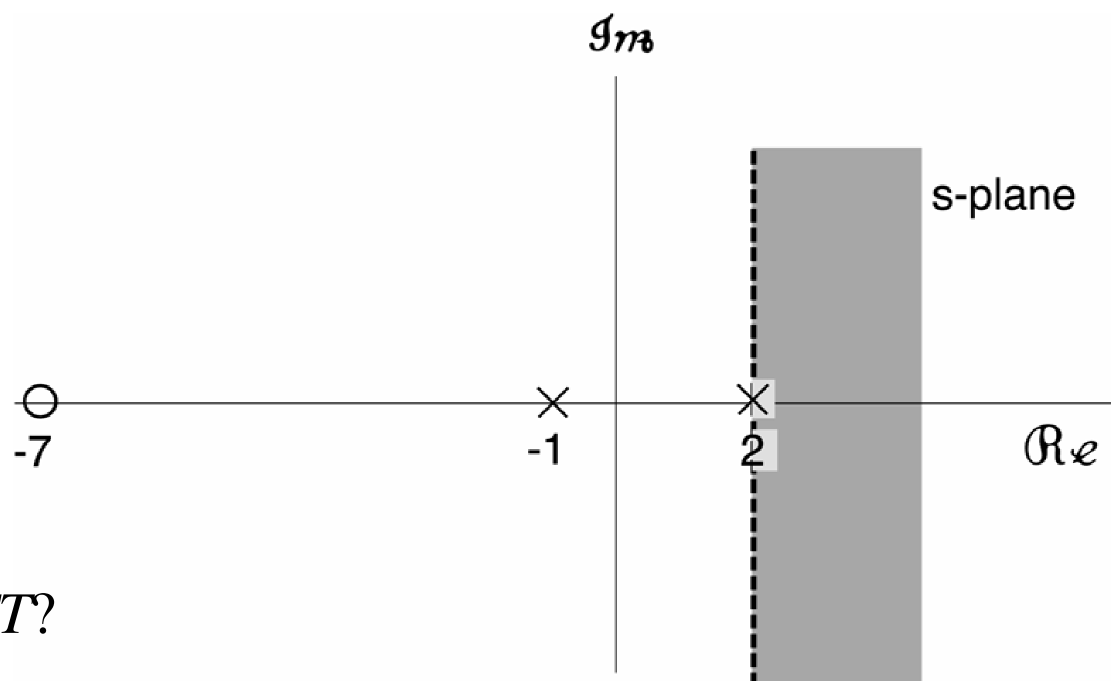
$$\Downarrow$$

$$X(s) = \frac{3}{s-2} - \frac{2}{s+1} = \frac{s+7}{(s-2)(s+1)} = \frac{s+7}{s^2-s-2} \quad \Re e\{s\} > 2$$

Notation:

× — *pole*

o — *zero*



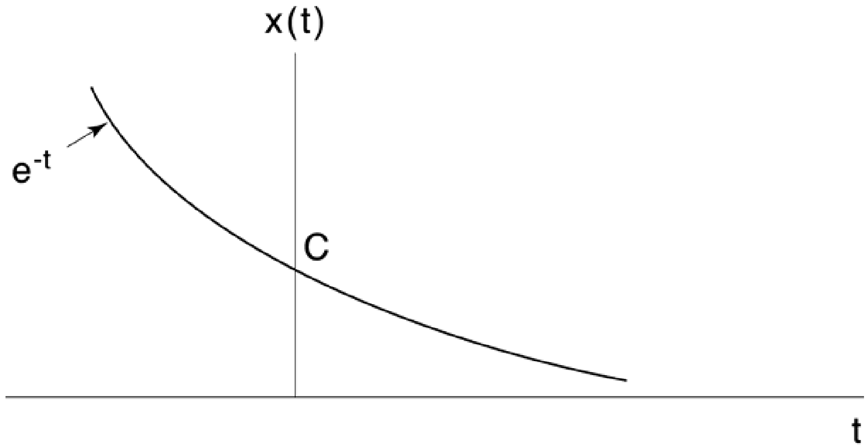

Q: Does $x(t)$ have *FT*?

# Laplace Transforms and ROCs

- Some signals do not have Laplace Transforms (have no ROC)

(a) $x(t) = Ce^{-t}$ for all $t$ since $\int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|dt = \infty$ for all $\sigma$

$$|x(t)e^{-\sigma t}| = Ce^{-(\sigma+1)t}$$




(b) $x(t) = e^{j\omega_0 t}$ for all $t$ 

*FT*: $X(j\omega) = 2\pi\delta(\omega - \omega_0)$

$$\int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|dt = \int_{-\infty}^{\infty} e^{-\sigma t}dt = \infty \text{ for all } \sigma$$

*X*(*s*) is defined only in ROC; we don't allow impulses in LTs

سیگنال‌ها و سیستم‌ها

تبدیل لاپلاس (۱)

