



## سیستم‌های چند عاملی

درس ۲۶

# ارضای قید توزیع شده

Distributed Constraint Satisfaction

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<http://courses.fouladi.ir/mas>

## مسئله‌ی ارضای قید

### معرفی

#### مسئله‌ی ارضای قید

#### Constraint Satisfaction Problem (CSP)

مجموعه‌ای از **متغیرها** با **دامنه** مشخص وجود دارند.

هدف تعیین مقدار برای این متغیرهاست به گونه‌ای که یک مجموعه از **قیدها** ارضاشوند.

انواع زیادی از مسائل می‌توانند به صورت CSP تعریف شوند.

مسائل CSP را می‌توان توسط الگوریتم‌های عمومی جستجو حل کرد،  
اما به دلیل ساختار خاص این مسائل، الگوریتم‌های کارآمدتری برای آنها وجود دارد.

## مسئله‌ی ارضای قید

### تعریف ریاضی

$\{X_1, X_2, \dots, X_n\}$	مجموعه‌ای از متغیرها	X	متغیرها Variables	مؤلفه‌ای سه‌گانه تعریف مسئله‌ی «ارضای قید»
$\{D_1, D_2, \dots, D_n\}$	مجموعه‌ای از دامنه‌ها (مقادیر مجاز متغیرها) هر دامنه به صورت $\{v_1, v_2, \dots, v_k\}$	D	دامنه‌ها Domains	
	مجموعه‌ای از قیدها: مشخص‌کننده مقادیر مجاز ترکیبات متغیرها هر قید به صورت رابطه‌ای روی تعدادی متغیر (نمایش لیستی رابطه / رابطه‌ی انتزاعی)	C	قیدها Constraints	

The constraints are defined by predicates,  $p_k(x_{k1}, x_{k2}, \dots, x_{kj})$

where each  $p_k$  is the function

$$p_k : D_{k1} \times D_{k2} \times \dots \times D_{kj} \rightarrow \{0, 1\}$$

## انتساب

### انتساب مقدار به متغیر و انواع آن

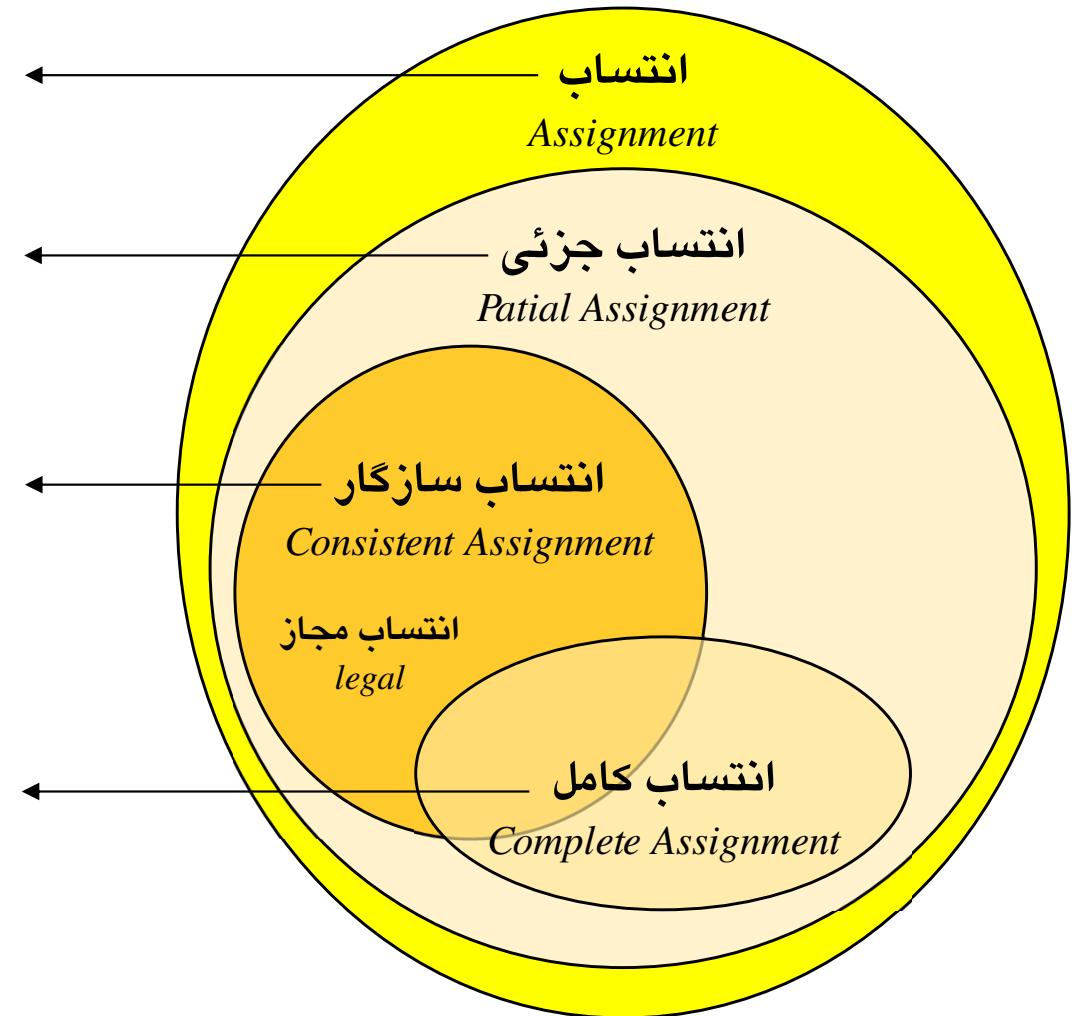
#### ASSIGNMENT

نسبت‌دهی مقادیر به تمام یا بعضی از متغیرها

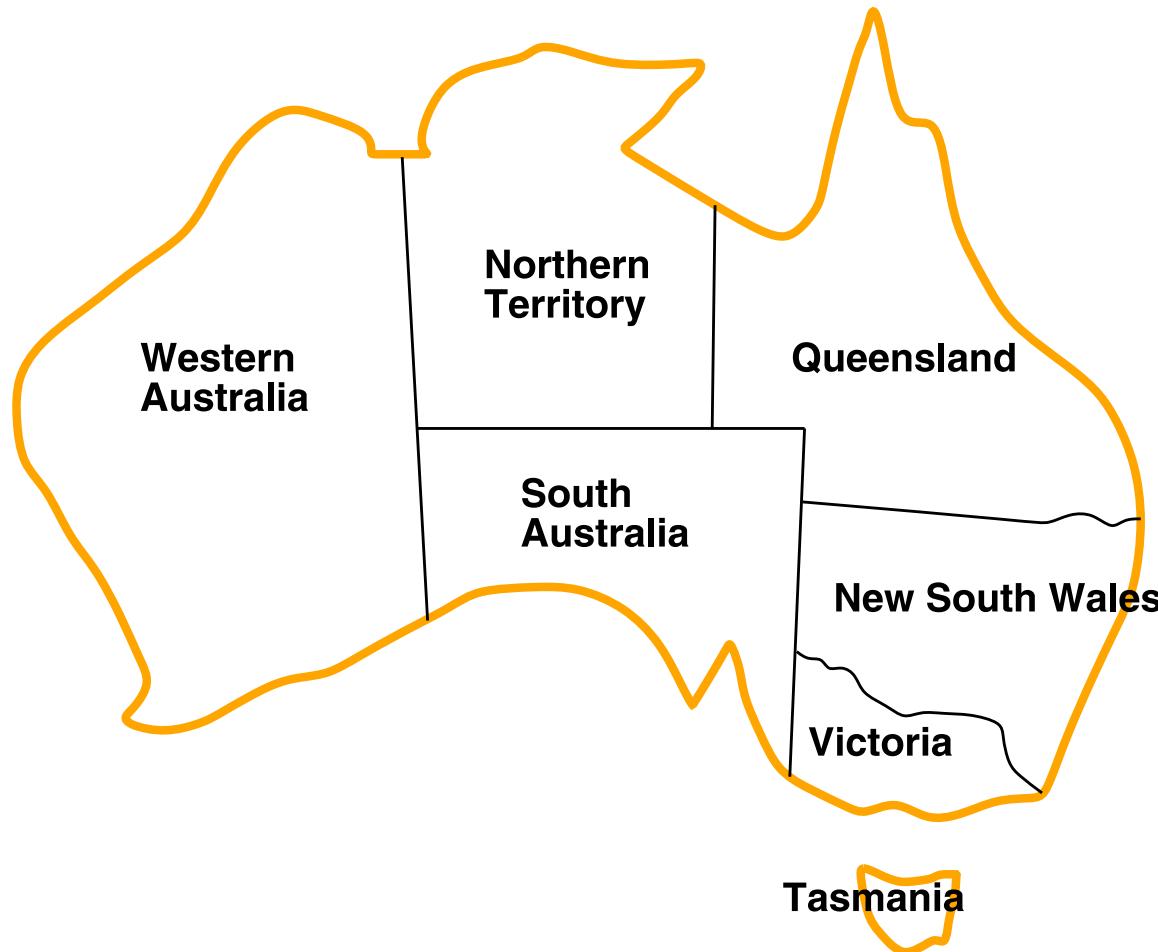
یک انتساب که در آن بعضی متغیرها  
نسبت‌دهی شده باشند.

