سیگنال‌ها و سیستم‌ها

درس ۱۶

تبدیل فوریه‌ی گسسته–زمان (۲)

The Discrete-Time Fourier Transform (2)

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http://courses.fouladi.ir/sigsys

طرح درس

COURSE OUTLINE

خصوصیات تبدیل فوریه‌ی گسسته–زمان و مثال‌ها
DTFT Properties and Examples
دوگانی در سری فوریه و تبدیل فوریه
Duality in FS & FT
روابط میان بازنمایی‌های فوریه
Relations among Fourier Representations

سیگنال‌ها و سیستم‌ها

تبدیل فوریه‌ی گسسته-زمان (۲)

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خصوصیات تبدیل فوریه‌ی گسسته-زمان و مثال‌ها

Convolution Property Example

$$h[n] = \alpha^n u[n], \quad x[n] = \beta^n u[n] \qquad |\alpha|, |\beta| < 1$$

$$H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}, \quad X(e^{j\omega}) = \frac{1}{1 - \beta e^{-j\omega}}$$

$$\Downarrow$$

$$y[n] = h[n] * x[n] \longleftrightarrow Y(e^{j\omega}) = \left(\frac{1}{1 - \alpha e^{-j\omega}}\right)\left(\frac{1}{1 - \beta e^{-j\omega}}\right)$$

- ratio of polynomials in $e^{-j\omega}$

$$\beta \neq \alpha : \; Y(e^{j\omega}) \overset{\text{PFE}}{=} \frac{A}{1 - \alpha e^{-j\omega}} + \frac{B}{1 - \beta e^{-j\omega}}$$

A, B - determined by partial fraction expansion

$$y[n] = A\alpha^n u[n] + B\beta^n u[n]$$

$$\beta = \alpha : \; Y(e^{j\omega}) = \left(\frac{1}{1 - \alpha e^{-j\omega}}\right)^2$$

$$nx[n] \leftrightarrow j\frac{dX(e^{j\omega})}{d\omega}$$

$$y[n] = (n+1)\alpha^n u[n]$$

DT LTI System Described by LCCDE's

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

From time-shifting property: $\quad x[n-k] \longleftrightarrow e^{-j\omega k} X(e^{j\omega})$

$$\Downarrow$$

$$\sum_{k=0}^{N} a_k e^{-jk\omega} Y(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} X(e^{j\omega})$$

$$\Downarrow$$

$$Y(e^{j\omega}) = \underbrace{\left[\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}} \right]}_{H(e^{j\omega})} X(e^{j\omega})$$

— Rational function of $e^{-j\omega}$, use PFE to get $h[n]$

Example: First-order recursive system

$$y[n] - \alpha y[n-1] = x[n], \quad |\alpha| < 1$$

with the condition of initial rest ⇔ *causal*

$$(1 - \alpha e^{-j\omega})Y(e^{j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \alpha e^{-j\omega}}$$

$$\Downarrow$$

$$h[n] = \alpha^n u[n]$$

DTFT Multiplication Property

$$y[n] = x_1[n] \cdot x_2[n] \longleftrightarrow Y(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$$

$$= \frac{1}{2\pi} X_1(e^{j\omega}) \otimes X_2(e^{j\omega})$$

$\hookrightarrow$ Periodic Convolution

Derivation:

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_1[n] \cdot x_2[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta}) e^{j\theta n} d\theta \right) x_2[n] e^{-j\omega n}$$

$$= \frac{1}{2\pi}\int_{2\pi} (X_1(e^{j\theta}) \underbrace{\sum_{n=-\infty}^{\infty} x_2[n] e^{-j(\omega-\theta)n}}_{X_2(e^{j(\omega-\theta)})}) d\theta$$

$$= \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) d\theta$$

Calculating Periodic Convolutions

Suppose we integrate from $-\pi$ to π:

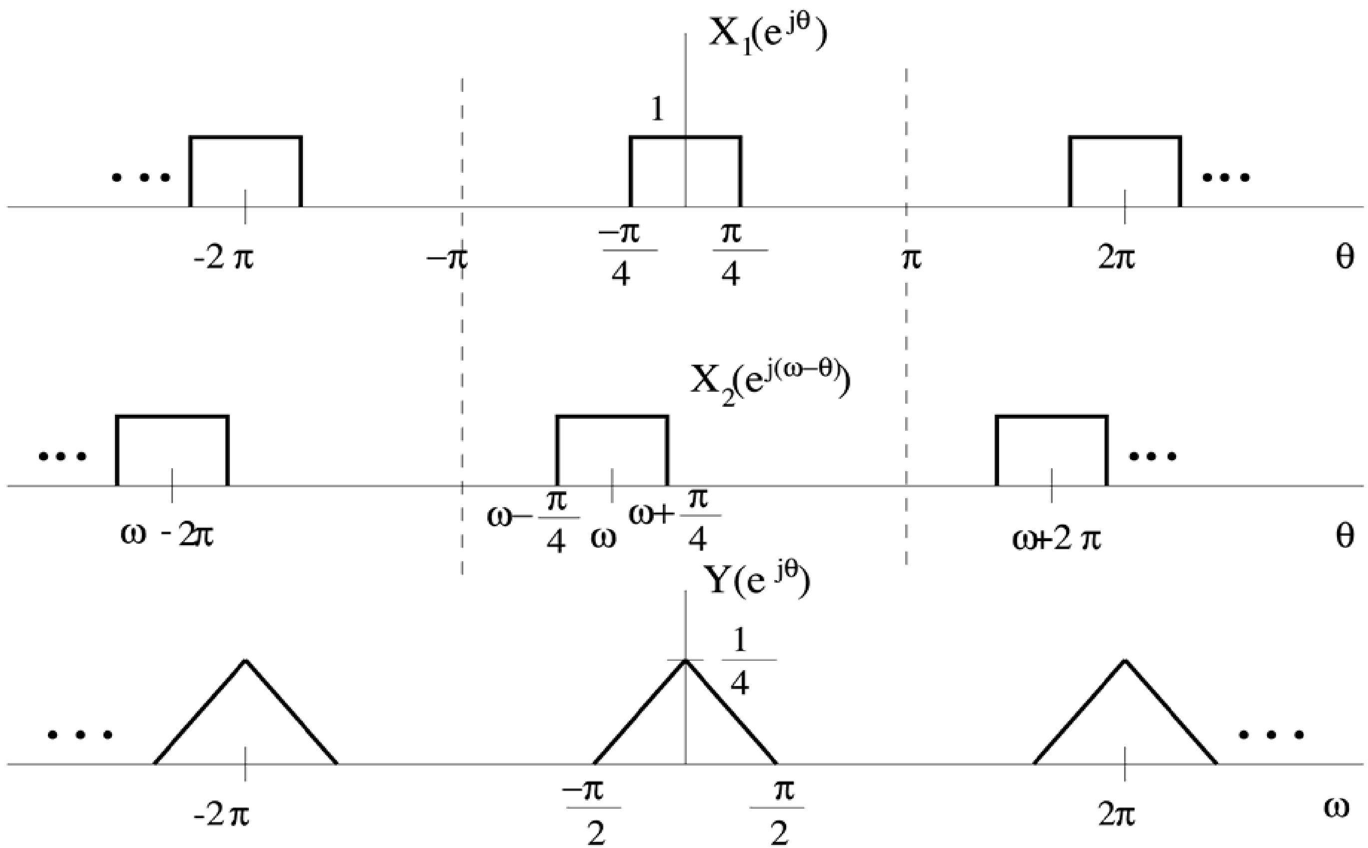
$$
\begin{aligned}
Y(e^{j\omega}) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta
\end{aligned}
$$

where

$$
\hat{X}_1(e^{j\theta}) = \begin{cases} X_1(e^{j\theta}), & |\theta| \leq \pi \\ 0, & \text{otherwise} \end{cases}
$$

Example: $y[n] = \left(\frac{\sin(\pi n/4)}{\pi n}\right)^2 = x_1[n] \cdot x_2[n], \quad x_1[n] = x_2[n] = \frac{\sin(\pi n/4)}{\pi n}$

$$Y(e^{j\omega}) = \frac{1}{2\pi} X_1(e^{j\omega}) \otimes X_2(e^{j\omega})$$



تبدیل فوریه‌ی گسسته–زمان (۲)

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دوگانی در سری فوریه و تبدیل فوریه

Duality in Fourier Analysis

Fourier Transform is highly symmetric

CTFT: *Both* time and frequency are continuous and in general *aperiodic*

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Same except for these differences

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\Downarrow$$

Suppose *f*(•) and *g*(•) are two functions related by

$$f(r) = \int_{-\infty}^{\infty} g(\tau)e^{-jr\tau}\,d\tau$$

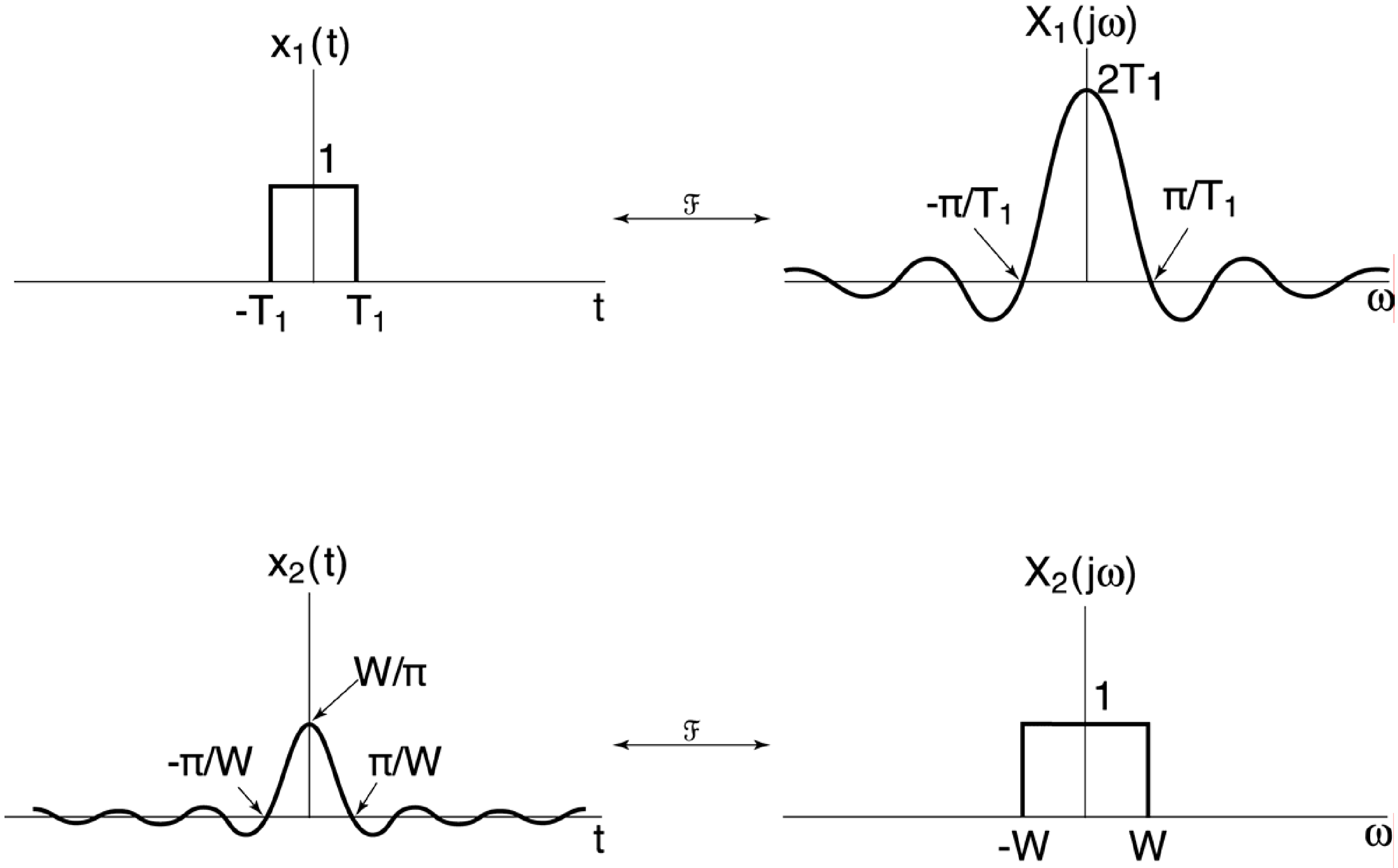
Then

Let $\tau = t$ and $r = \omega$: $x_1(t) = g(t) \longleftrightarrow X_1(j\omega) = f(\omega)$

Let $\tau = -\omega$ and $r = t$: $x_2(t) = f(t) \longleftrightarrow X_2(j\omega) = 2\pi g(-\omega)$

Example of CTFT duality

Square pulse in either time or frequency domain



DTFS

$$\textit{Discrete \& periodic} \text{ in time} \longleftrightarrow \textit{Periodic \& discrete} \text{ in frequency}$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = x[n+N], \quad \omega_0 = \frac{2\pi}{N}$$

$$a_k = \frac{1}{N} \sum_{k=<N>} x[n] e^{-jk\omega_0 n} = a_{k+N}$$

Duality in DTFS

Suppose $f[\cdot]$ and $g[\cdot]$ are two functions related by

$$f[m] = \frac{1}{N} \sum_{r=<N>} g[r] e^{-jr\omega_0 m}$$

$$\Rightarrow g[r] = \sum_{m=<N>} f[m] e^{jr\omega_0 m}$$

Then

Let $m = n$ and $r = -k$: $\qquad x_1[n] = f[n] \longleftrightarrow a_k = \frac{1}{N} g[-k]$

Let $r = n$ and $m = k$: $\qquad x_2[n] = g[n] \longleftrightarrow a_k = f[k]$

Duality between CTFS and DTFT

CTFS

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = x(t+T), \quad \omega_0 = \frac{2\pi}{T}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

Periodic in time $\leftrightarrow$ *Discrete* in frequency

DTFT

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = X\left(e^{j(\omega+2\pi)}\right)$$

Discrete in time $\leftrightarrow$ *Periodic* in frequency

CTFS-DTFT Duality

Suppose $f(\cdot)$ is a CT signal and $g[\cdot]$ a DT sequence related by

$$f(\tau) = \sum_{m=-\infty}^{+\infty} g[m] e^{jm\tau} = f(\tau + 2\pi)$$

Then

$$x(t) = f(t) \longleftrightarrow a_k = g[k]$$

(periodic with period 2π)

$$x[n] = g[n] \longleftrightarrow X(e^{j\omega}) = f(-\omega)$$

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روابط میان بازنمایی‌های فوریه

چهار بازنمایی فوریه

Four Fourier Representations

We have discussed four closely related Fourier representations.