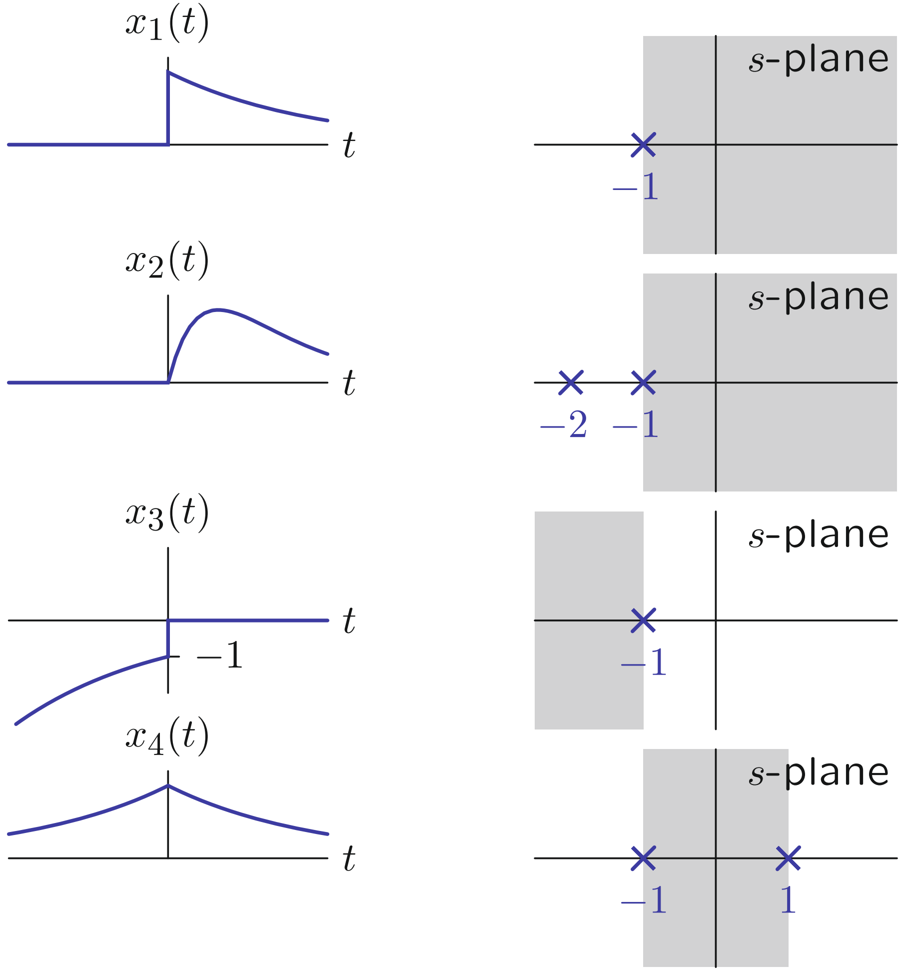
# ۴

# خصوصیات نواحی همگرایی

# تبدیل لاپلاس

## تعبیر حوزه‌ی زمان ناحیه‌ی همگرایی

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$

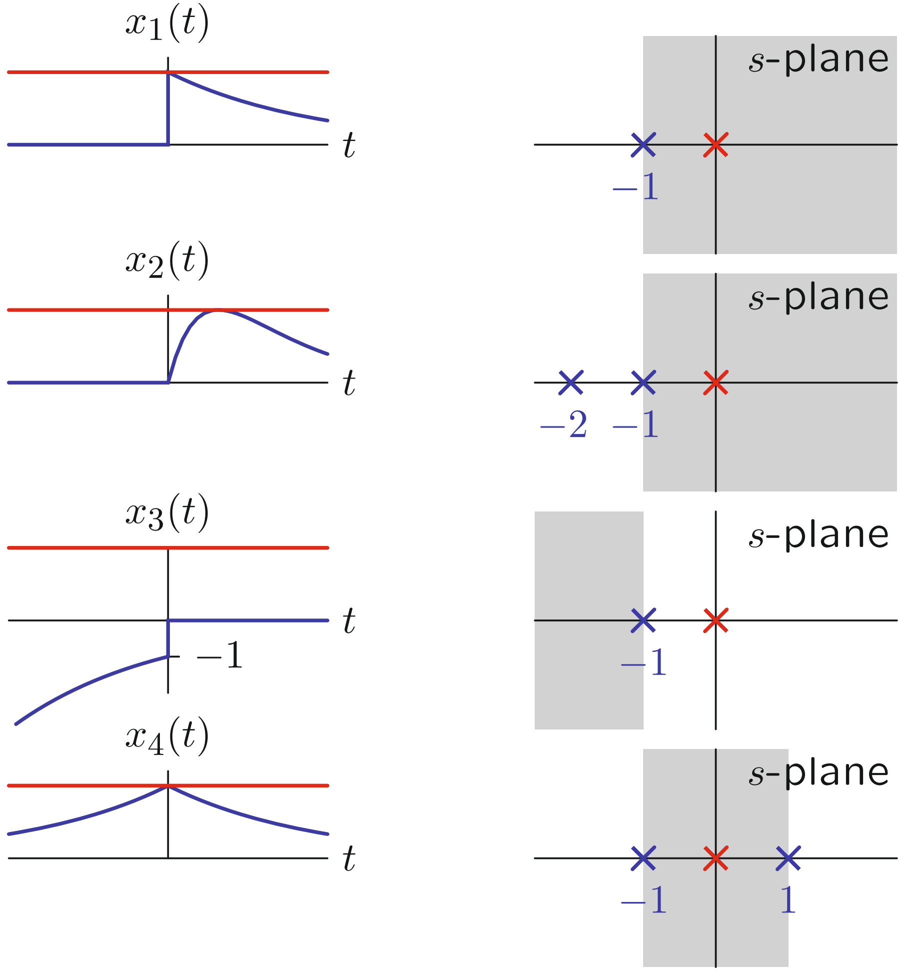



Prepared by Kazim Fouladi | Spring 2024 | 6th Edition
کاظم فولادی

# تبدیل لاپلاس

## تعبیر حوزه‌ی زمان ناحیه‌ی همگرایی

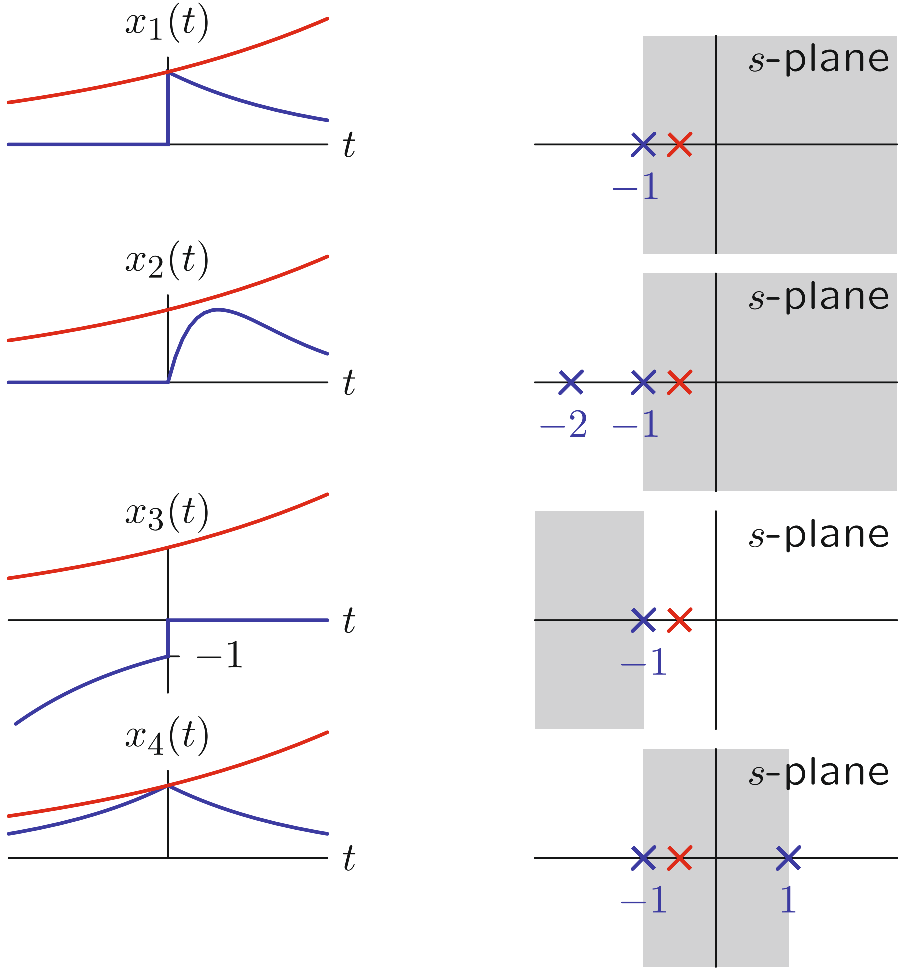
$$X(s)=\int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$



# تبدیل لاپلاس

## تعبیر حوزه‌ی زمان ناحیه‌ی همگرایی

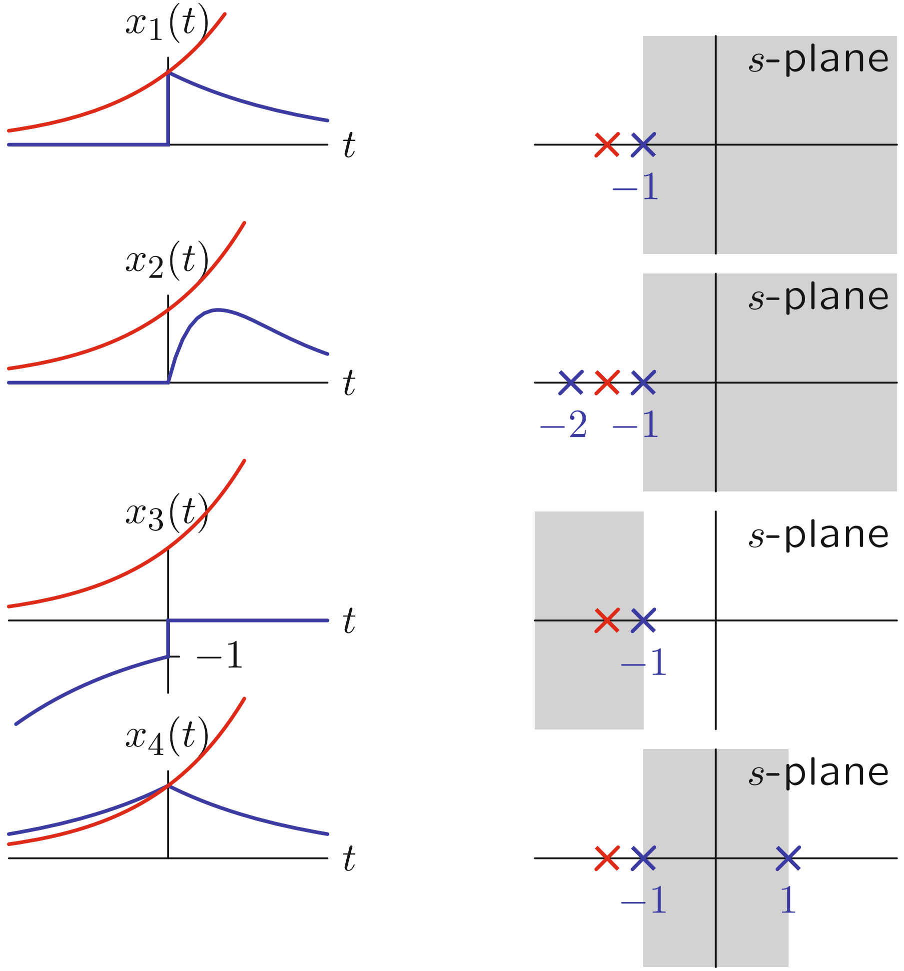
$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$



# تبدیل لاپلاس

## تعبیر حوزه‌ی زمان ناحیه‌ی همگرایی

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$



## Properties of the ROC

- The ROC can take on only a small number of different forms

1) The ROC consists of a collection of lines parallel to the $j\omega$-axis in the $s$-plane (i.e. the ROC only depends on $\sigma$).
   Why?

$$\int_{-\infty}^{\infty} |x(t)e^{-st}|dt = \int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|dt < \infty \text{ depends only on } \sigma = \Re e\{s\}$$