یک انتساب که هیچ قیدی را نقض نکرده باشد

یک انتساب که در آن همهٔ متغیرها  
نسبت‌دهی شده باشند.



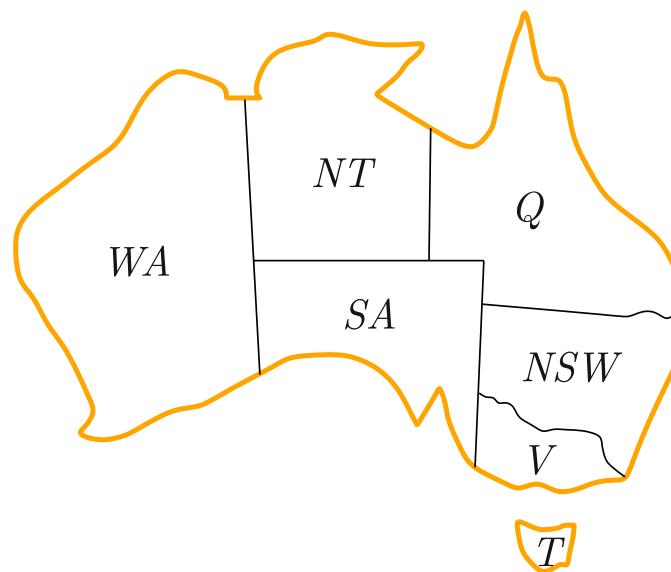
## مسئله‌ی «رنگ‌آمیزی نقشه»

MAP-COLORING

انتساب رنگ به استان‌ها به طوری که استان‌های مجاور هم رنگ نباشند.

## مسئله‌ی ارضای قید

مثال: مسئله‌ی رنگ‌آمیز نقشه



*WA, NT, Q, NSW, V, SA, T*

استان‌ها

X

متغیرها  
Variables

D

دامنه‌ها  
Domains

C

قیدها  
Constraints

$D_i = \{red, blue, green\}$

رنگ‌ها

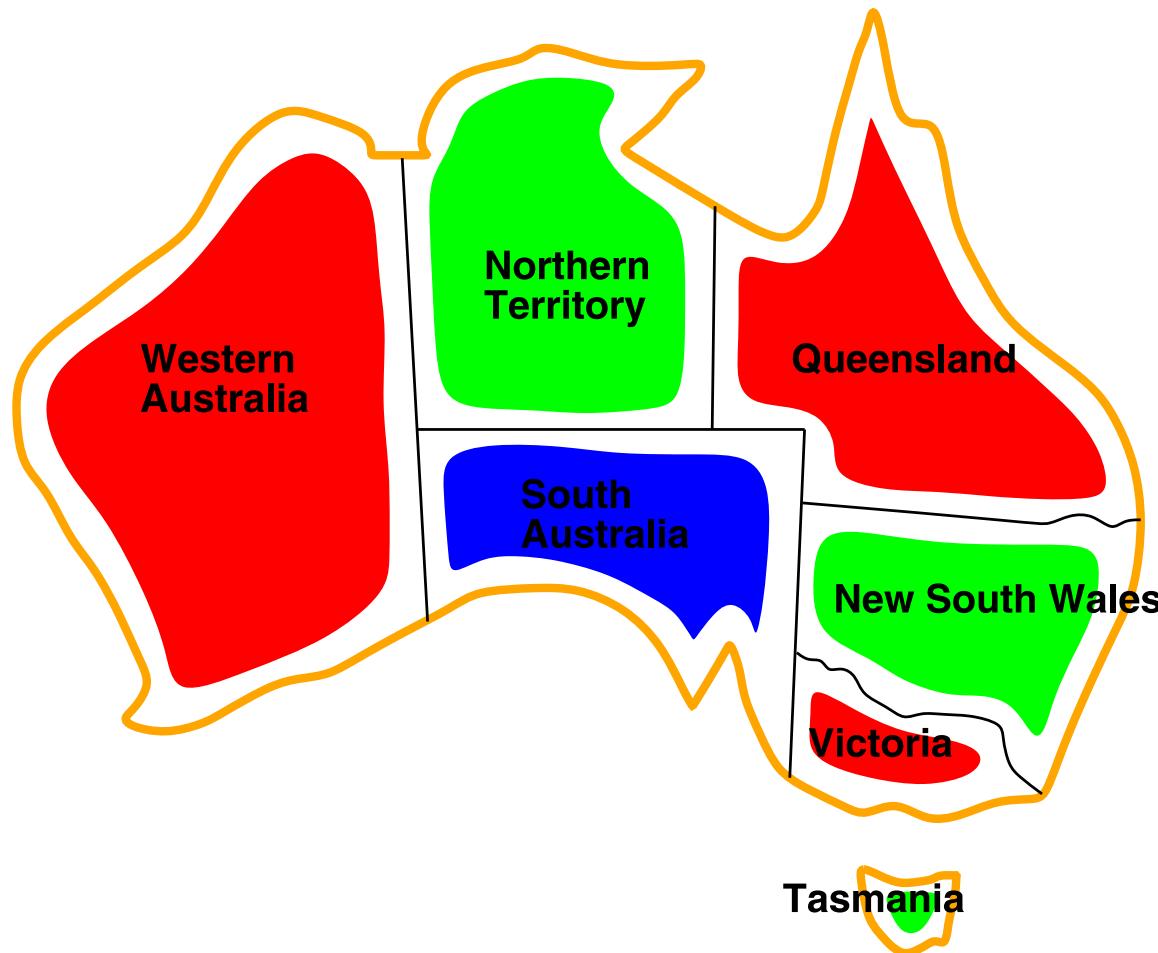
ناحیه‌های مجاور باید رنگ‌های متفاوت داشته باشند.

e.g.,  $WA \neq NT, V \neq NSW, \dots$

مؤلفه‌های تعریف مسئله‌ی «ارضای قید»

## مسئله‌ی ارضی قید

مثال: مسئله‌ی رنگ‌آمیز نقشه: راه حل



راه حل‌ها: انتساب‌هایی هستند که همه‌ی قیدها را ارضا کنند، مانند:

$$\{WA = \text{red}, NT = \text{green}, Q = \text{red}, NSW = \text{green}, V = \text{red}, SA = \text{blue}, T = \text{green}\}$$