DT Fourier Series

$$a_k = a_{k+N} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = x[n+N] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$$

DT Fourier transform

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(e^{j\Omega}) e^{j\Omega n} d\Omega$$

CT Fourier Series

$$a_k = \frac{1}{T} \int_T x(t) e^{-j\frac{2\pi}{T}kt} dt$$

$$x(t) = x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

CT Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

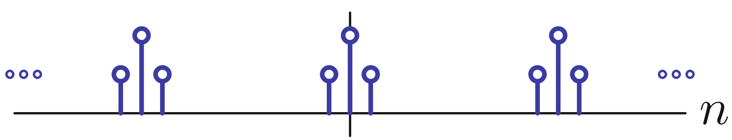
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

چهار نوع «زمان»

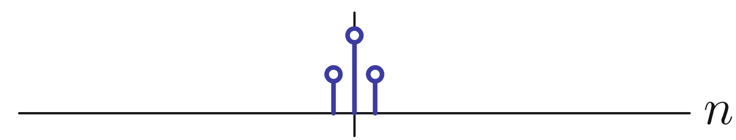
Four Types of "Time"

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

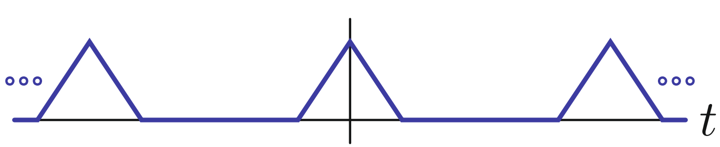
DT Fourier Series



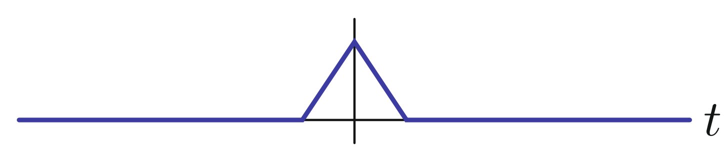
DT Fourier transform



CT Fourier Series



CT Fourier transform



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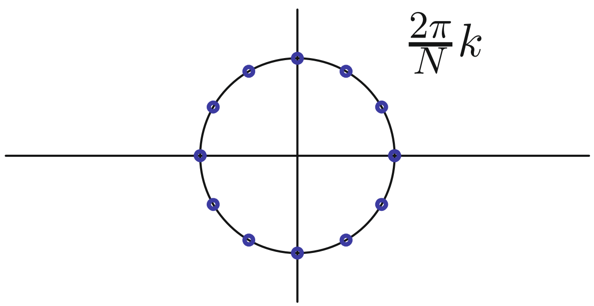
چهار نوع «فرکانس»

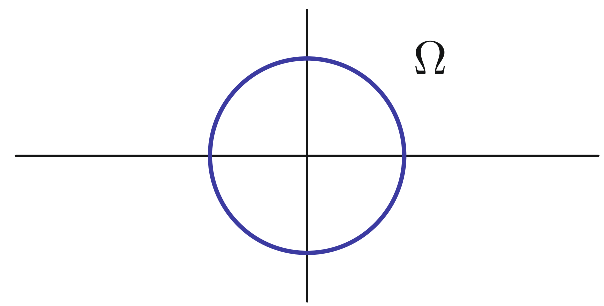
Four Types of "Frequency"

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

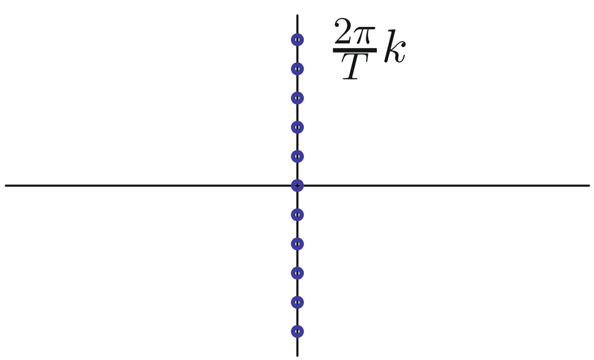
DT Fourier Series



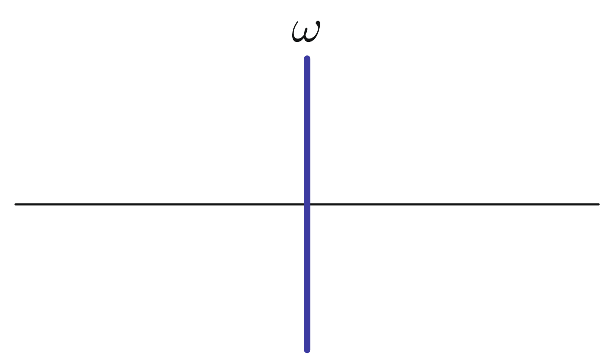
DT Fourier transform



CT Fourier Series



CT Fourier transform

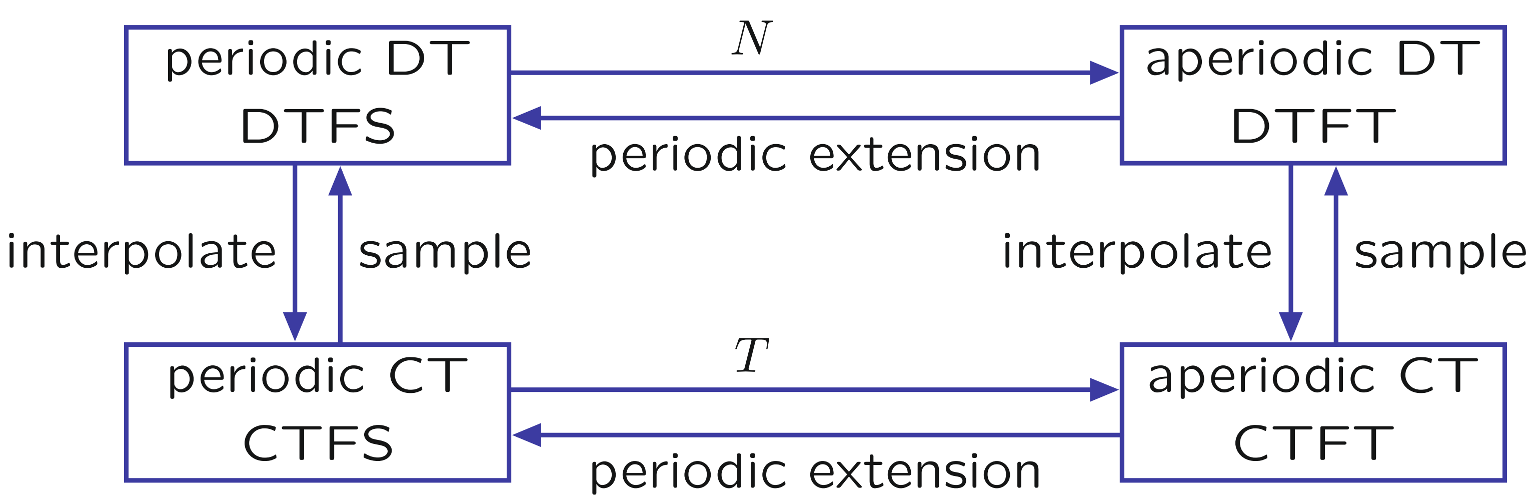


روابط میان بازنمایی‌های فوریه

Relations among Fourier Representations

Different Fourier representations are related because they apply to signals that are related.

DTFS (discrete-time Fourier series):	periodic DT
DTFT (discrete-time Fourier transform):	aperiodic DT
CTFS (continuous-time Fourier series):	periodic CT
CTFT (continuous-time Fourier transform):	aperiodic CT



روابط میان بازنمایی‌های فوریه

رابطه‌ی میان سری فوریه و تبدیل فوریه

Relations among Fourier Representations

A periodic signal can be represented by a Fourier series or by an equivalent Fourier transform.

Series: represent periodic signal as weighted sum of harmonics

$$x(t) = x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} \; ; \qquad \omega_0 = \frac{2\pi}{T}$$

The Fourier transform of a sum is the sum of the Fourier transforms:

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Therefore periodic signals can be equivalently represented as Fourier transforms (with impulses!).

روابط میان بازنمایی‌های فوریه

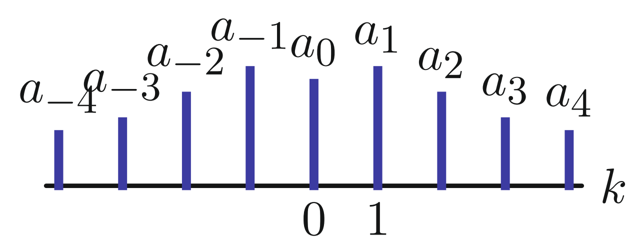
رابطه‌ی میان سری فوریه و تبدیل فوریه

Relations among Fourier Representations

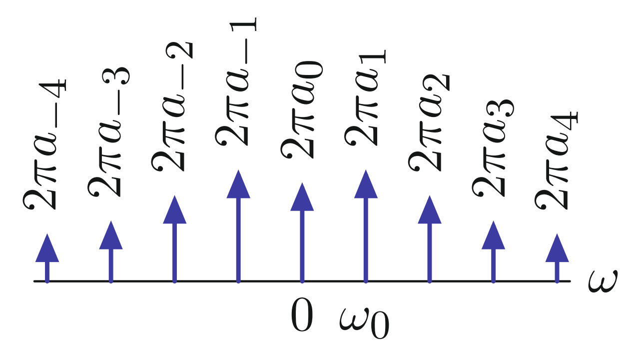
A periodic signal can be represented by a Fourier series or by an equivalent Fourier transform.

$$x(t) = x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Fourier Series



Fourier Transform



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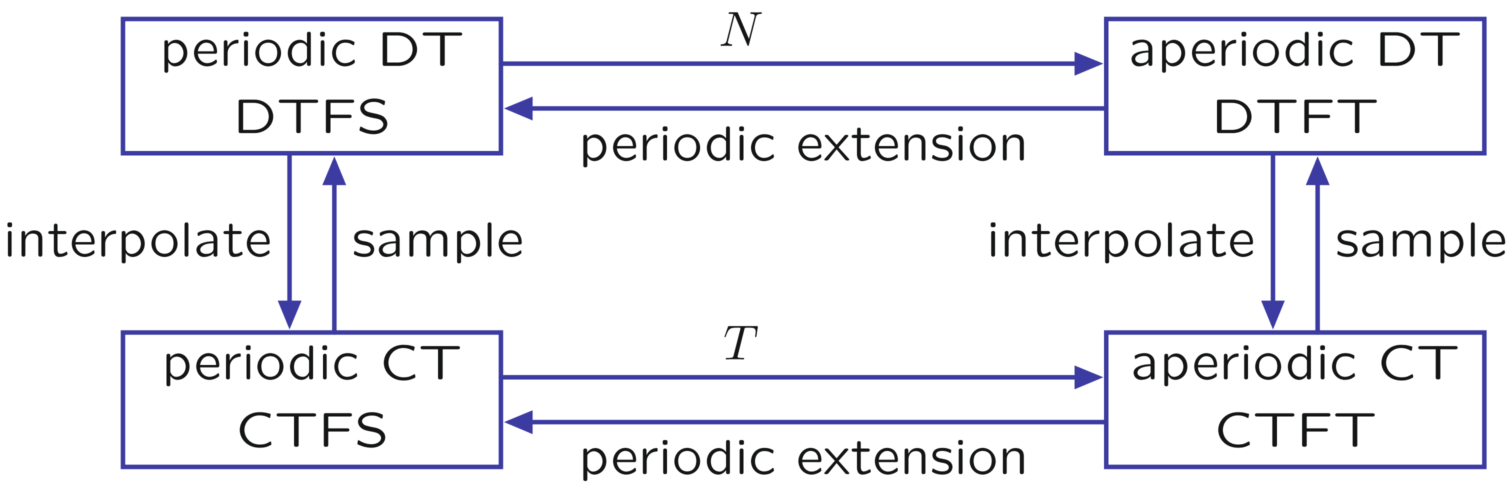
روابط میان بازنمایی‌های فوریه

Relations among Fourier Representations

Explore other relations among Fourier representations.