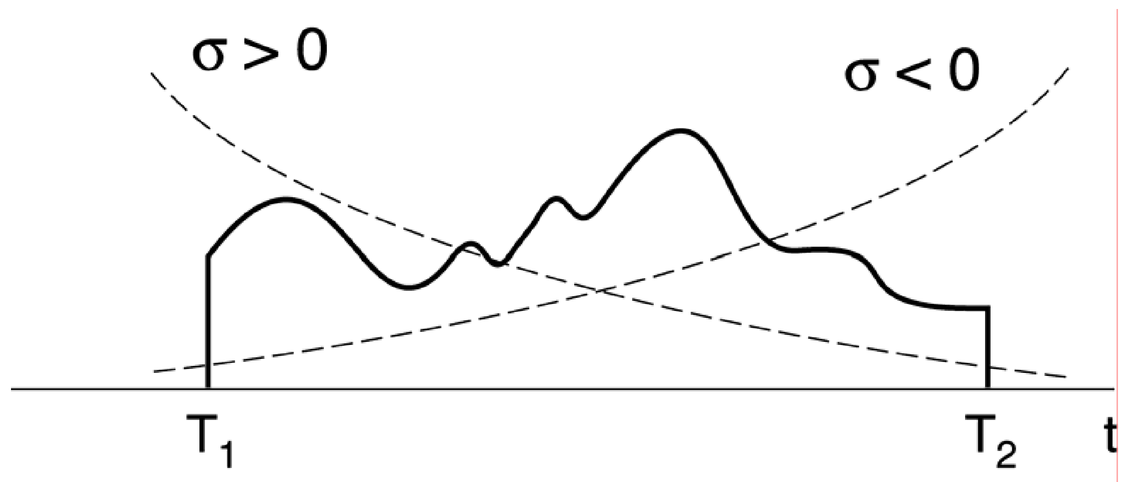
2) If $X(s)$ is rational, then the ROC does not contain any poles.
   Why?

Poles are places where $D(s) = 0$

$$X(s) = \frac{N(s)}{D(s)} = \infty \quad \text{Not convergent.}$$

## More Properties

3) If $x(t)$ is of finite duration and is absolutely integrable, then the ROC is the entire $s$-plane.

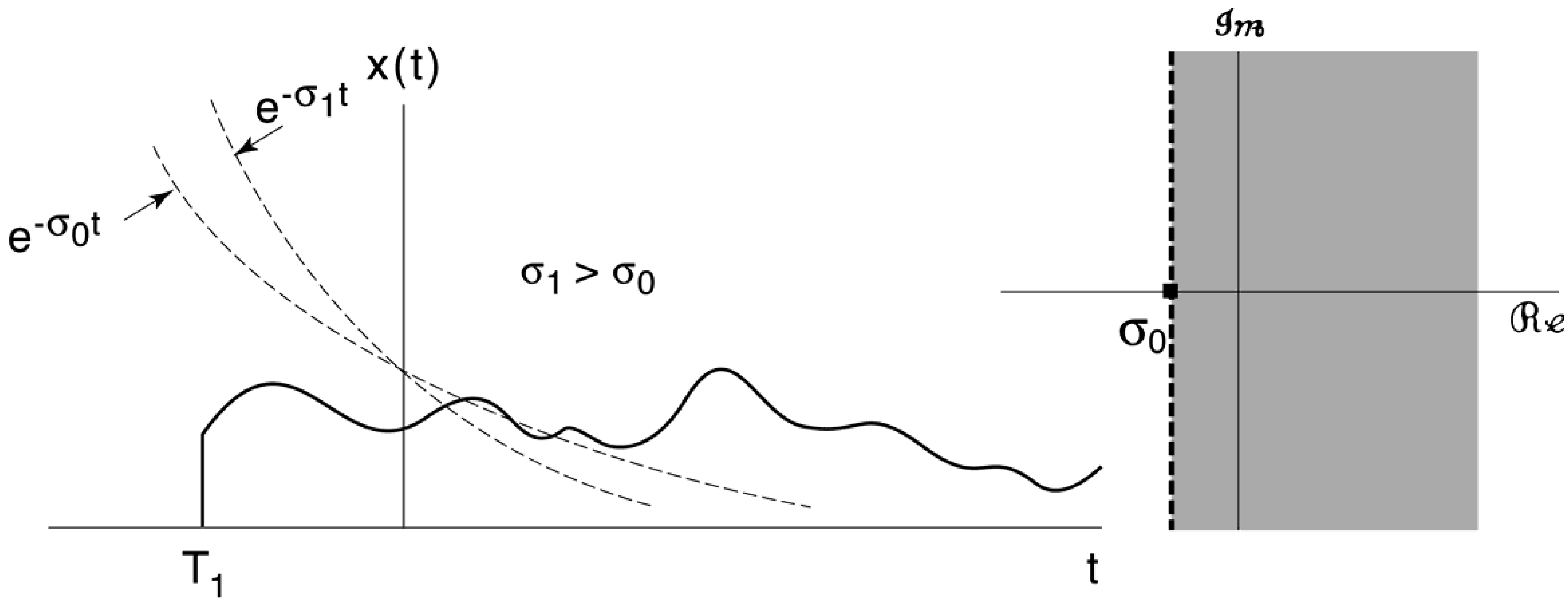



$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt = \underbrace{\int_{T_1}^{T_2} x(t)e^{-st}dt}_{\text{A finite integration interval}}$$