## گراف قید

### CONSTRAINT GRAPH

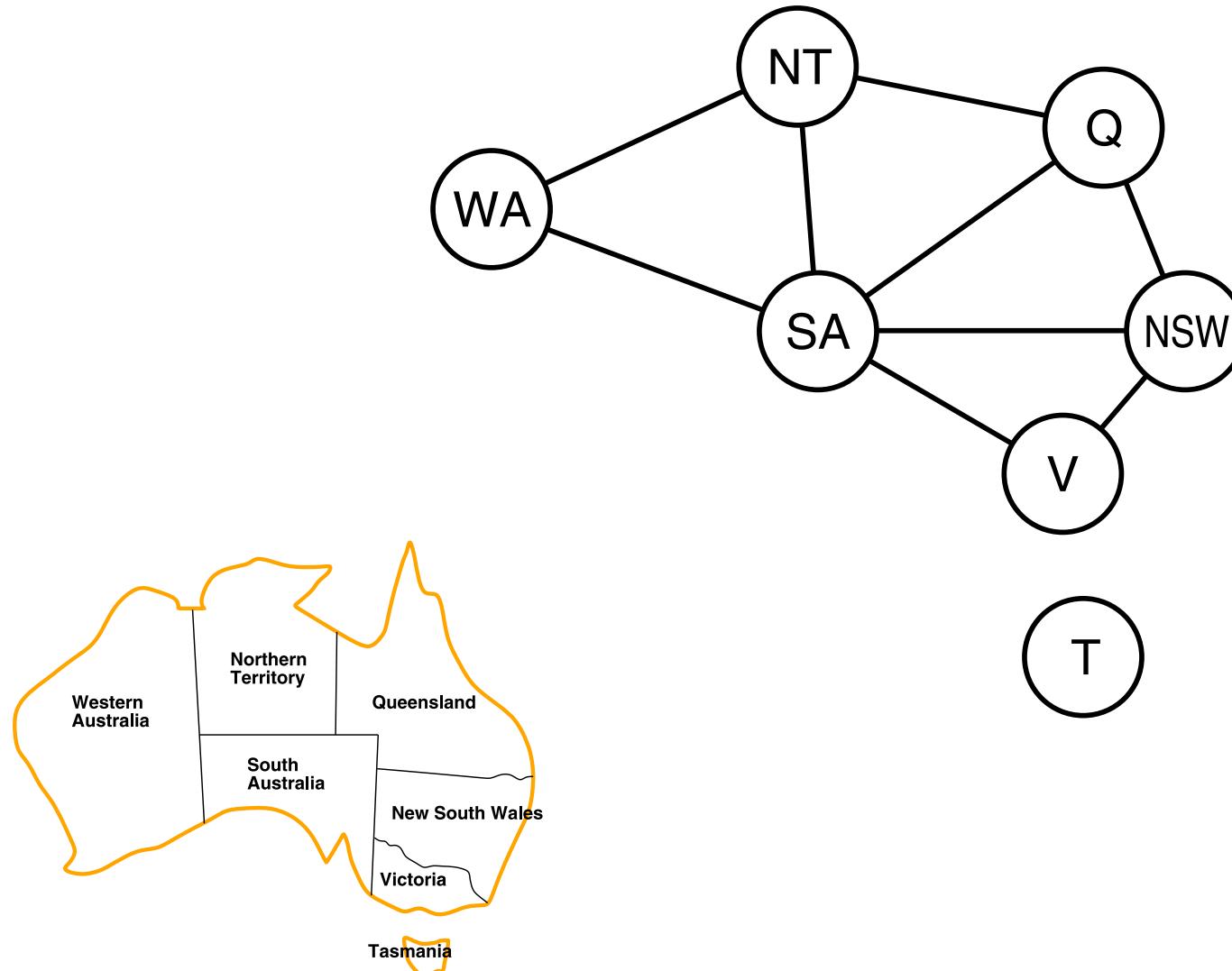
گراف قید <i>Constraint Graph</i>	
گره‌ها <i>Nodes</i>	کمان‌ها <i>Arcs</i>
نشان‌دهنده‌ی متغیرها	نشان‌دهنده‌ی قیدها

قابل استفاده برای CSP دودویی

الگوریتم‌های CSP همه‌منظوره، از ساختار گراف برای تسریع جستجو استفاده می‌کنند.

## گراف قید

مثال

CONSTRAINT GRAPH

## حل مسئله‌ی ارضای قید با جستجوی عمق-اول

A centralized depth first search algorithm for the CSP. Variable  $g$  holds the partial assignment of variable values. The algorithm is called with  $\text{DEPTH-FIRST-SEARCH-CSP}(1, \emptyset)$ .

**DEPTH-FIRST-SEARCH-CSP( $i, g$ )**

```

1  if  $i > n$ 
2    then return  $g$ 
3  for  $v \in D_i$ 
4    do if setting  $x_i \leftarrow v$  does not violate any constraint in  $P$  given  $g$ 
5      then  $g' \leftarrow \text{DEPTH-FIRST-SEARCH-CSP}(i + 1, g + \{x_i \leftarrow v\})$ 
6      if  $g' \neq \emptyset$ 
7        then return  $g'$ 
8
9 return  $\emptyset$ 
```

## حل مسئله‌ی اراضی قید توزیع شده

### اراضی قید چندعاملی

هر عامل مسئول مقدار یکی از متغیرها و قیدهای مربوط به آن است.

## حل مسئله‌ی ارضای قید با الگوریتم فیلترینگ

### FILTERING ALGORITHM

FILTERING()

```
1 for  $j \in \{\text{neighbors of } i\}$             $\triangleright i$  is this agent.  
2   do REVISE( $x_i, x_j$ )
```

HANDLE-NEW-DOMAIN( $j, D'$ )

```
1  $D_j \leftarrow D'$   
2 REVISE( $x_i, x_j$ )
```

REVISE( $x_i, x_j$ )

```
1 old-domain  $\leftarrow D_i$   
2 for  $v_i \in D_i$   
3   do if there is no  $v_j \in D_j$  consistent with  $v_i$   
4     then  $D_i \leftarrow D_i - v_i$   
5 if old-domain  $\neq D_i$   
6   then  $\forall_{k \in \{\text{neighbors of } i\}} k.\text{HANDLE-NEW-DOMAIN}(i, D_i)$ 
```

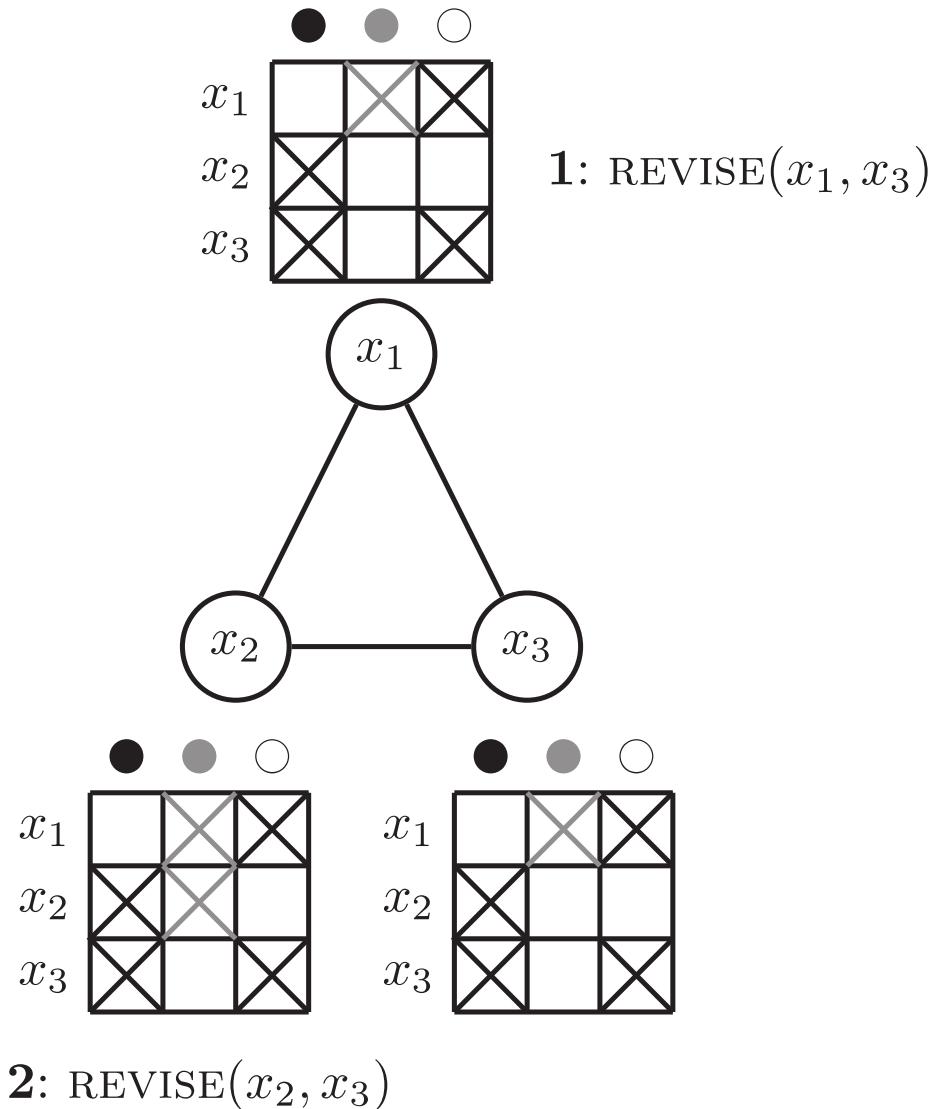
The filtering algorithm.  
Each agent  $i$  executes  
FILTERING().

هر عامل دامنه‌اش را به عامل‌های همسایه‌اش اطلاع می‌دهد  
و سپس مقادیری که نمی‌توانند قیدها را ارضاء کنند،  
از دامنه‌اش حذف می‌کند.