Start with an aperiodic CT signal. Determine its Fourier transform.

Convert the signal so that it can be represented by alternate Fourier representations and compare.

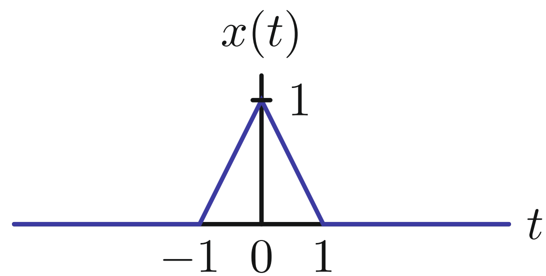


روابط میان بازنمایی‌های فوریه

شروع با تبدیل فوریه‌ی پیوسته–زمان

START WITH THE CT FOURIER TRANSFORM

Determine the Fourier transform of the following signal.



Could calculate Fourier transform from the definition.

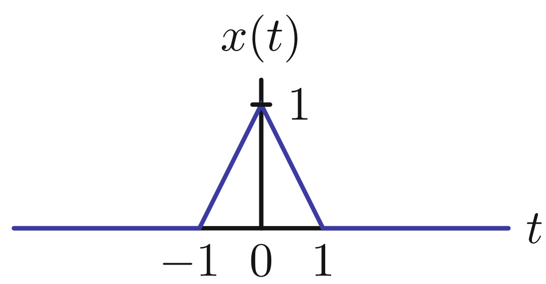
$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$$

روابط میان بازنمایی‌های فوریه

شروع با تبدیل فوریه‌ی پیوسته–زمان

START WITH THE CT FOURIER TRANSFORM

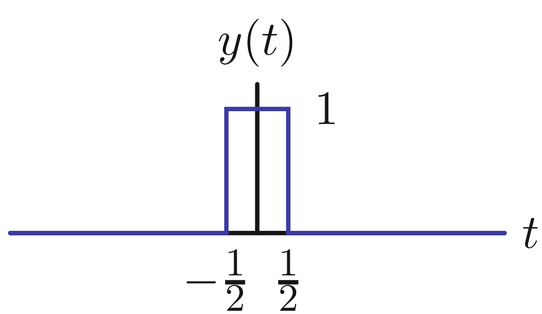
Determine the Fourier transform of the following signal.

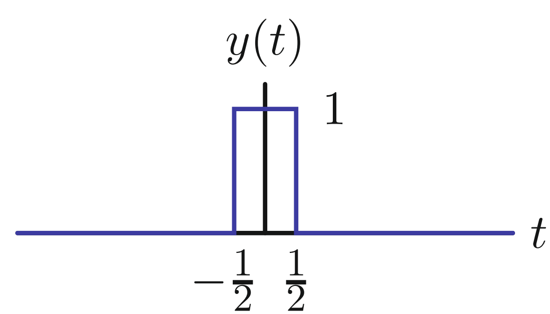


Could calculate Fourier transform from the definition.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$$

Easier to calculate $x(t)$ by convolution of two square pulses:



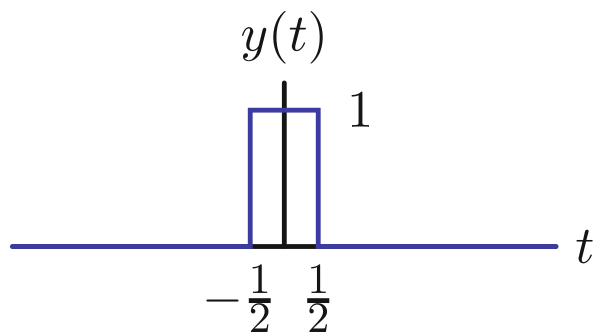


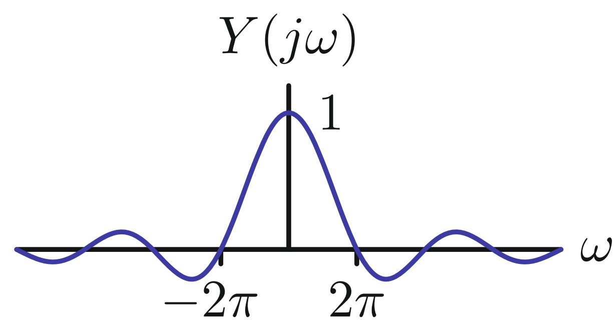
روابط میان بازنمایی‌های فوریه

شروع با تبدیل فوریه‌ی پیوسته–زمان

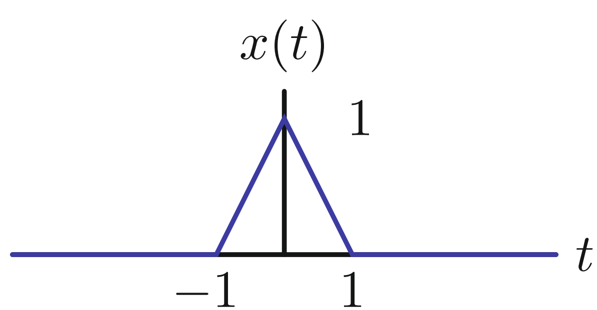
Start with the CT Fourier Transform

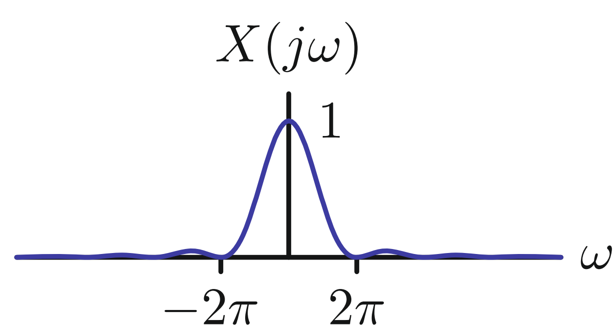
The transform of $y(t)$ is $\frac{2\sin(\omega/2)}{\omega}$





so the transform of $x(t) = (y * y)(t)$ is $X(j\omega) = Y(j\omega) \times Y(j\omega)$.





روابط میان بازنمایی‌های فوریه

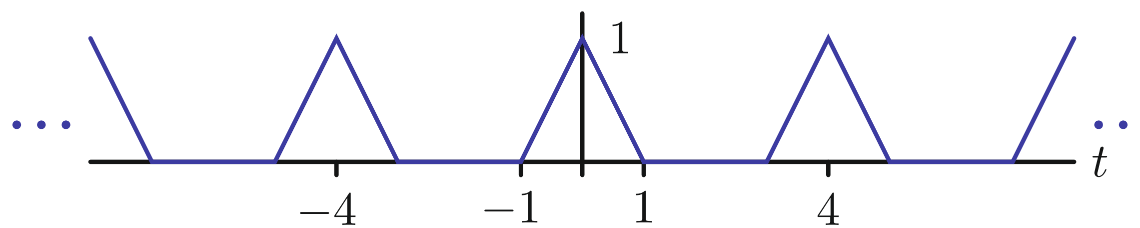
رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

What is the effect of making a signal periodic in time?

Find Fourier transform of periodic extension of $x(t)$ to period $T = 4$.

$$z(t) = \sum_{k=-\infty}^{\infty} x(t + 4k)$$



Could calculate $Z(j\omega)$ for the definition ... ugly.

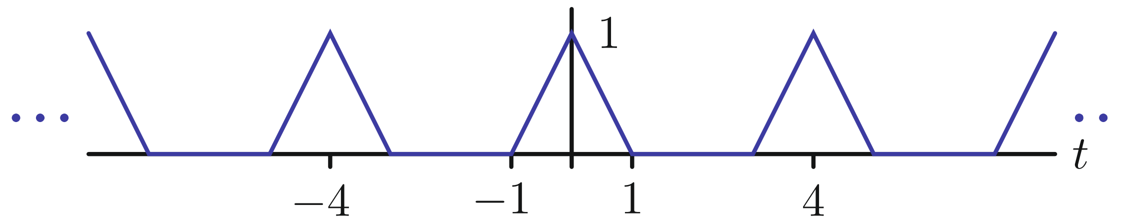
روابط میان بازنمایی‌های فوریه

رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

Easier to calculate $z(t)$ by convolving $x(t)$ with an impulse train.

$$z(t) = \sum_{k=-\infty}^{\infty} x(t+4k)$$



$$z(t) = \sum_{k=-\infty}^{\infty} x(t+4k) = (x * p)(t)$$

where

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t+4k)$$

Then

$$Z(j\omega) = X(j\omega) \times P(j\omega)$$

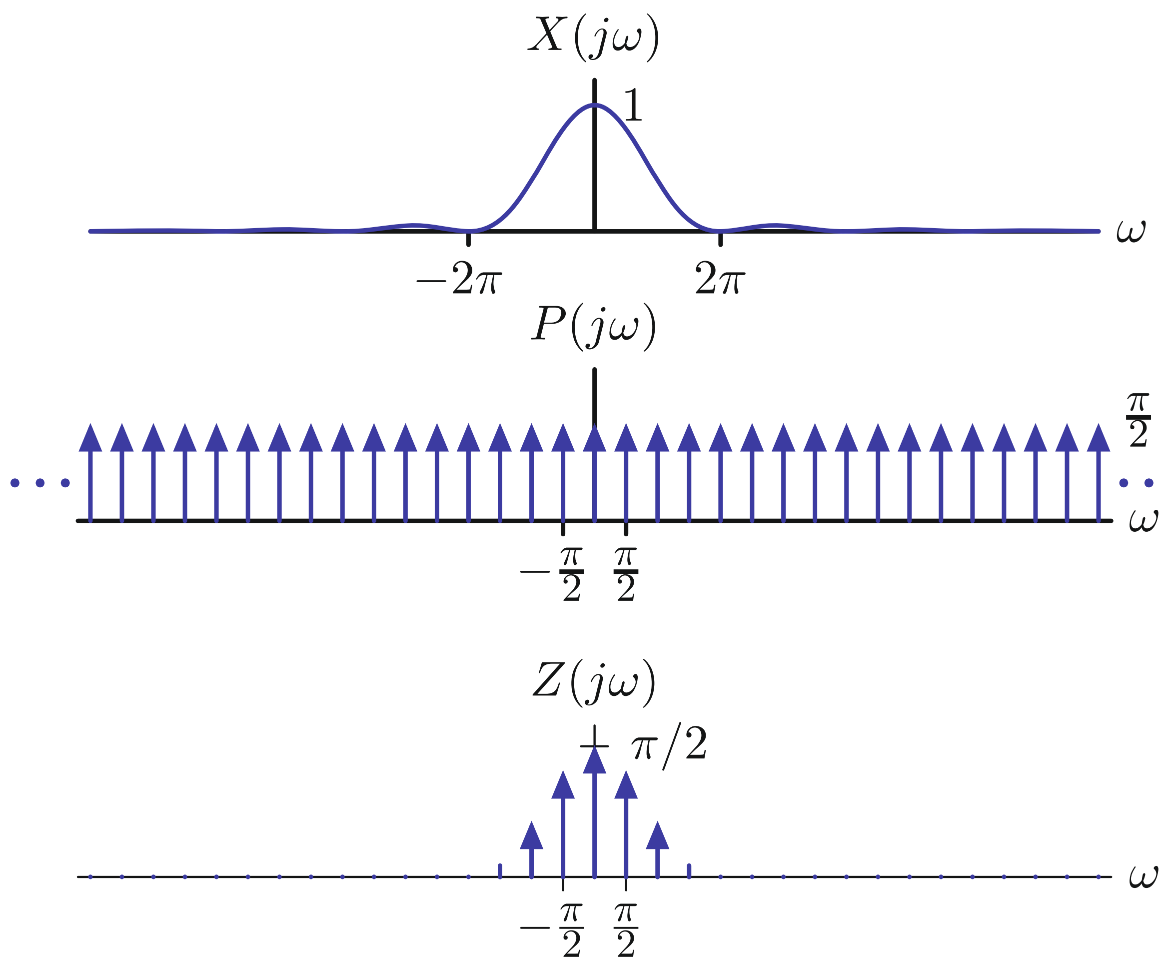
We already know $P(j\omega)$: it's also an impulse train!

روابط میان بازنمایی‌های فوریه

رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

Convolving in time corresponds to multiplying in frequency.