$$< \infty \quad \text{if} \int_{T_1}^{T_2} |x(t)|dt < \infty$$

# ROC Properties that Depend on Which Side You Are On - I

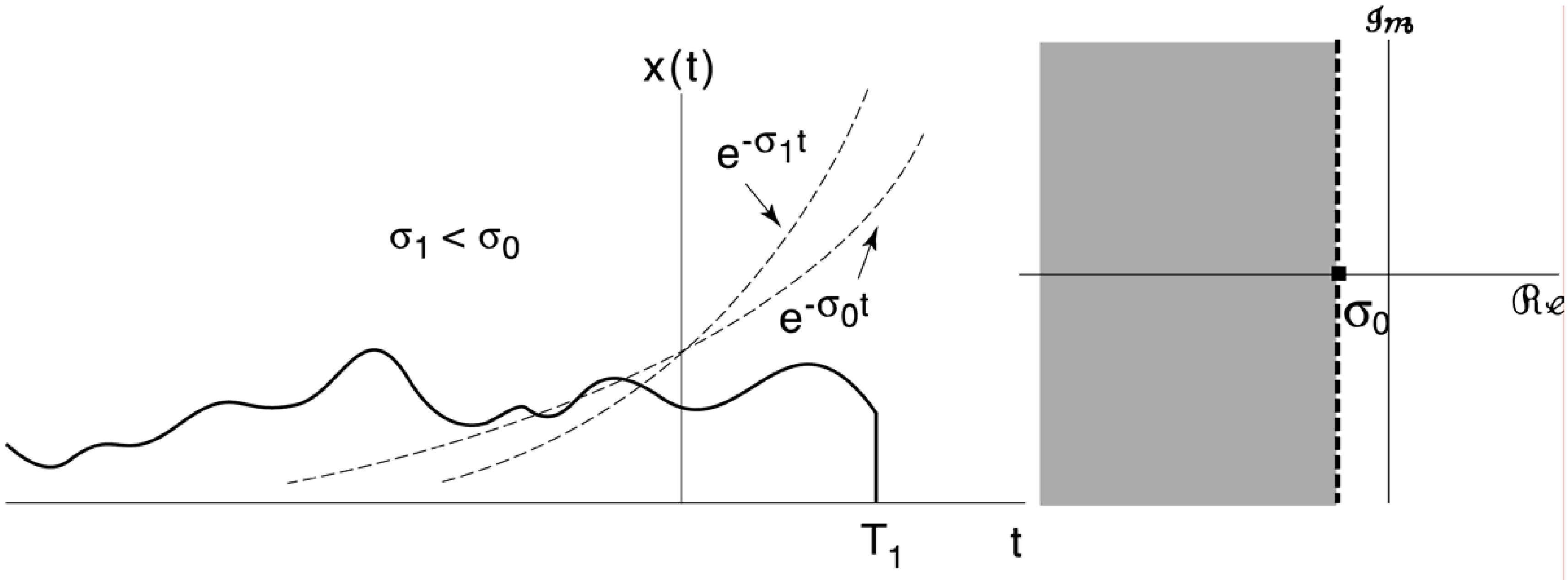
4) If $x(t)$ is right-sided (i.e. if it is zero *before* some time), and if $Re(s) = \sigma_o$ is in the ROC, then all values of $s$ for which $Re(s) > \sigma_o$ are also in the ROC.




ROC is a right half plane (RHP)

# ROC Properties that Depend on Which Side You Are On - II

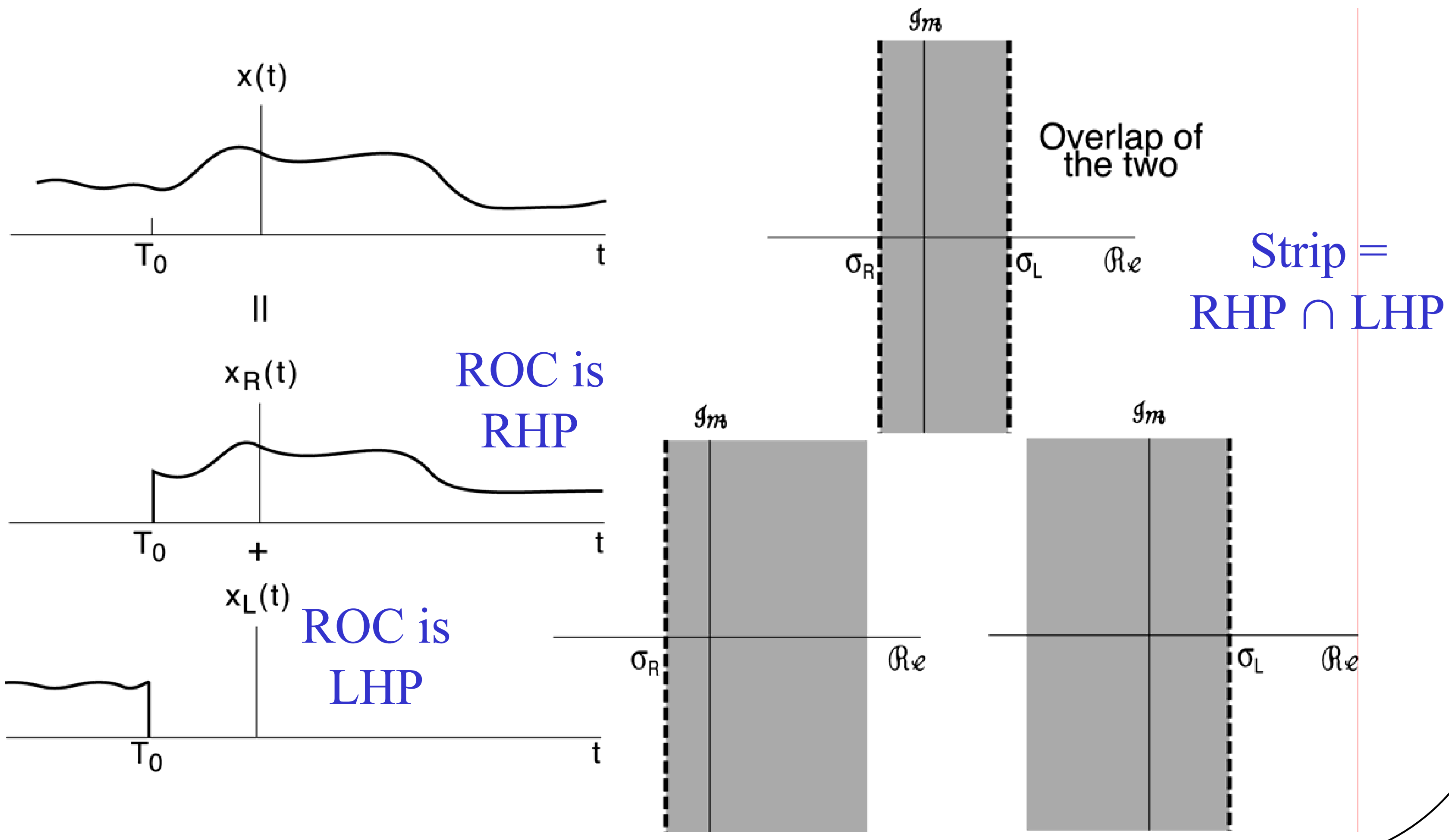
4) If $x(t)$ is left-sided (i.e. if it is zero *after* some time), and if $Re(s) = \sigma_o$ is in the ROC, then all values of $s$ for which $Re(s) < \sigma_o$ are also in the ROC.