## حل مسئله‌ی ارضای قید با الگوریتم فیلترینگ

مثال

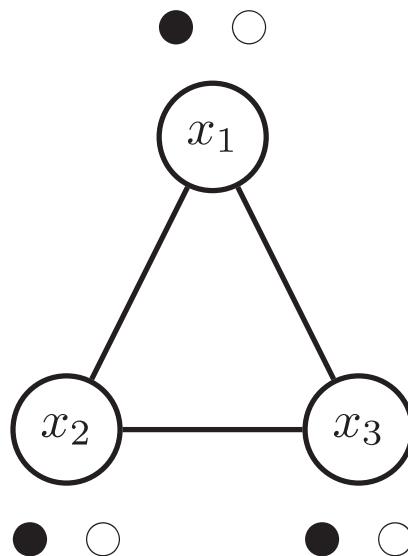
Filtering example. The agents start out with some prohibited colors as indicated by the black crosses. On the first step  $x_1$  does his REVISE and eliminates the color gray from consideration. It then tells everyone else about this. Then  $x_2$  does its revise and eliminates the color gray from its domain.



## حل مسئله‌ی ارضای قید با الگوریتم فیلترینگ

مثال

Example of a problem that does not have a solution and the filtering algorithm cannot that fact.



## حل مسئله‌ی ارضای قید با الگوریتم سازگاری مبتنی بر هاپیر رزولوشن

قاعده‌ی هاپیر رزولوشن

HYPER-RESOLUTION BASED CONSISTENCY ALGORITHM

$$\begin{array}{c}
 A_1 \vee A_2 \vee \cdots \vee A_m \\
 \neg(A_1 \wedge A_{11} \wedge \cdots) \\
 \neg(A_2 \wedge A_{21} \wedge \cdots) \\
 \vdots \\
 \neg(A_m \wedge A_{m1} \wedge \cdots) \\
 \hline
 \neg(A_{11} \wedge \cdots \wedge A_{21} \wedge \cdots \wedge A_{m1} \wedge \cdots).
 \end{array}$$

## حل مسئله‌ی ارضای قید با الگوریتم سازگاری مبتنی بر هاپیر رزولوشن

مثال

### HYPER-RESOLUTION BASED CONSISTENCY ALGORITHM

	$x_1$	$x_2$	$x_3$
	$x_1 = \circ \vee x_1 = \bullet$	$x_2 = \circ \vee x_2 = \bullet$	$x_3 = \circ \vee x_3 = \bullet$
	$\neg(x_1 = \circ \wedge x_2 = \circ)$	$\neg(x_1 = \circ \wedge x_2 = \circ)$	$\neg(x_1 = \circ \wedge x_2 = \circ)$
Time	$\neg(x_1 = \bullet \wedge x_2 = \bullet)$	$\neg(x_1 = \bullet \wedge x_2 = \bullet)$	$\neg(x_1 = \bullet \wedge x_2 = \bullet)$
1	$\neg(x_2 = \circ \wedge x_3 = \bullet)$	$\neg(x_2 = \circ \wedge x_3 = \bullet)$	$\neg(x_2 = \circ \wedge x_3 = \bullet)$
2	$\neg(x_2 = \bullet \wedge x_3 = \circ)$	$\neg(x_3 = \bullet)$	$\neg(x_2 = \bullet \wedge x_3 = \circ)$
3		$\neg(x_2 = \bullet \wedge x_3 = \circ)$	$\neg(x_3 = \circ)$
4		$\neg(x_3 = \circ)$	$\neg(x_3 = \bullet)$

Example databases for a sample run of the hyper-resolution based consistency algorithm as applied to the graph of figure 2.5. Only a few of the nogoods produced by the graph's constraints are shown due to space constraints. You can infer the rest. New statements are added starting at time 1.

## حل مسئله‌ی ارضای قید با الگوریتم سازکاری مبتنی بر هاپیر رزولوشن

### HYPER-RESOLUTION BASED CONSISTENCY ALGORITHM

**procedure ReviseHR( $NG_i, NG_j^*$ )**

**repeat**

$$NG_i \leftarrow NG_i \cup NG_j^*$$

let  $NG_i^*$  denote the set of new Nogoods that  $i$  can derive from  $NG_i$  and his domain using hyper-resolution

**if**  $NG_i^*$  is nonempty **then**

$$NG_i \leftarrow NG_i \cup NG_i^*$$

send the Nogoods  $NG_i^*$  to all neighbors of  $i$

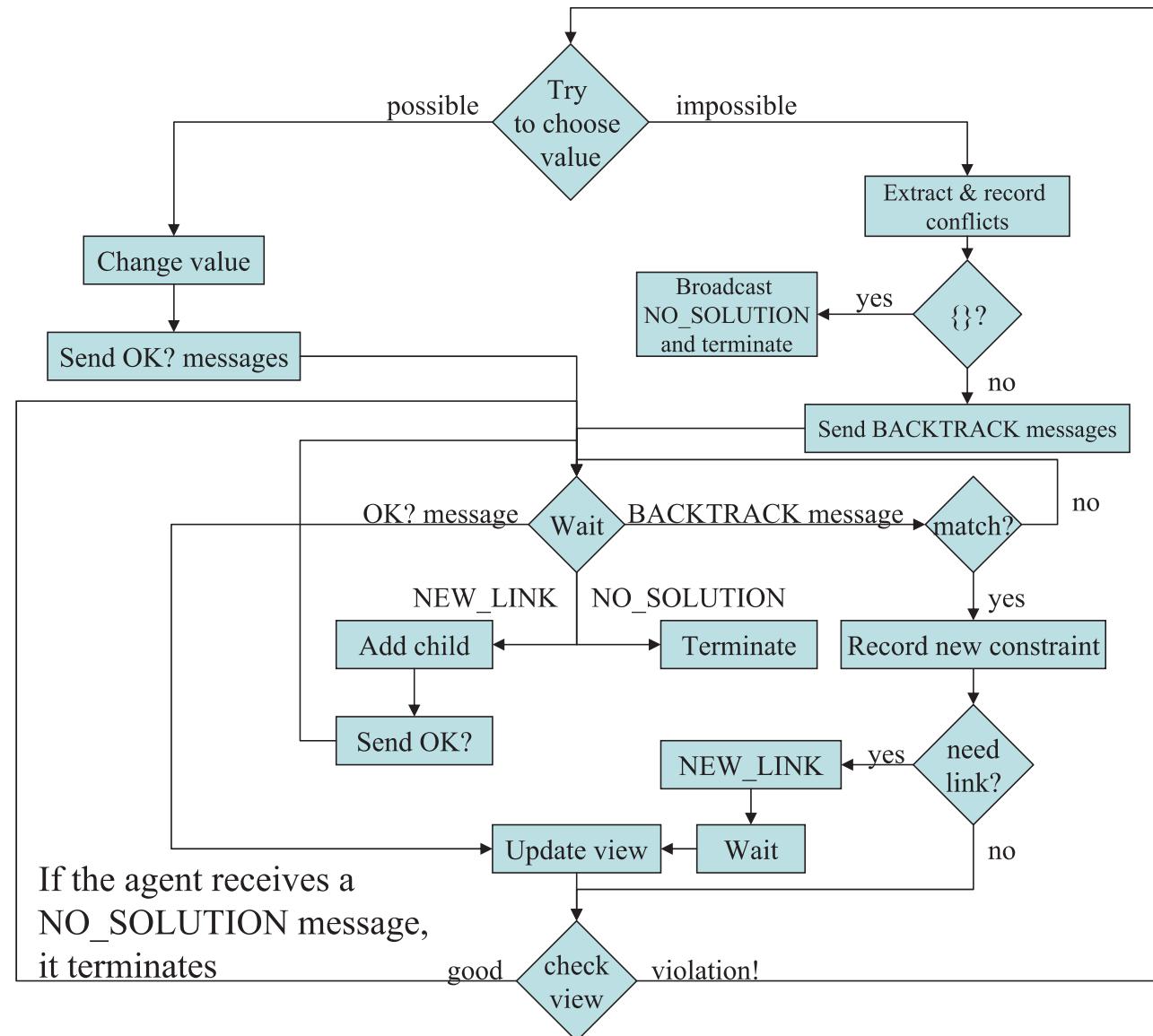
**if**  $\{\} \in NG_i^*$  **then**

└ stop

**until** there is no change in  $i$ 's set of Nogoods  $NG_i$

## الگوریتم عقب‌گرد ناهمکام

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

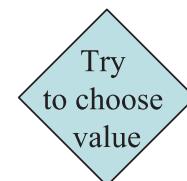


## الگوریتم عقب‌گرد ناهمگام

۱۶ از

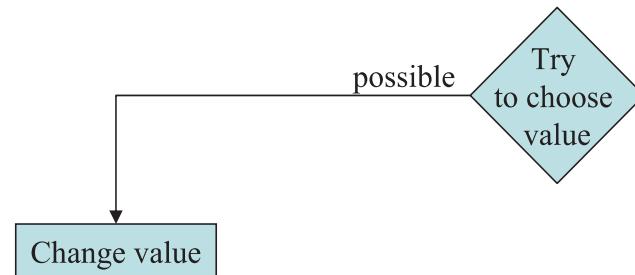
THE ASYNCHRONOUS BACKTRACKING ALGORITHM

عامل برای متغیرش یک مقدار انتخاب می‌کند که  
قیدهایی که مسئول برقراری آنهاست را ارضاء کند.