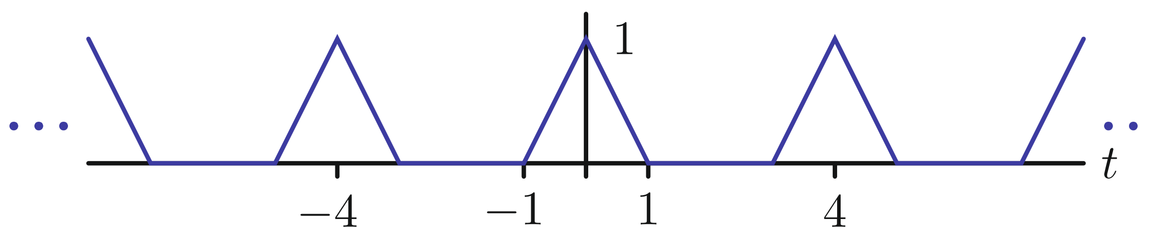
روابط میان بازنمایی‌های فوریه

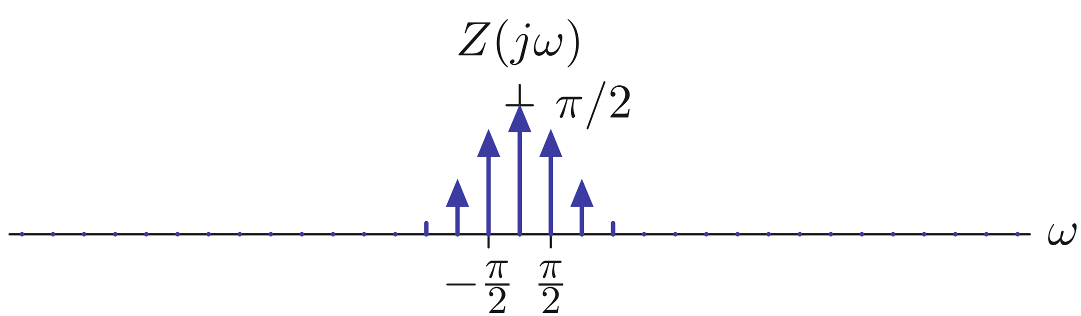
رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

The Fourier transform of a periodically extended function is a discrete function of frequency ω.

$$z(t) = \sum_{k=-\infty}^{\infty} x(t+4k)$$



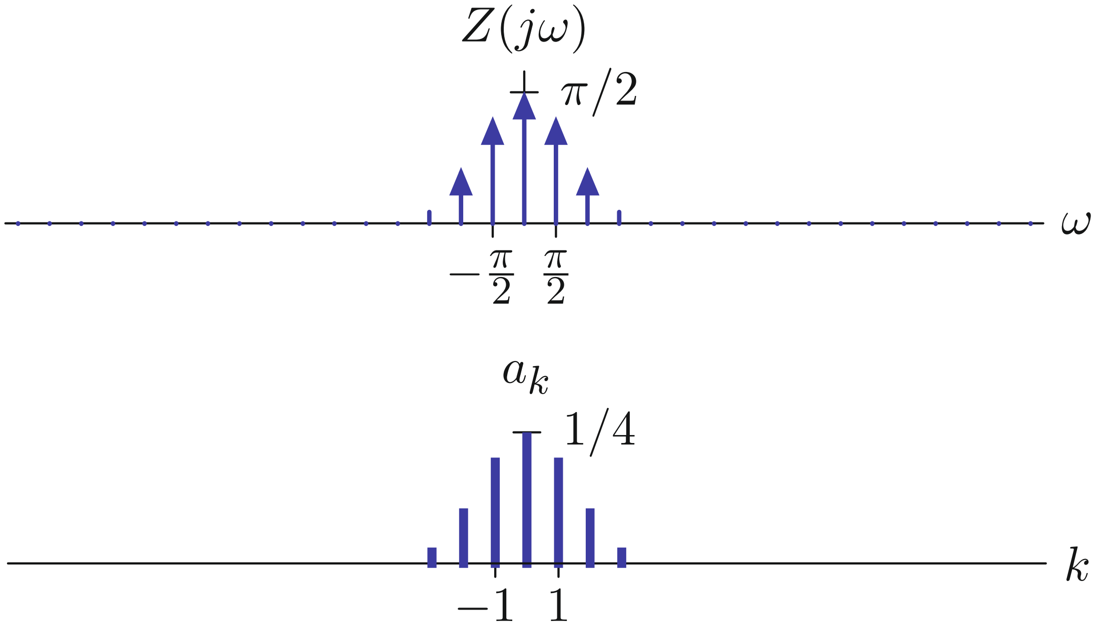


روابط میان بازنمایی‌های فوریه

رابطه‌ی بین تبدیل فوریه و سری فوریه

RELATION BETWEEN FOURIER TRANSFORM AND SERIES

The weight (area) of each impulse in the Fourier transform of a periodically extended function is 2π times the corresponding Fourier series coefficient.

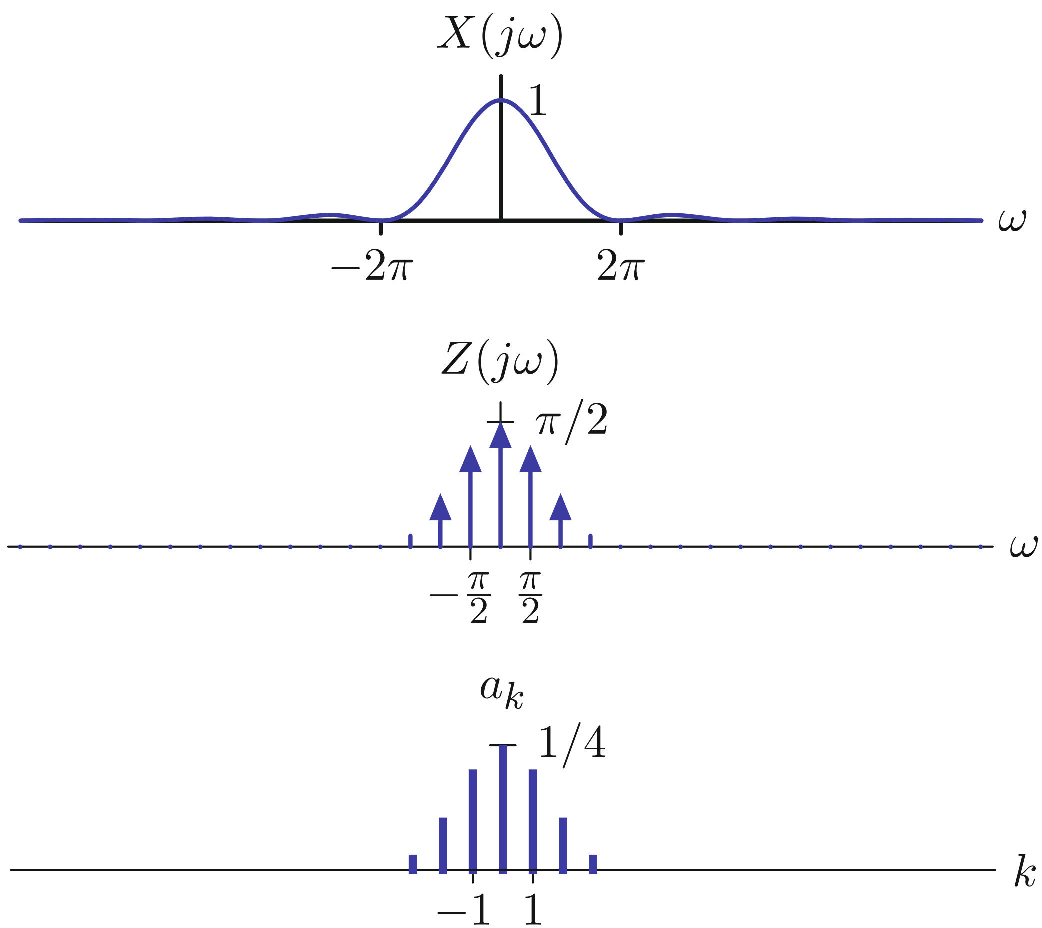


روابط میان بازنمایی‌های فوریه

رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

The effect of periodic extension of $x(t)$ to $z(t)$ is to sample the frequency representation.



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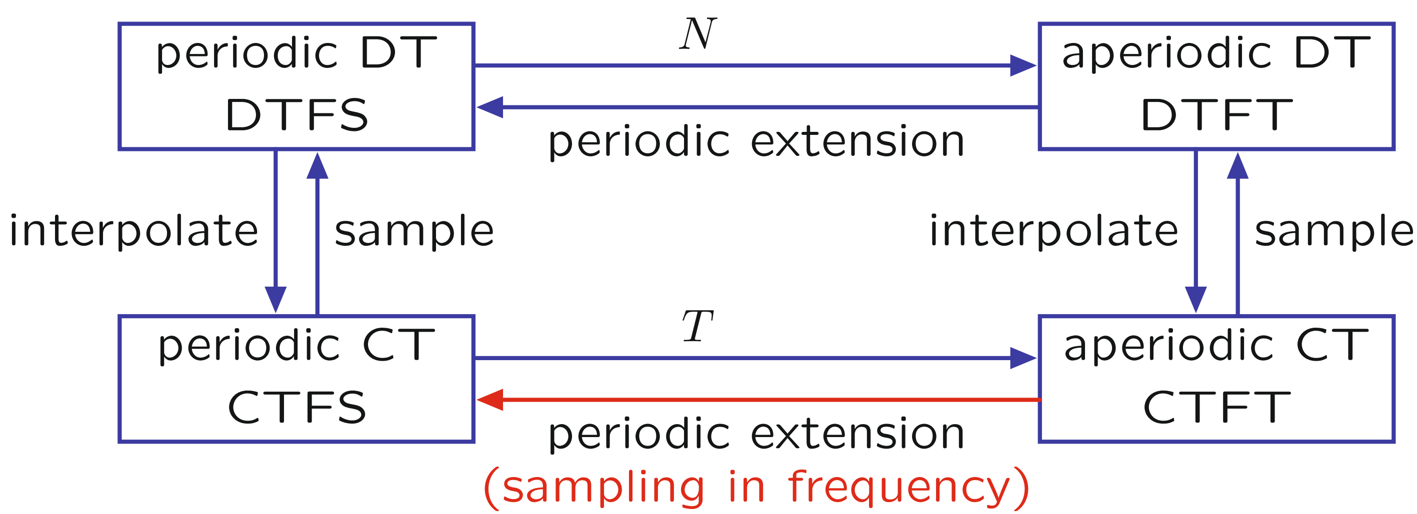
روابط میان بازنمایی‌های فوریه

رابطه‌ی بین تبدیل فوریه و سری فوریه

Relation between Fourier Transform and Series

Periodic extension of a CT signal produces a discrete function of frequency.

Periodic extension
  = convolving with impulse train in time
    = multiplying by impulse train in frequency
      sampling in frequency

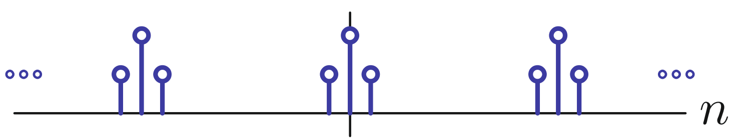


چهار نوع «زمان»

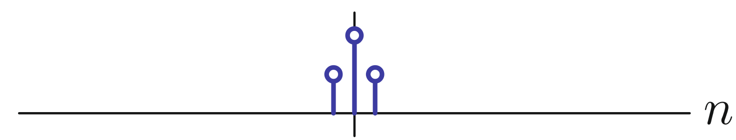
FOUR TYPES OF “TIME”

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

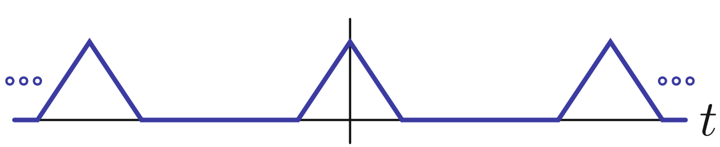
DT Fourier Series



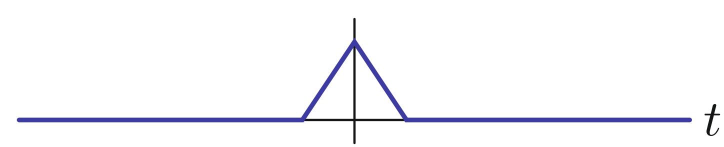
DT Fourier transform



CT Fourier Series



CT Fourier transform

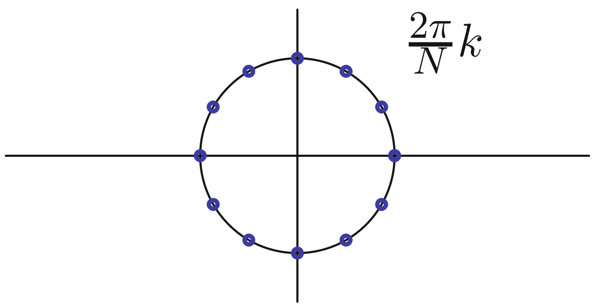


چهار نوع «فرکانس»

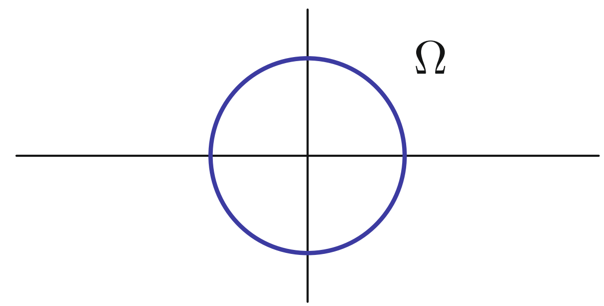
Four Types of “Frequency”