ROC is a left half plane (LHP)

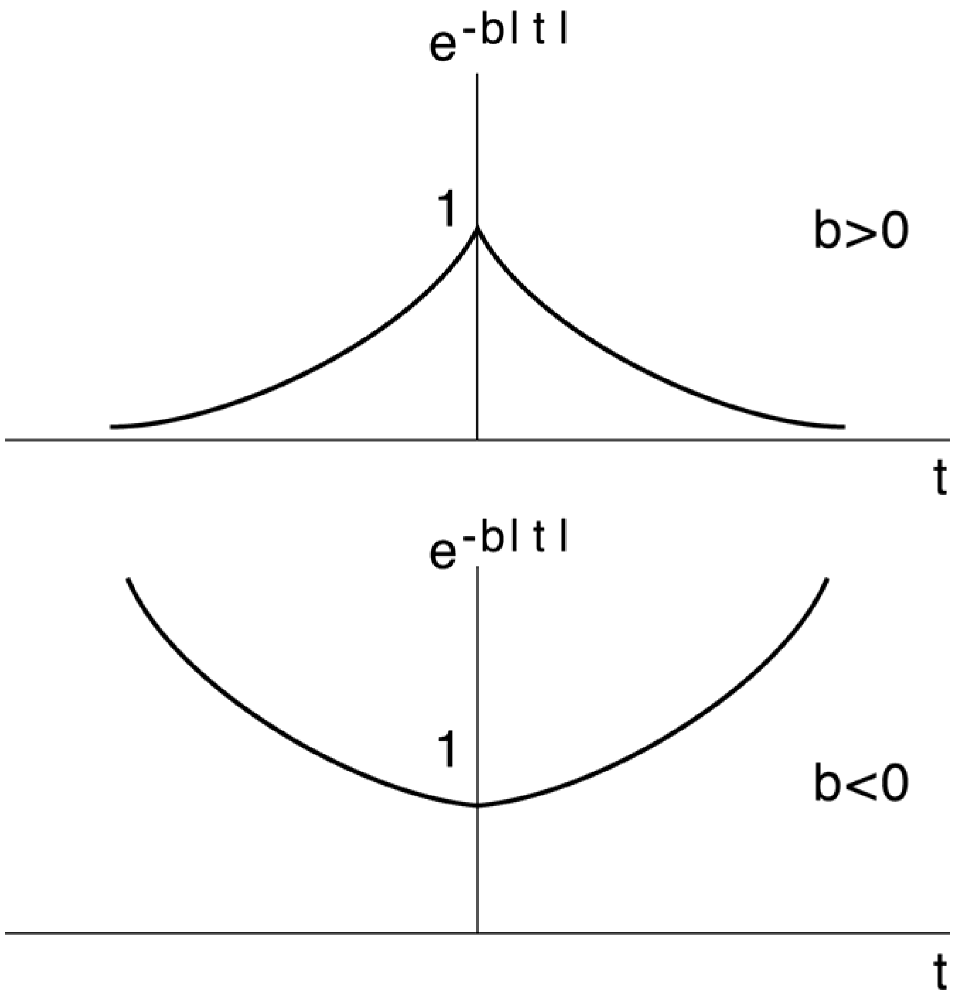
# Still More ROC Properties

6) If $x(t)$ is two-sided and if the line $Re(s) = \sigma_o$ is in the ROC, then the ROC consists of a strip in the $s$-plane that includes the line $Re(s) = \sigma_o$.



**Example:** $x(t) = e^{-b|t|}$

Intuition?




Okay: multiply by constant ($e^{0t}$) and will be integrable

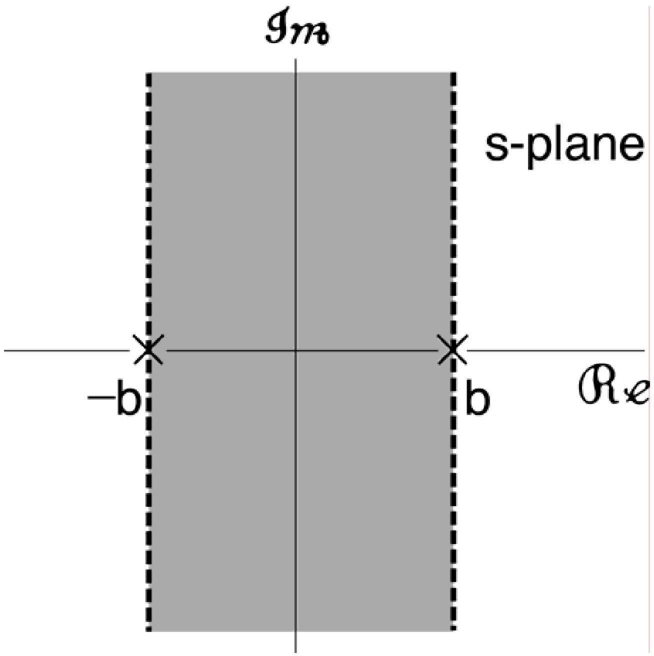
Looks bad: no $e^{\sigma t}$ will dampen both sides

**Example (continued):**

$$x(t) = e^{bt}u(-t) + e^{-bt}u(t)$$

$$e^{bt}u(-t) \Downarrow -\frac{1}{s-b}, \; \Re e\{s\} < b \qquad e^{-bt}u(t) \Downarrow \frac{1}{s+b}, \; \Re e\{s\} > -b$$

Overlap if $b > 0 \Rightarrow X(s) = \frac{-2b}{s^2 - b^2}$, with ROC:



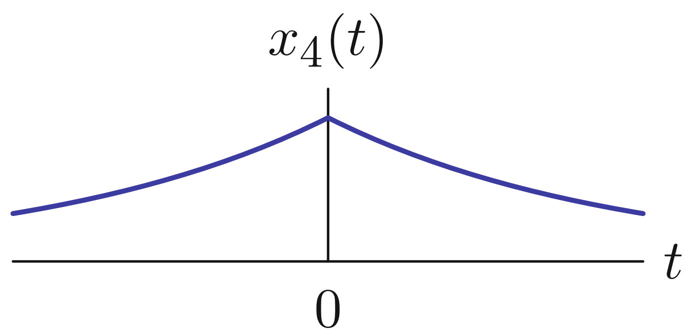

What if $b < 0$? ⇒ No overlap ⇒ No Laplace Transform

# تبدیل لاپلاس

## مثال

## LAPLACE TRANSFORM

$$x_4(t) = e^{-|t|}$$




$$X_4(s) = \int_{-\infty}^{\infty} e^{-|t|} e^{-st} dt$$

$$= \int_{-\infty}^{0} e^{(1-s)t} dt + \int_{0}^{\infty} e^{-(1+s)t} dt$$

$$= \left.\frac{e^{(1-s)t}}{(1-s)}\right|_{-\infty}^{0} + \left.\frac{e^{-(1+s)t}}{-(1+s)}\right|_{0}^{\infty}$$

$$= \underbrace{\frac{1}{1-s}}_{\mathrm{Re}(s)<1} + \underbrace{\frac{1}{1+s}}_{\mathrm{Re}(s)>-1}$$

$$= \frac{1+s+1-s}{(1-s)(1+s)} = \frac{2}{1-s^2}$$

$$; \quad -1 < \mathrm{Re}(s) < 1$$

The ROC is the intersection of $\mathrm{Re}(s) < 1$ and $\mathrm{Re}(s) > -1$.

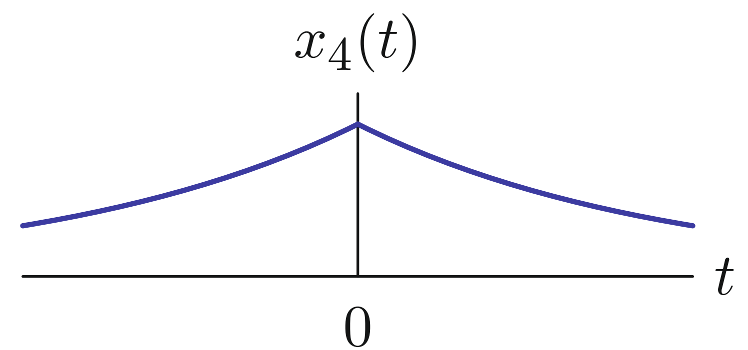
# تبدیل لاپلاس

مثال

## LAPLACE TRANSFORM

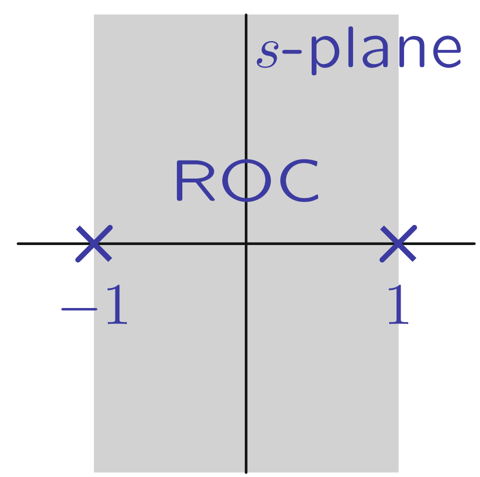
ناحیه‌ی همگرایی سیگنالی که دوسمتی باشد، یک نوار عمودی است.

$$x_4(t) = e^{-|t|}$$




$$X_4(s) = \frac{2}{1 - s^2}$$

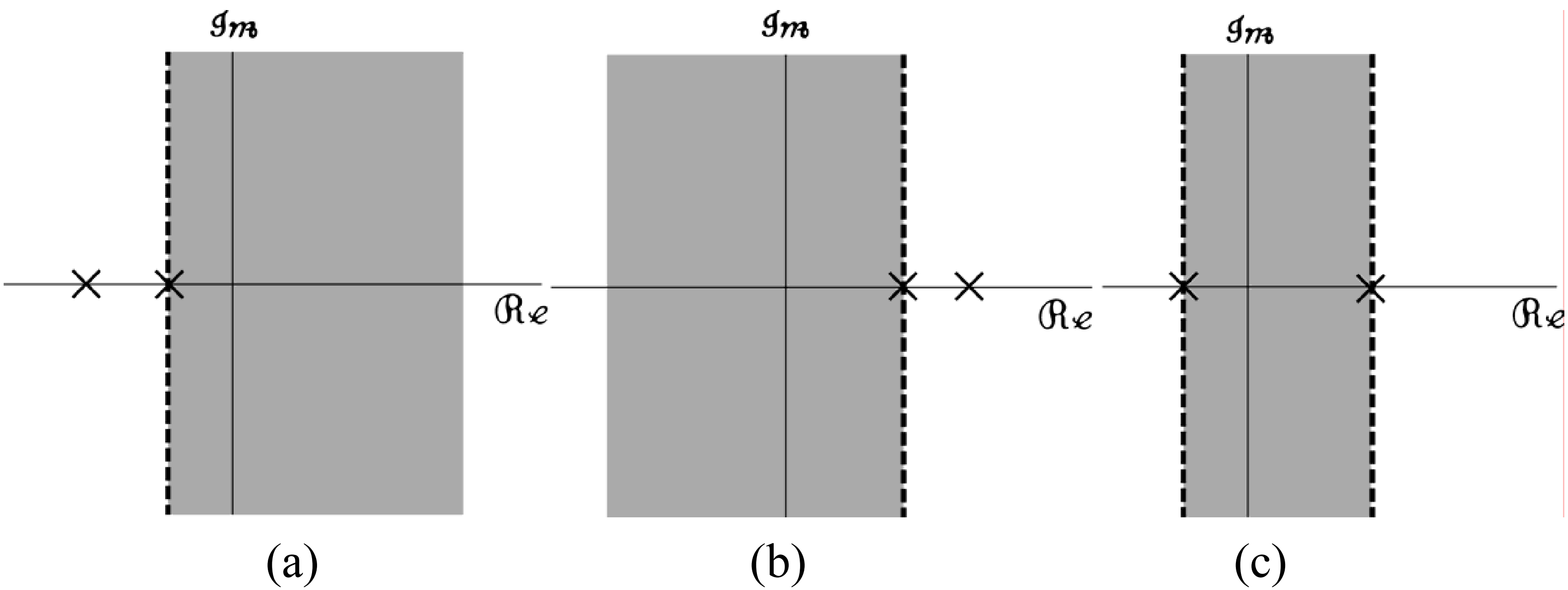
$$-1 < \mathrm{Re}(s) < 1$$



# Properties, Properties

7) If $X(s)$ is rational, then its ROC is bounded by poles or extends to infinity. In addition, no poles of $X(s)$ are contained in the ROC.