## الگوریتم عقب‌گرد ناهمکام

از ۱۶

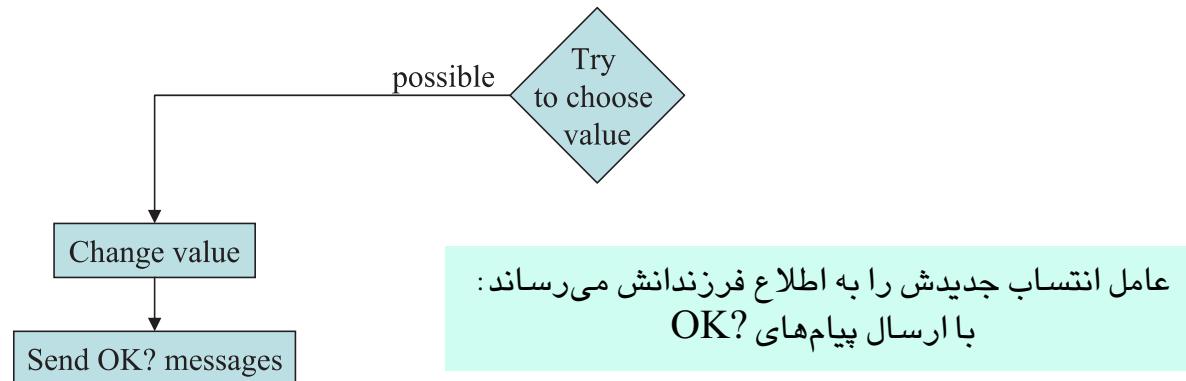
THE ASYNCHRONOUS BACKTRACKING ALGORITHM

اگر حداقل یک مقدار وجود دارد که آن قیدها را ارضاء نماید، عامل یکی را برمی‌گزیند و نسبت‌دهی آن به متغیرش را انجام می‌دهد.

# الگوریتم عقب‌گرد ناهمکام

۱۶ از ۳

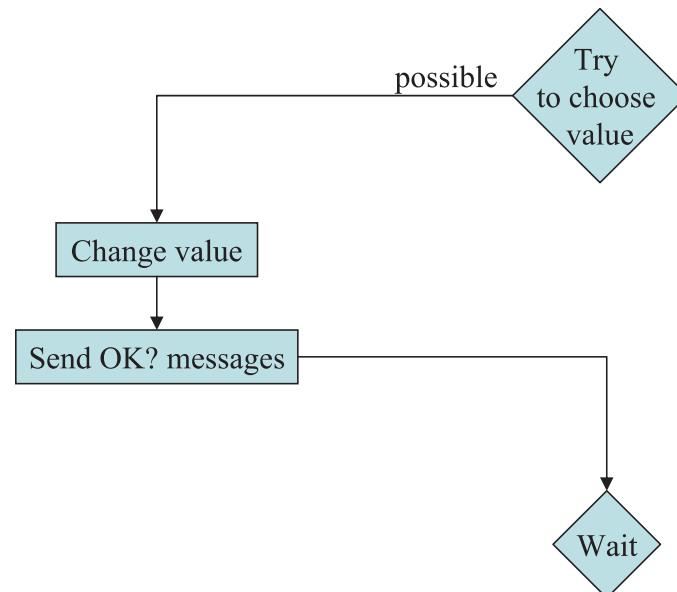
## THE ASYNCHRONOUS BACKTRACKING ALGORITHM



# الگوریتم عقب‌گرد ناهمکام

۱۶ از ۴

## THE ASYNCHRONOUS BACKTRACKING ALGORITHM

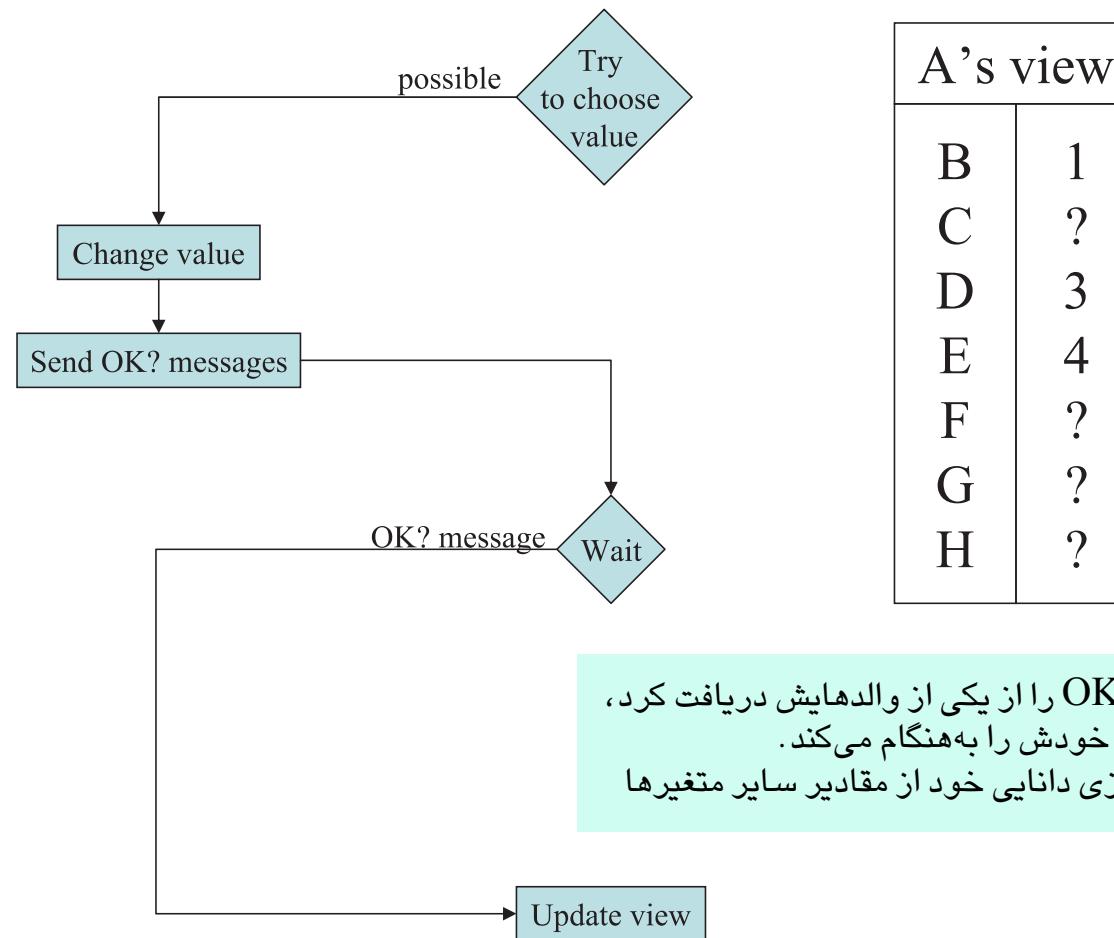


عامل سپس منتظر پاسخ به پیام OK خودش از سوی فرزندانش می‌ماند.  
 (همچنین دیگر پیام‌های OK از سوی والدهایش)