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

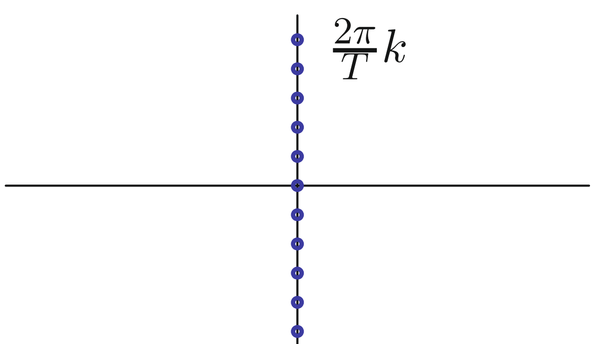
DT Fourier Series



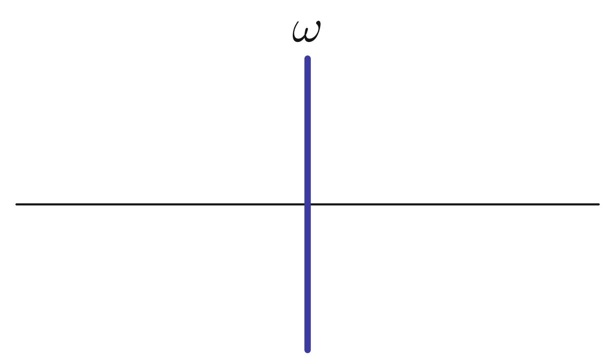
DT Fourier transform



CT Fourier Series



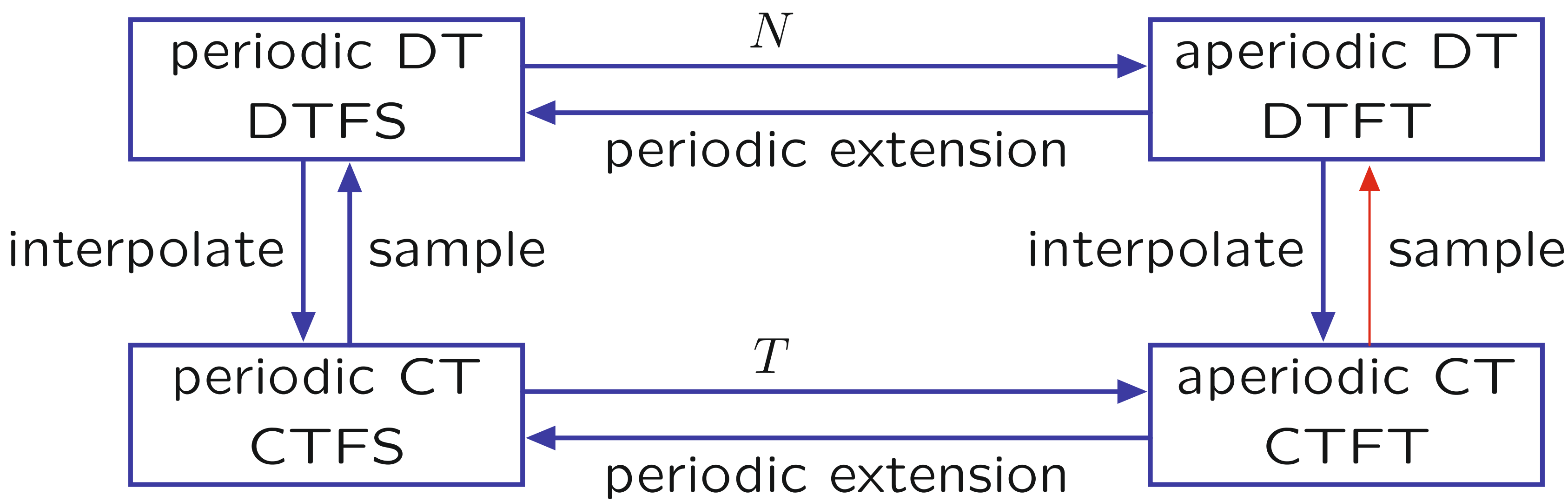
CT Fourier transform



روابط میان بازنمایی‌های فوریه

Relations among Fourier Representations

Compare to sampling in time.



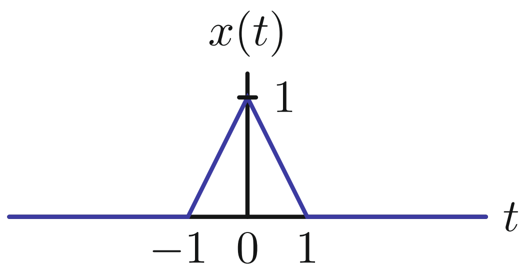
روابط میان بازنمایی‌های فوریه

روابط میان تبدیل‌های پیوسته–زمان و گسسته– زمان

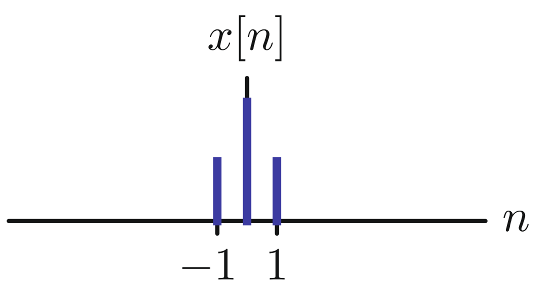
Relations between CT and DT transforms

Sampling a CT signal generates a DT signal.

$$x[n] = x(nT)$$



Take $T = \frac{1}{2}$.



What is the effect on the frequency representation?

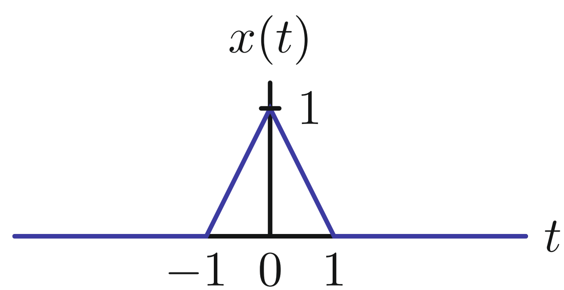
روابط میان بازنمایی‌های فوریه

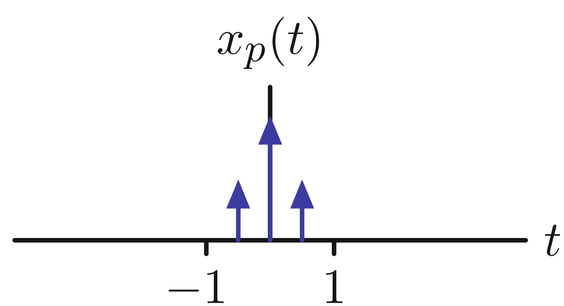
روابط میان تبدیل‌های پیوسته–زمان و گسسته–زمان

Relations between CT and DT transforms

We can generate a signal with the same shape by multiplying $x(t)$ by an impulse train with $T = \frac{1}{2}$.

$$x_p(t) = x(t) \times p(t) \quad \text{where} \quad p(t) = \sum_{k=-\infty}^{\infty} \delta(t + kT)$$



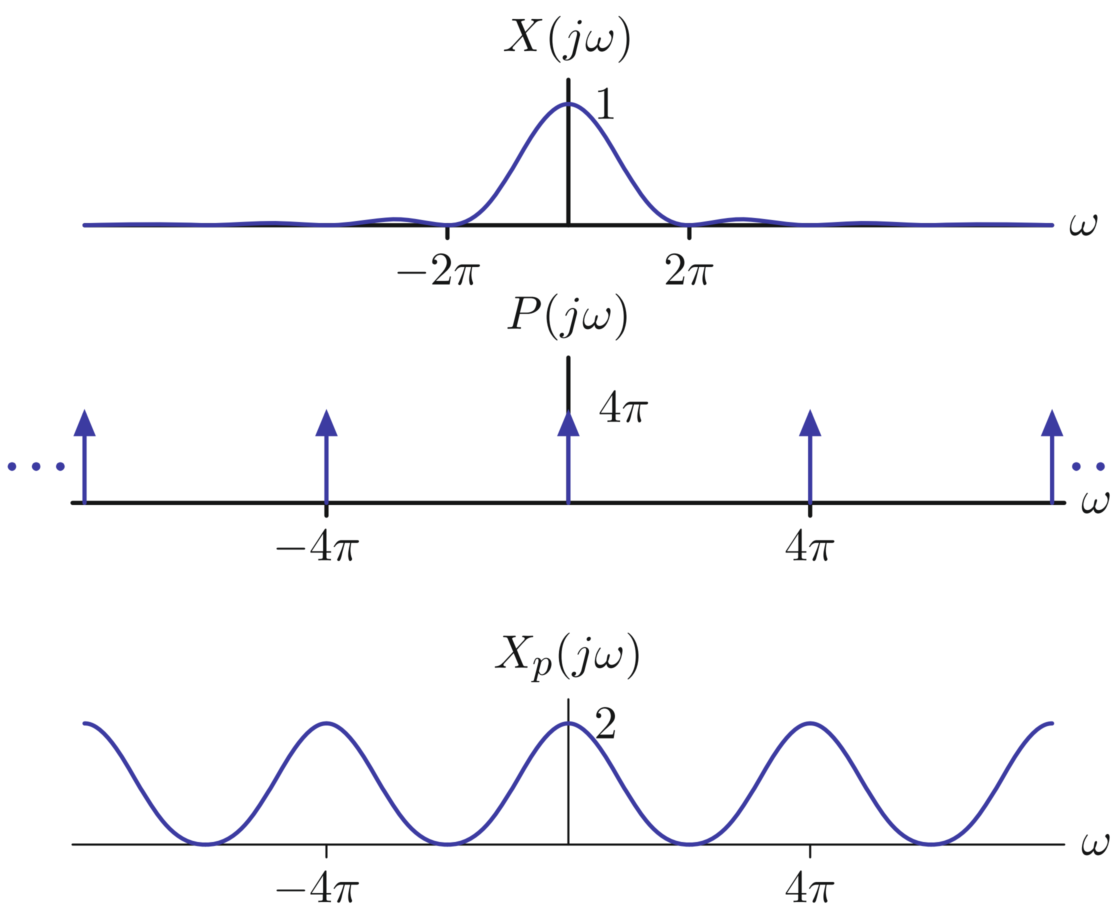


روابط میان بازنمایی‌های فوریه

روابط میان تبدیل‌های پیوسته–زمان و گسسته– زمان

RELATIONS BETWEEN CT AND DT TRANSFORMS

Multiplying $x(t)$ by an impulse train in time is equivalent to convolving $X(j\omega)$ by an impulse train in frequency (then $\div 2\pi$).

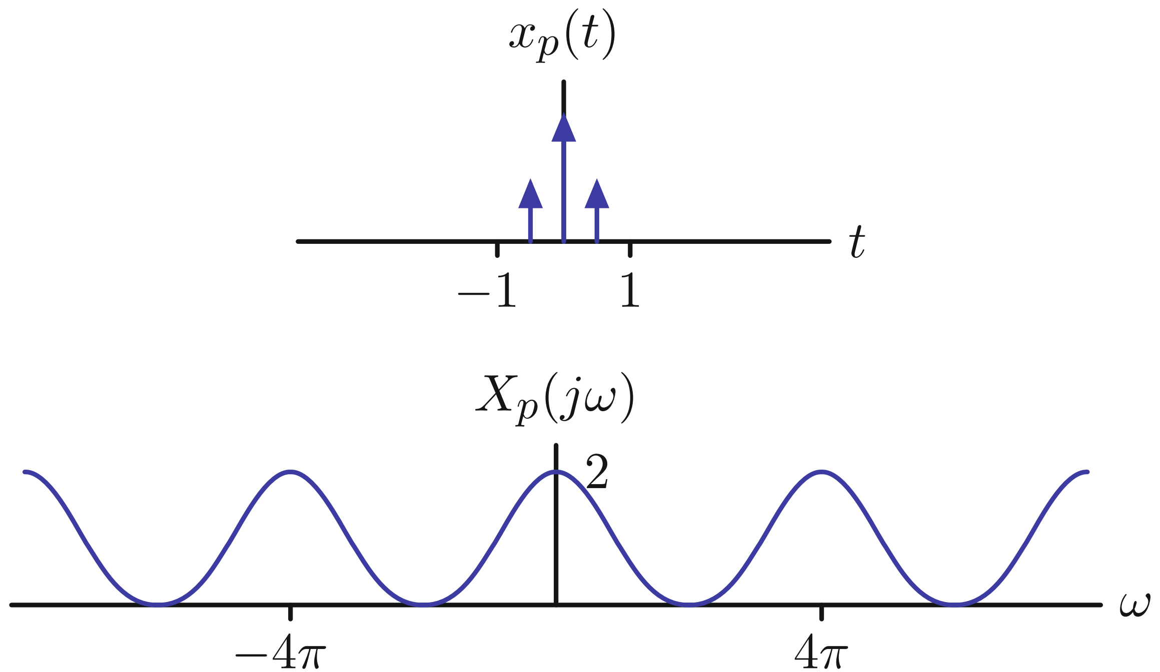


روابط میان بازنمایی‌های فوریه

روابط میان تبدیل‌های پیوسته–زمان و گسسته– زمان

RELATIONS BETWEEN CT AND DT TRANSFORMS

The Fourier transform of the “sampled” signal $x_p(t)$ is periodic in ω with period 4π.