8) Suppose $X(s)$ is rational, then
   (a) If $x(t)$ is right-sided, the ROC is to the right of the rightmost pole.
   (b) If $x(t)$ is left-sided, the ROC is to the left of the leftmost pole.



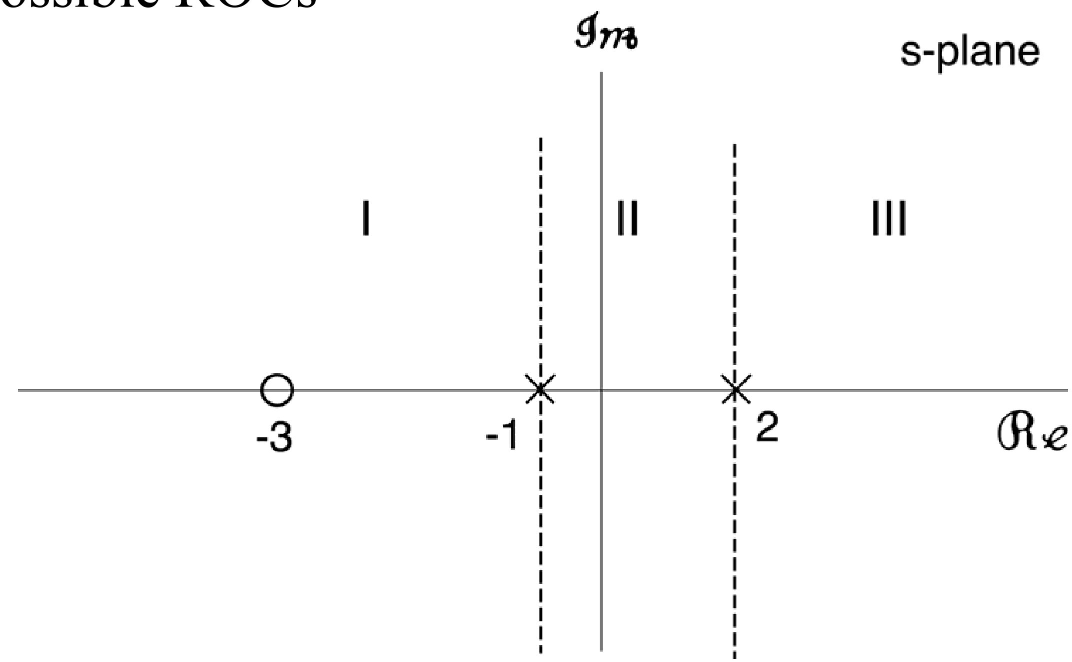

9) If ROC of $X(s)$ includes the $j\omega$-axis, then *FT* of $x(t)$ exists.

9) If ROC of $X(s)$ includes the $j\omega$-axis, then *FT* of $x(t)$ exists.

**Example:**

$$X(s) = \frac{(s+3)}{(s+1)(s-2)}$$

Three possible ROCs




| | ROC: | | Fourier Transform exists? |
|---|---|---|---|
| $x(t)$ is right-sided | ROC: | III | No |
| $x(t)$ is left-sided | ROC: | I | No |
| $x(t)$ extends for all time | ROC: | II | Yes |

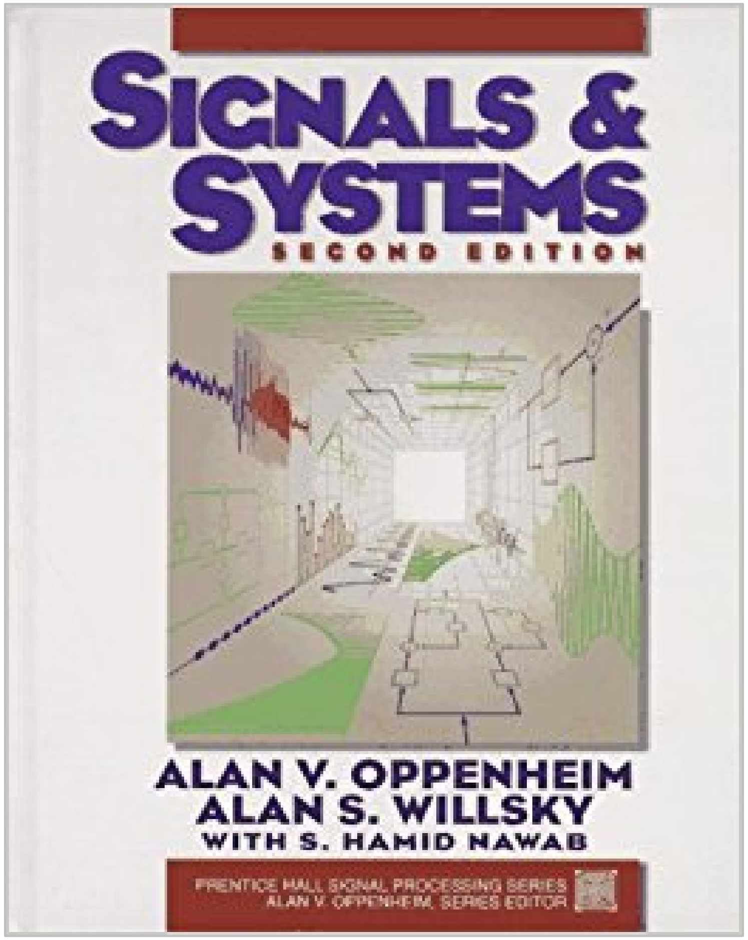
# سیگنال‌ها و سیستم‌ها

تبدیل لاپلاس (۱)

# ۵

# منابع

# منبع اصلی




A.V. Oppenheim, A.S. Willsky, S.H. Nawab,
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**Chapter 9**