## الگوریتم عقب‌گرد ناهمکام

۱۶ از ۵

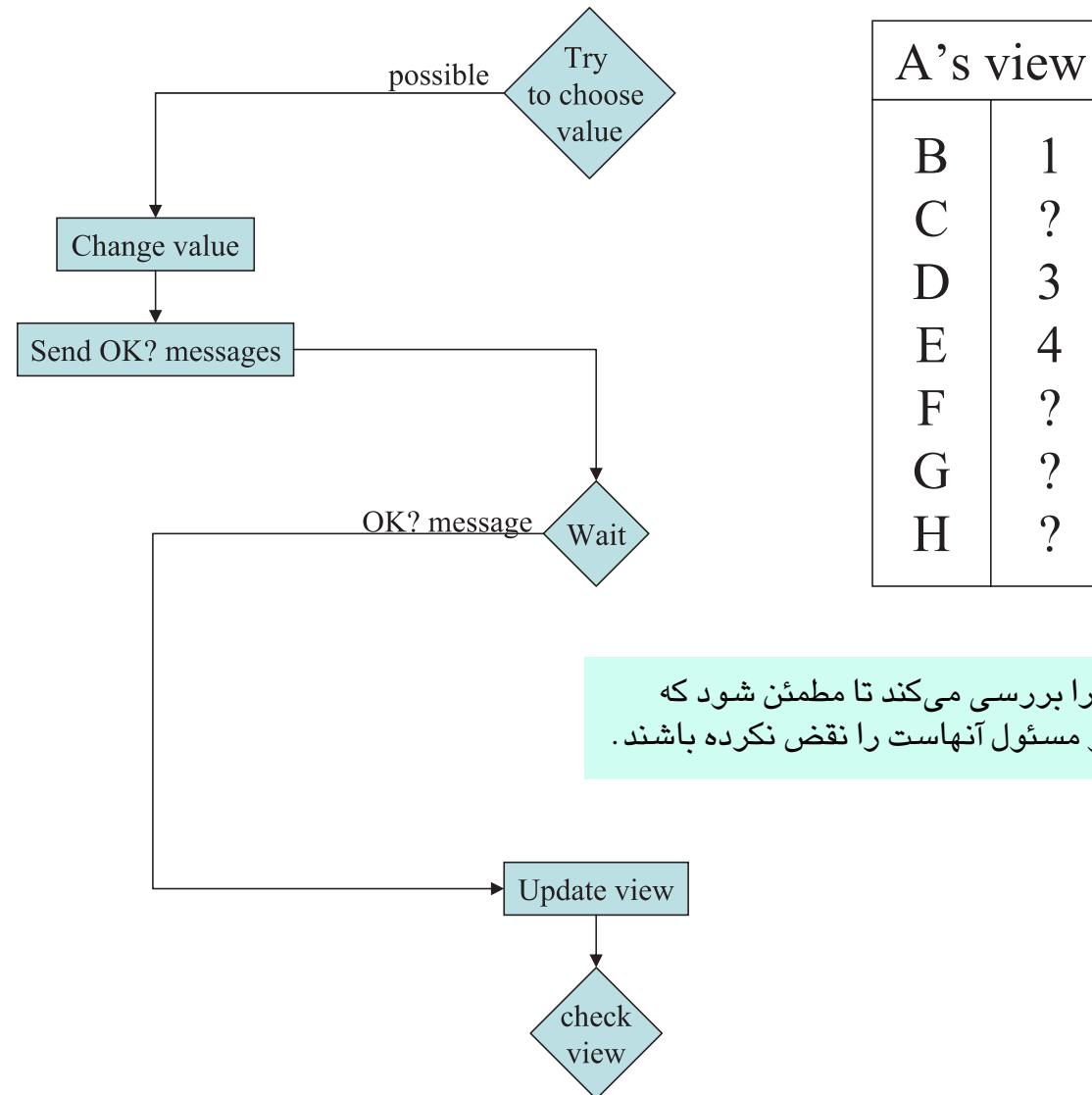
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از

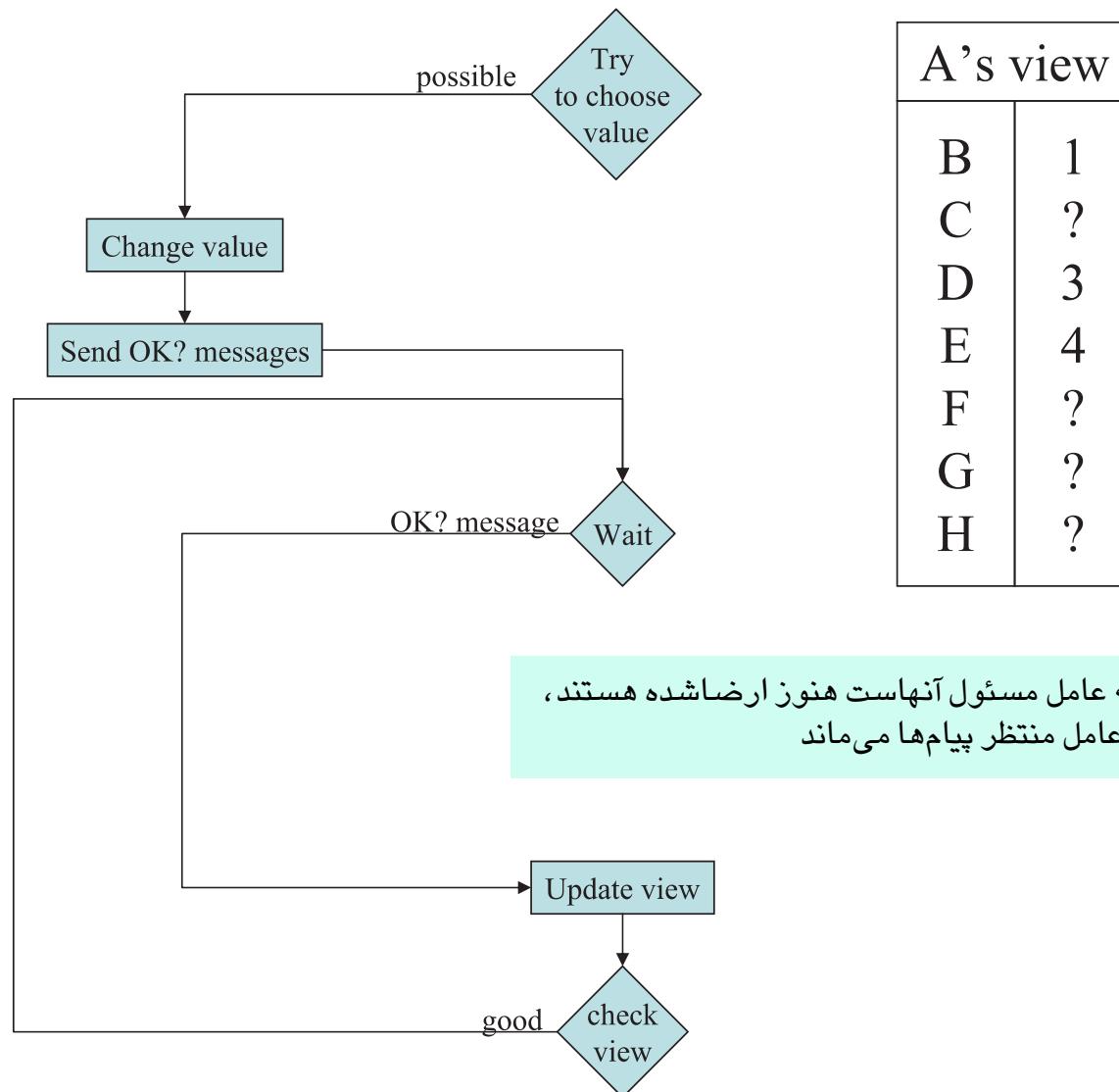
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از ۷

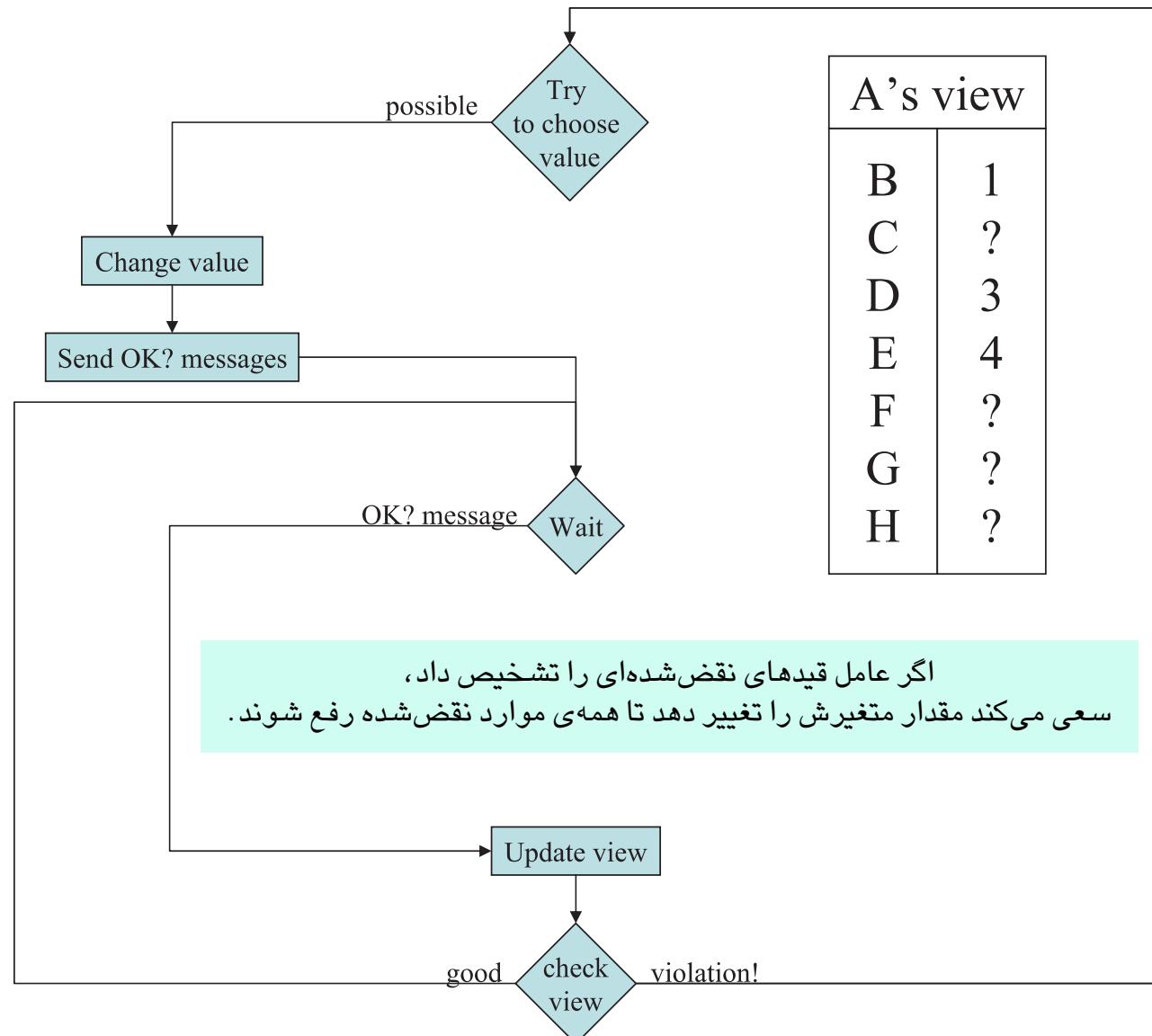
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