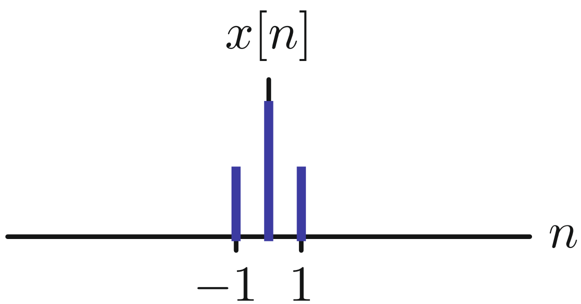


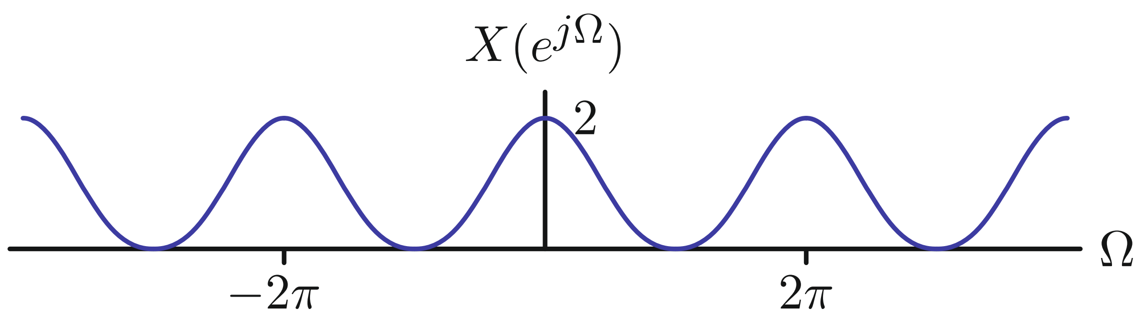
روابط میان بازنمایی‌های فوریه

روابط میان تبدیل‌های پیوسته–زمان و گسسته– زمان

RELATIONS BETWEEN CT AND DT TRANSFORMS

The Fourier transform of the “sampled” signal $x_p(t)$ has the same shape as the DT Fourier transform of $x[n]$.



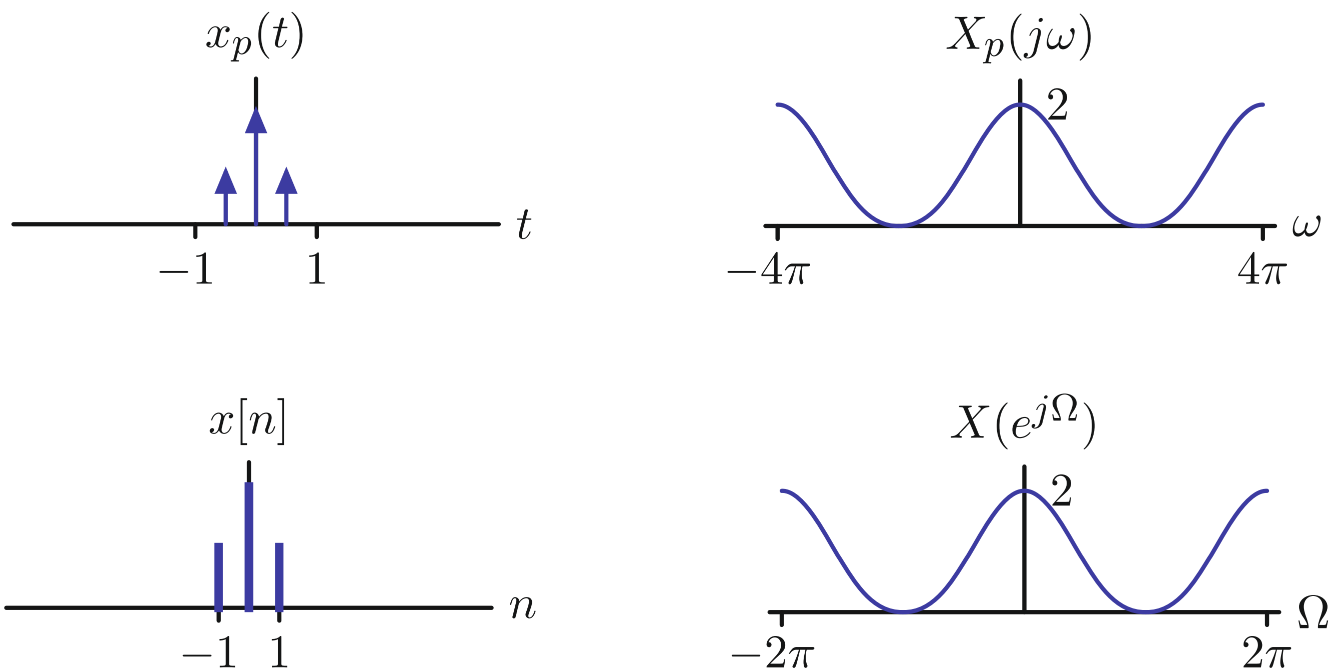


روابط میان بازنمایی‌های فوریه

تبدیل فوریه‌ی گسسته– زمان

DT Fourier transform

The CT Fourier transform of a “sampled” signal ($x_p(t)$) is equal to the DT Fourier transform of the samples ($x[n]$) where $\Omega = \omega T$, i.e., $X(j\omega) = X(e^{j\Omega})\Big|_{\Omega=\omega T}$.



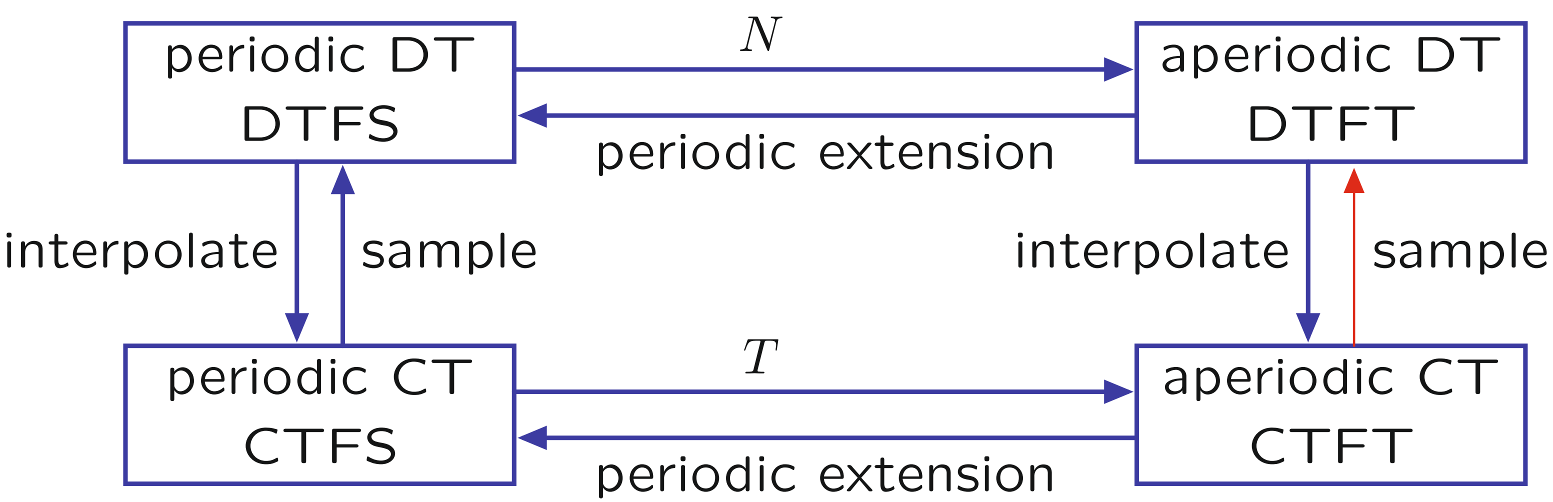
$\Omega = \omega T = \frac{1}{2}\omega$

روابط میان بازنمایی‌های فوریه

روابط میان تبدیل‌های پیوسته–زمان و گسسته– زمان

RELATION BETWEEN CT AND DT FOURIER TRANSFORMS

The CT Fourier transform of a "sampled" signal ($x_p(t)$) is equal to the DT Fourier transform of the samples ($x[n]$) where $\Omega = \omega T$, i.e., $X(j\omega) = X(e^{j\Omega})\Big|_{\Omega=\omega T}$.

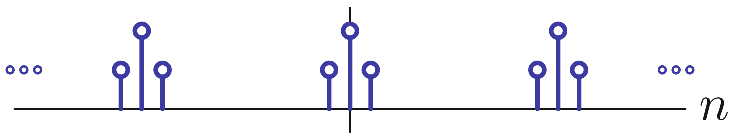


چهار نوع «زمان»

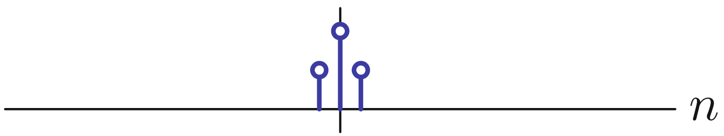
FOUR TYPES OF "TIME"

discrete vs. continuous ($\updownarrow$) and periodic vs aperiodic ($\leftrightarrow$)

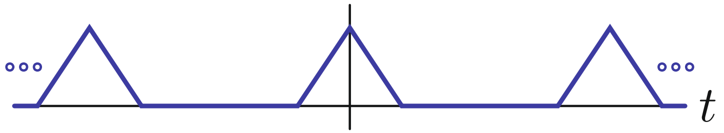
DT Fourier Series



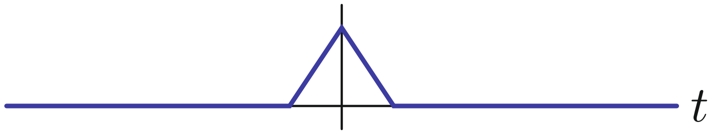
DT Fourier transform



CT Fourier Series



CT Fourier transform



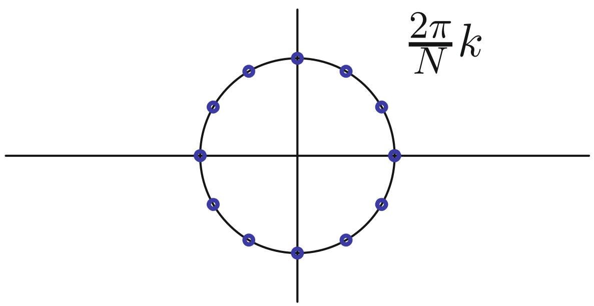
Prepared by Kazim Fouladi | Spring 2024 | 6th Edition

چهار نوع «فرکانس»

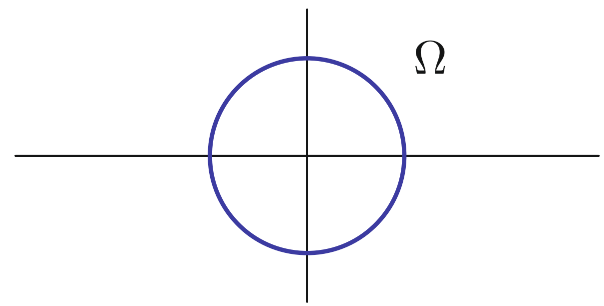
Four Types of "Frequency"

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

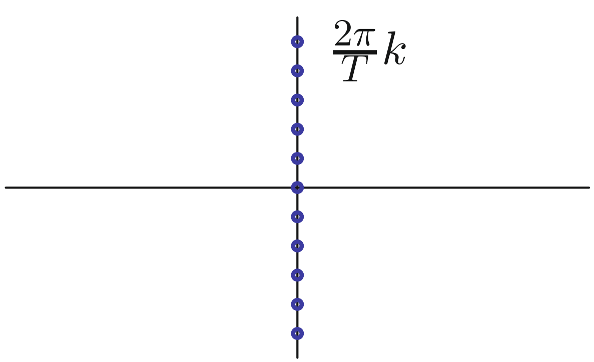
DT Fourier Series



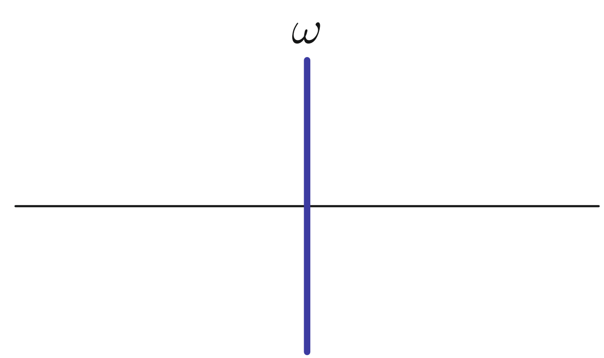
DT Fourier transform



CT Fourier Series



CT Fourier transform

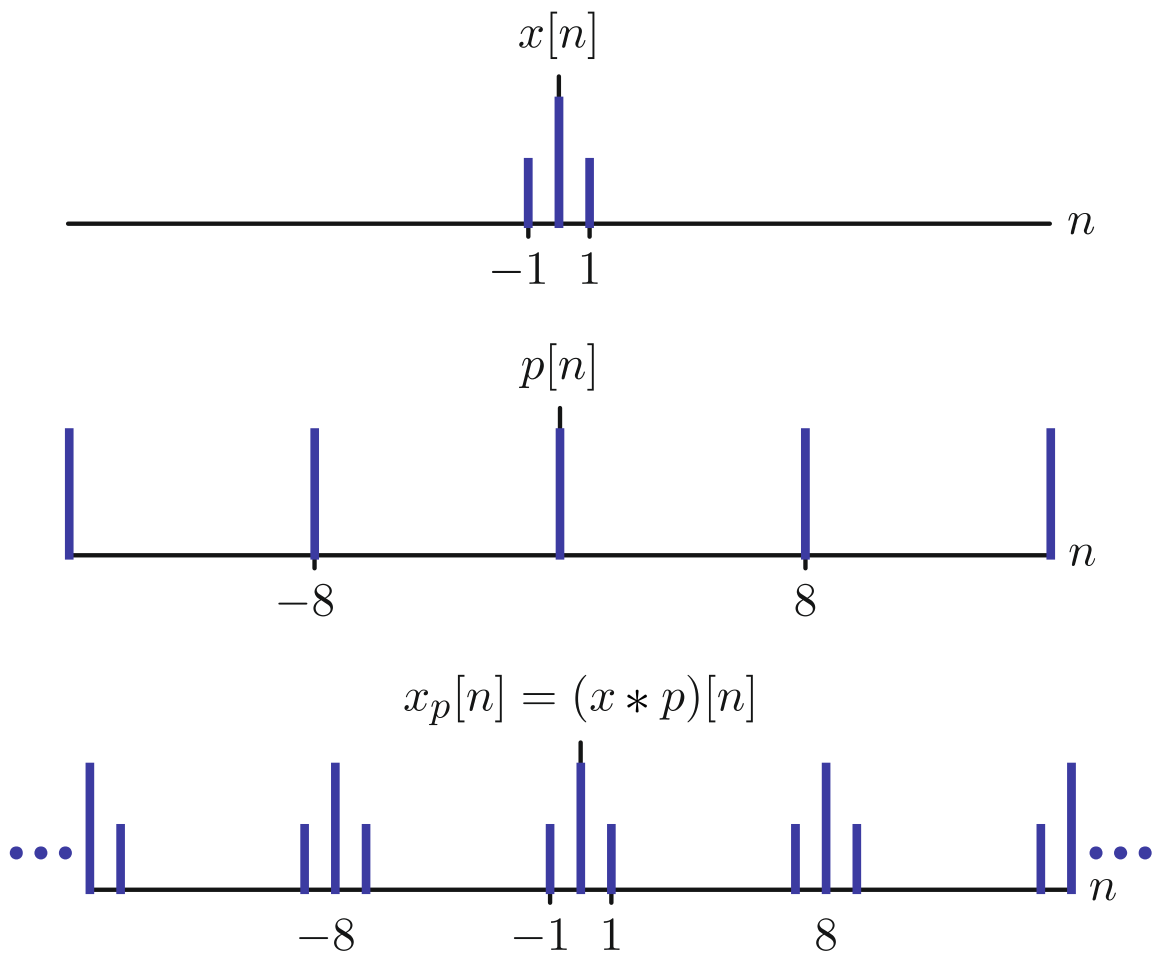


روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل فوریه و سری فوریه‌ی گسسته–زمان