از ۱۶۸

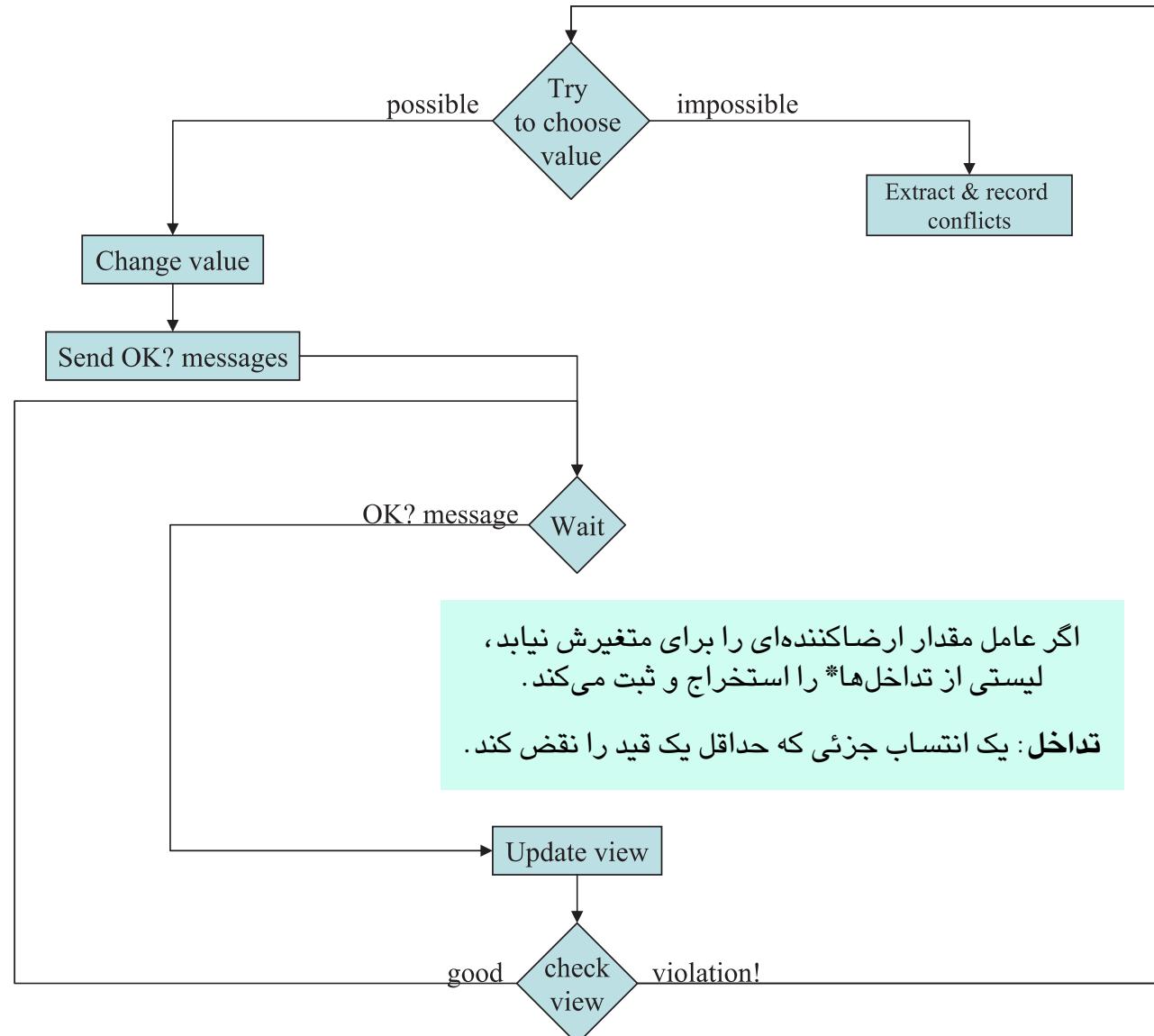
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از

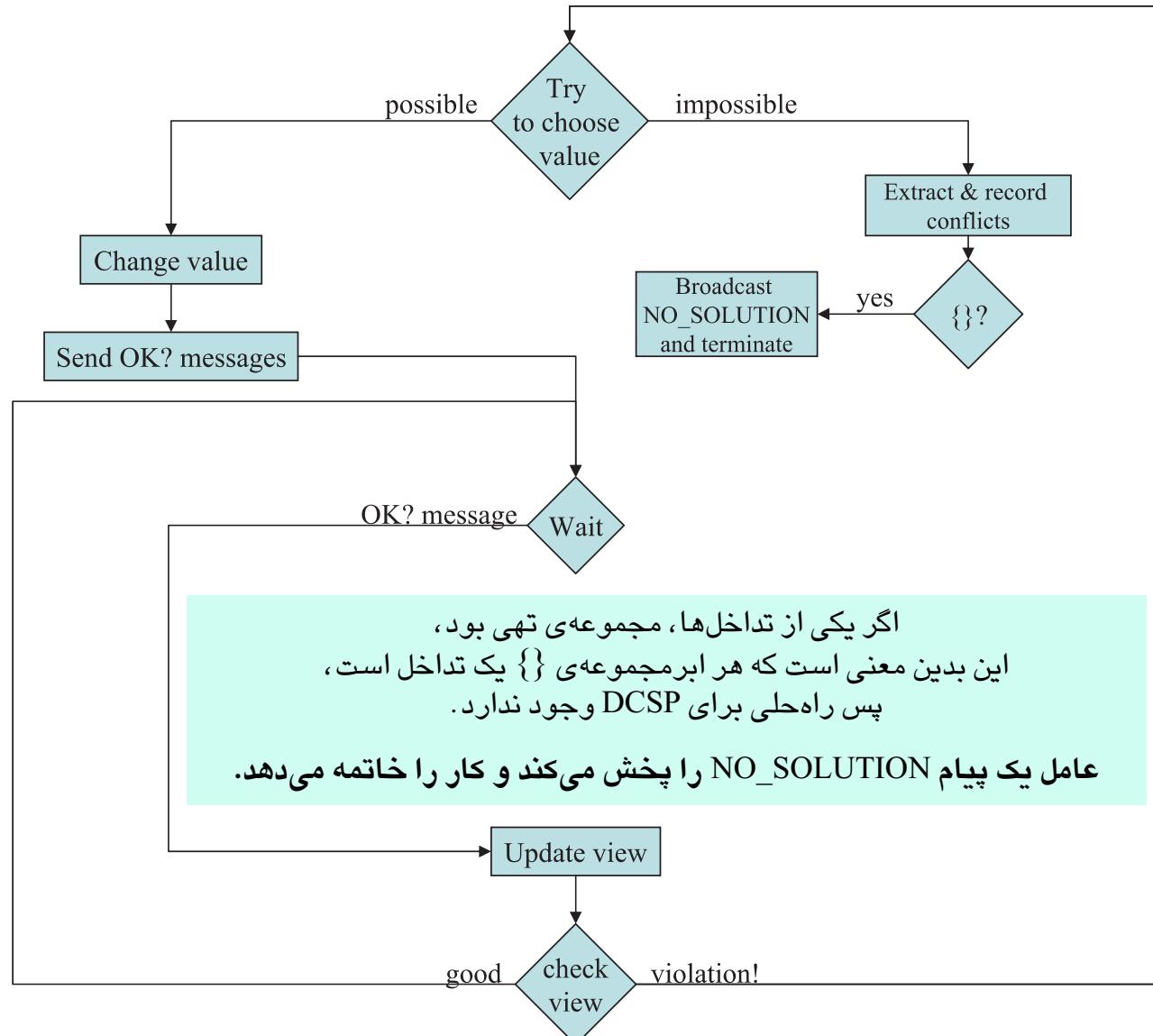
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۰ از ۱۶