Relation Between DT Fourier Transform and Series

Periodic extension of a DT signal is equivalent to convolution of the signal with an impulse train.

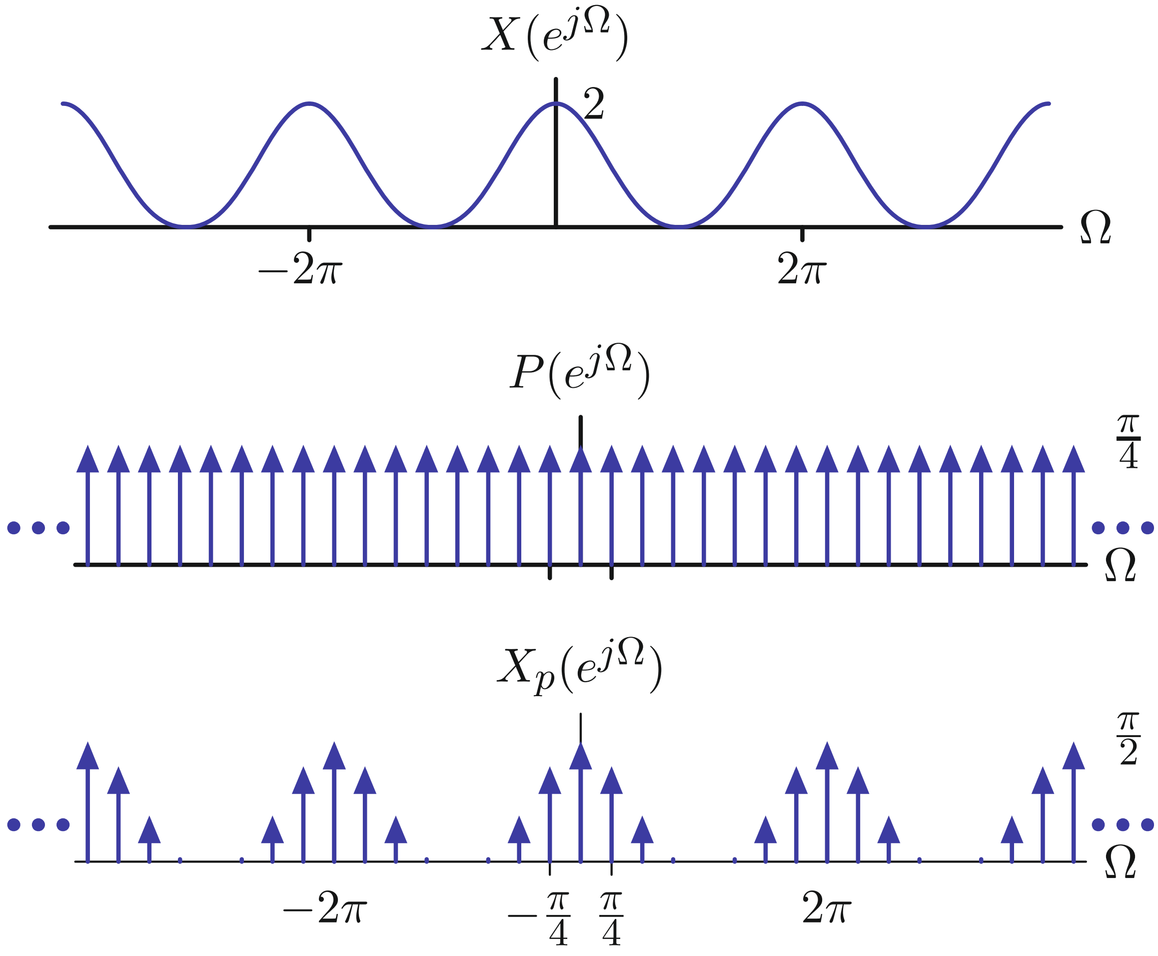


روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل فوریه و سری فوریه‌ی گسسته–زمان

Relation Between DT Fourier Transform and Series

Convolution by an impulse train in time is equivalent to multiplication by an impulse train in frequency.

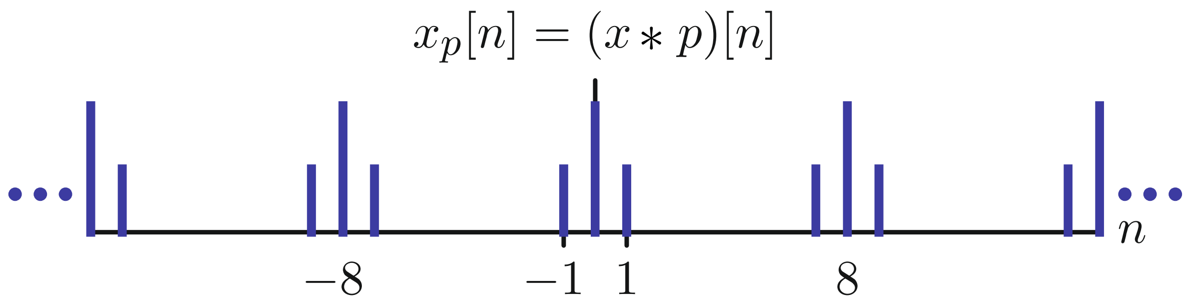


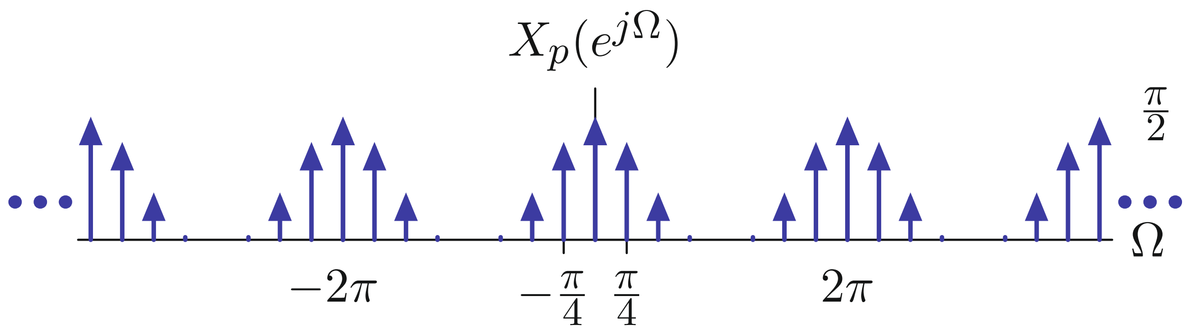
روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل فوریه و سری فوریه‌ی گسسته–زمان

RELATION BETWEEN DT FOURIER TRANSFORM AND SERIES

Periodic extension of a discrete signal ($x[n]$) results in a signal ($x_p[n]$) that is both periodic and discrete. Its transform ($X_p(e^{j\Omega})$) is also periodic and discrete.



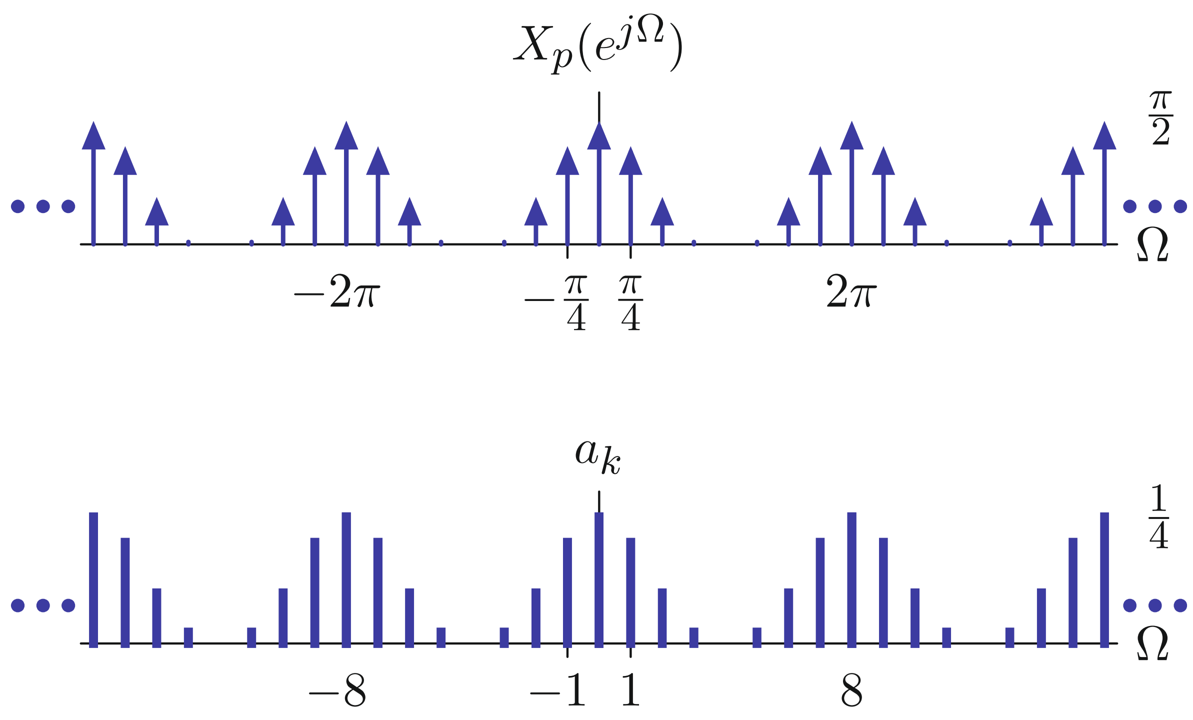


روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل فوریه و سری فوریه‌ی گسسته–زمان

Relation Between DT Fourier Transform and Series

The weight of each impulse in the Fourier transform of a periodically extended function is 2π times the corresponding Fourier series coefficient.

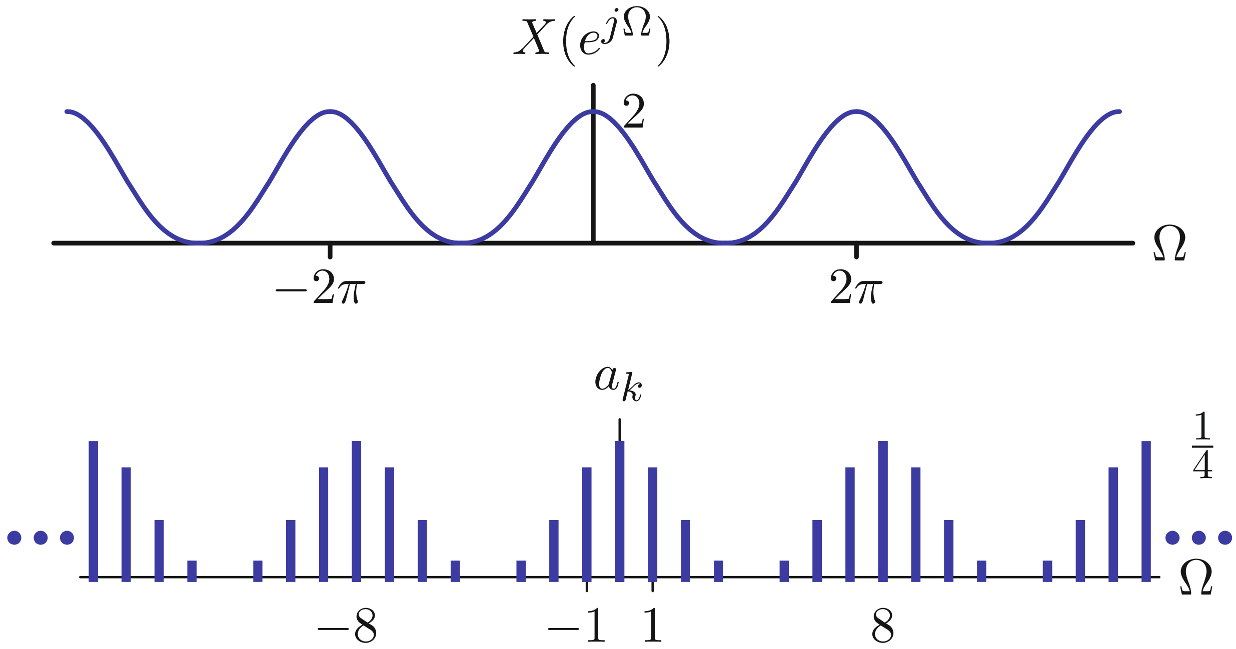


روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل‌های فوریه و سری فوریه

Relation between Fourier Transforms and Series

The effect of periodic extension was to sample the frequency representation.

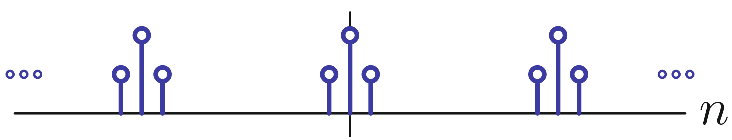


چهار نوع «زمان»

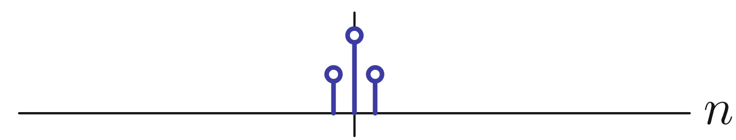
Four Types of "Time"

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

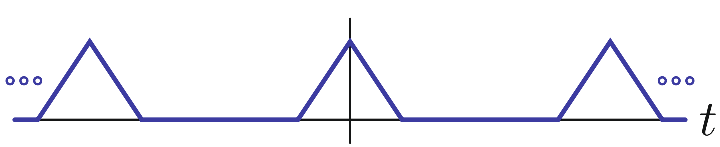
DT Fourier Series



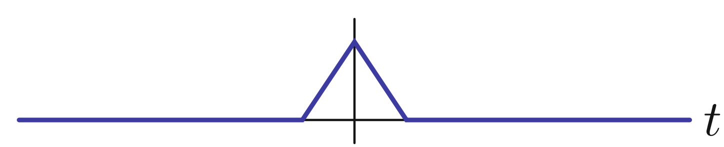
DT Fourier transform



CT Fourier Series



CT Fourier transform

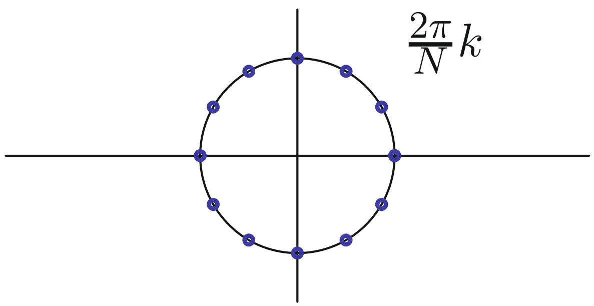


چهار نوع «فرکانس»

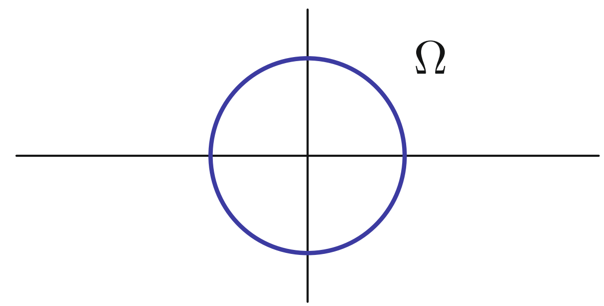
Four Types of "Frequency"

discrete vs. continuous (↕) and periodic vs aperiodic (↔)

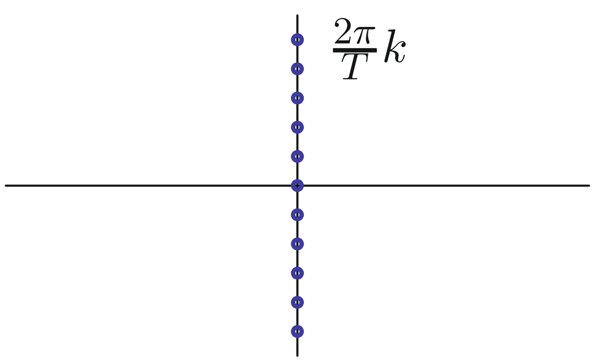
DT Fourier Series



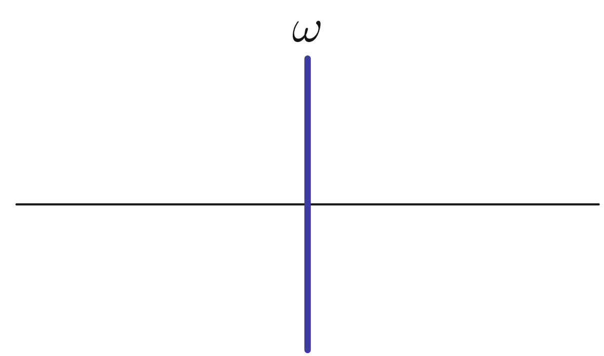
DT Fourier transform



CT Fourier Series



CT Fourier transform



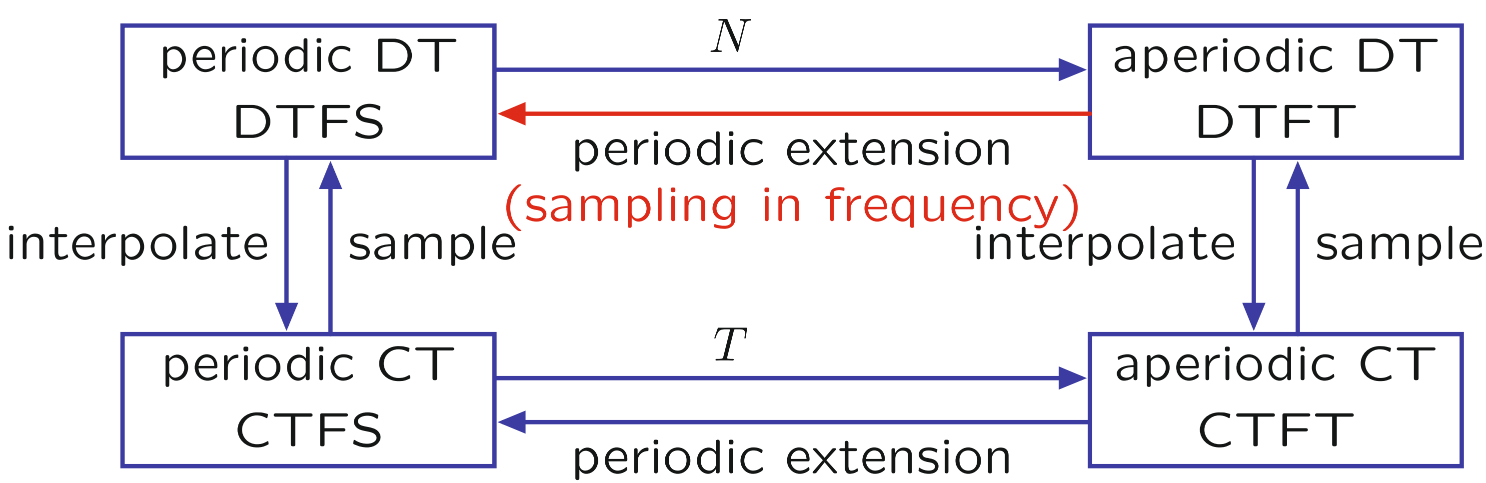
روابط میان بازنمایی‌های فوریه

رابطه‌ی میان تبدیل‌های فوریه و سری فوریه

Relation between Fourier Transforms and Series

Periodic extension of a DT signal produces a discrete function of frequency.

Periodic extension
= convolving with impulse train in time
= multiplying by impulse train in frequency
sampling in frequency

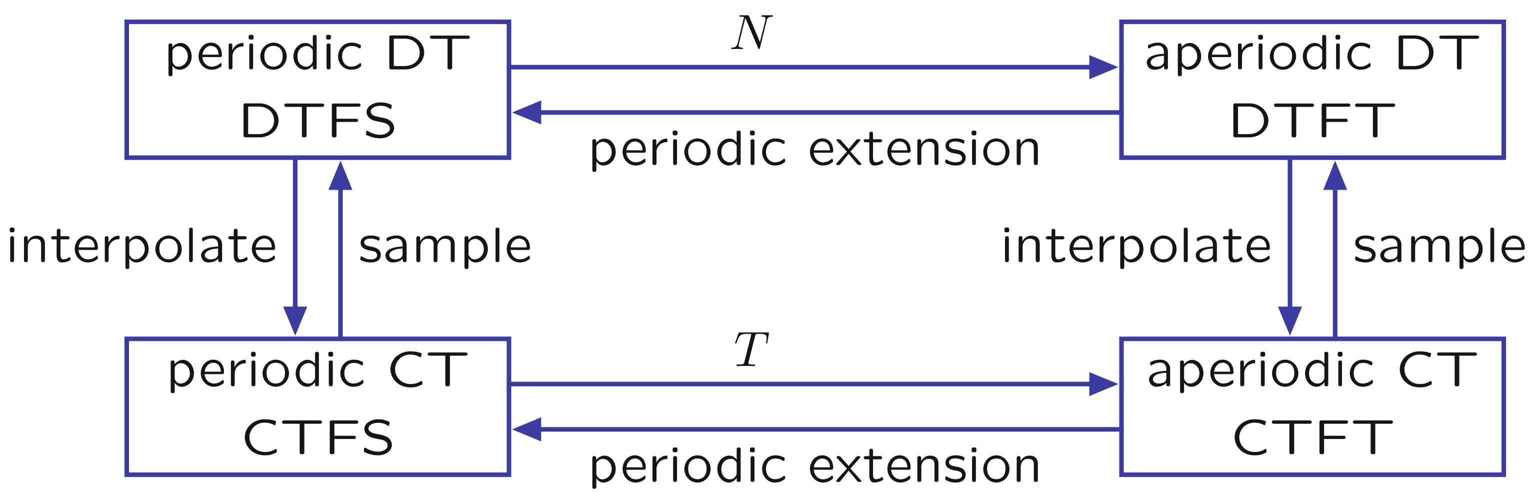


روابط میان بازنمایی‌های فوریه

Relations among Fourier Representations

Different Fourier representations are related because they apply to signals that are related.

DTFS (discrete-time Fourier series):	periodic DT
DTFT (discrete-time Fourier transform):	aperiodic DT
CTFS (continuous-time Fourier series):	periodic CT
CTFT (continuous-time Fourier transform):	aperiodic CT



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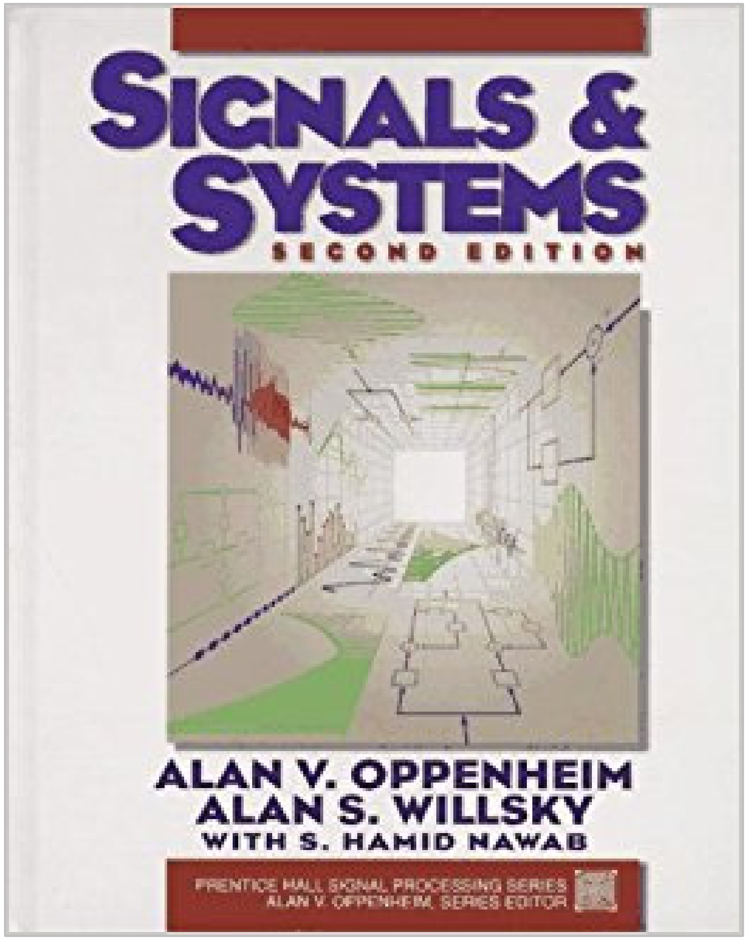
منابع

منبع اصلی



A.V. Oppenheim, A.S. Willsky, S.H. Nawab,
Signals and Systems,
Second Edition, Prentice Hall, 1997.

Chapter 5