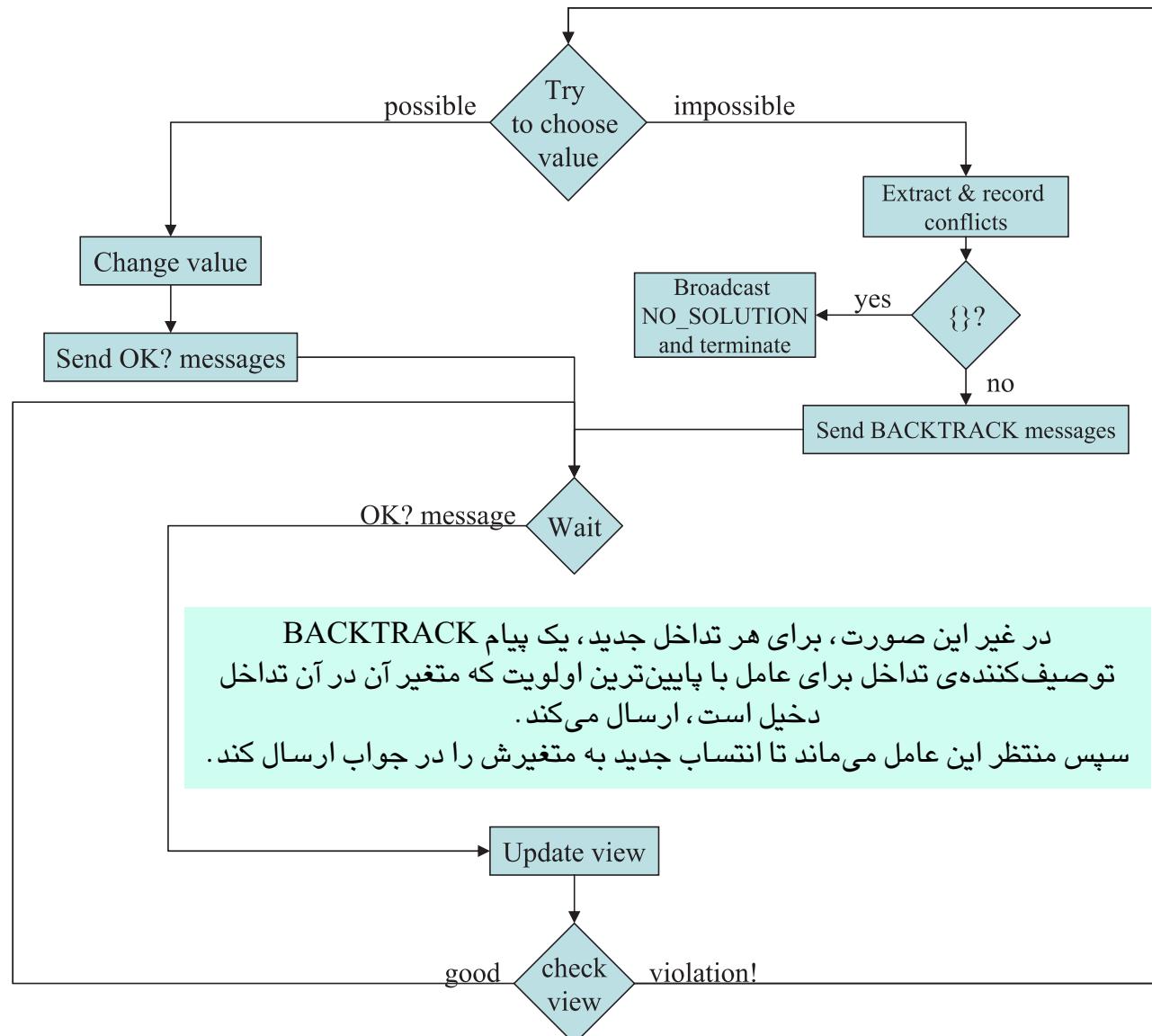
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از ۱۱

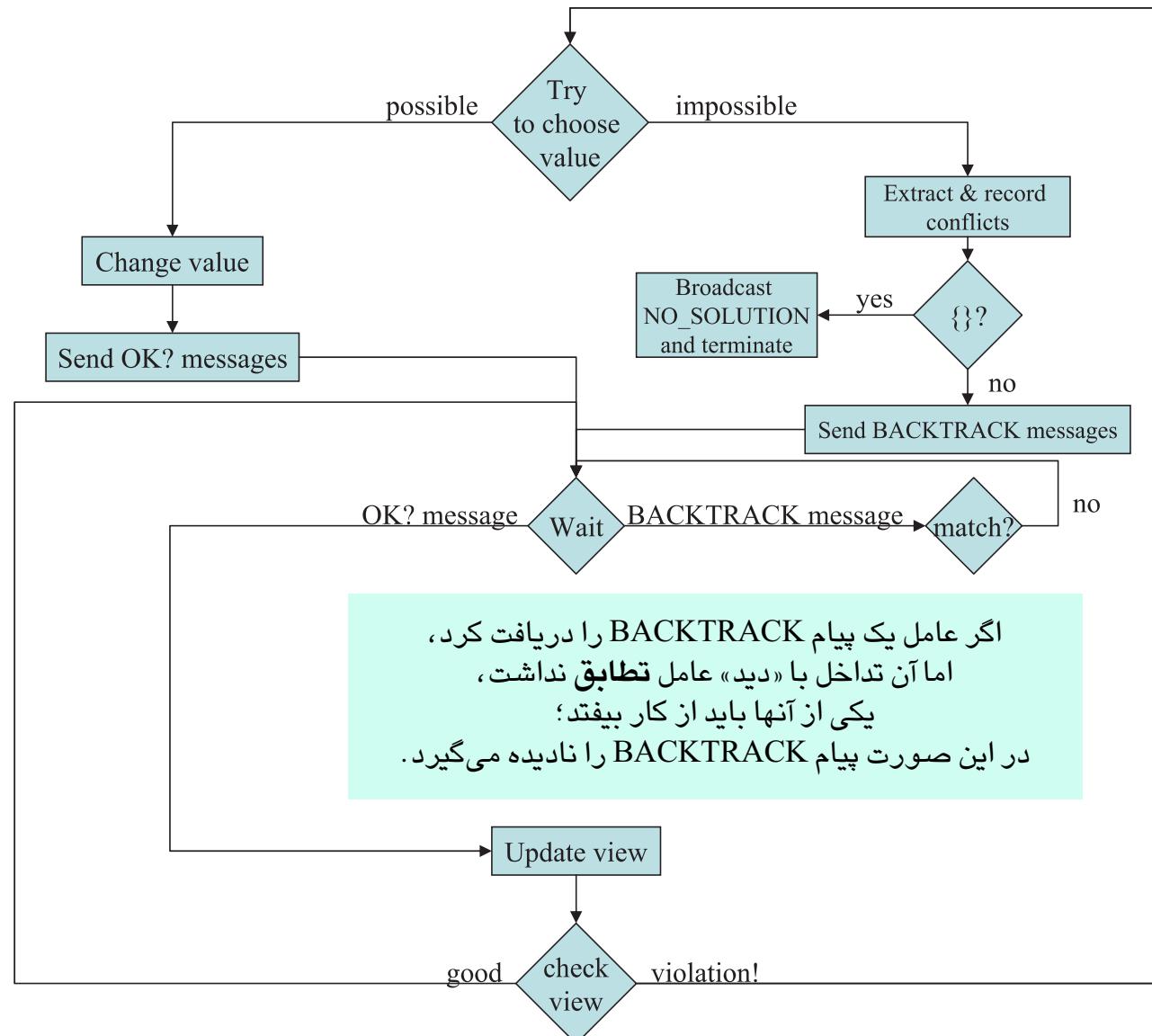
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوريتم عقب گرد ناهمگام

۱۲ از ۱۶

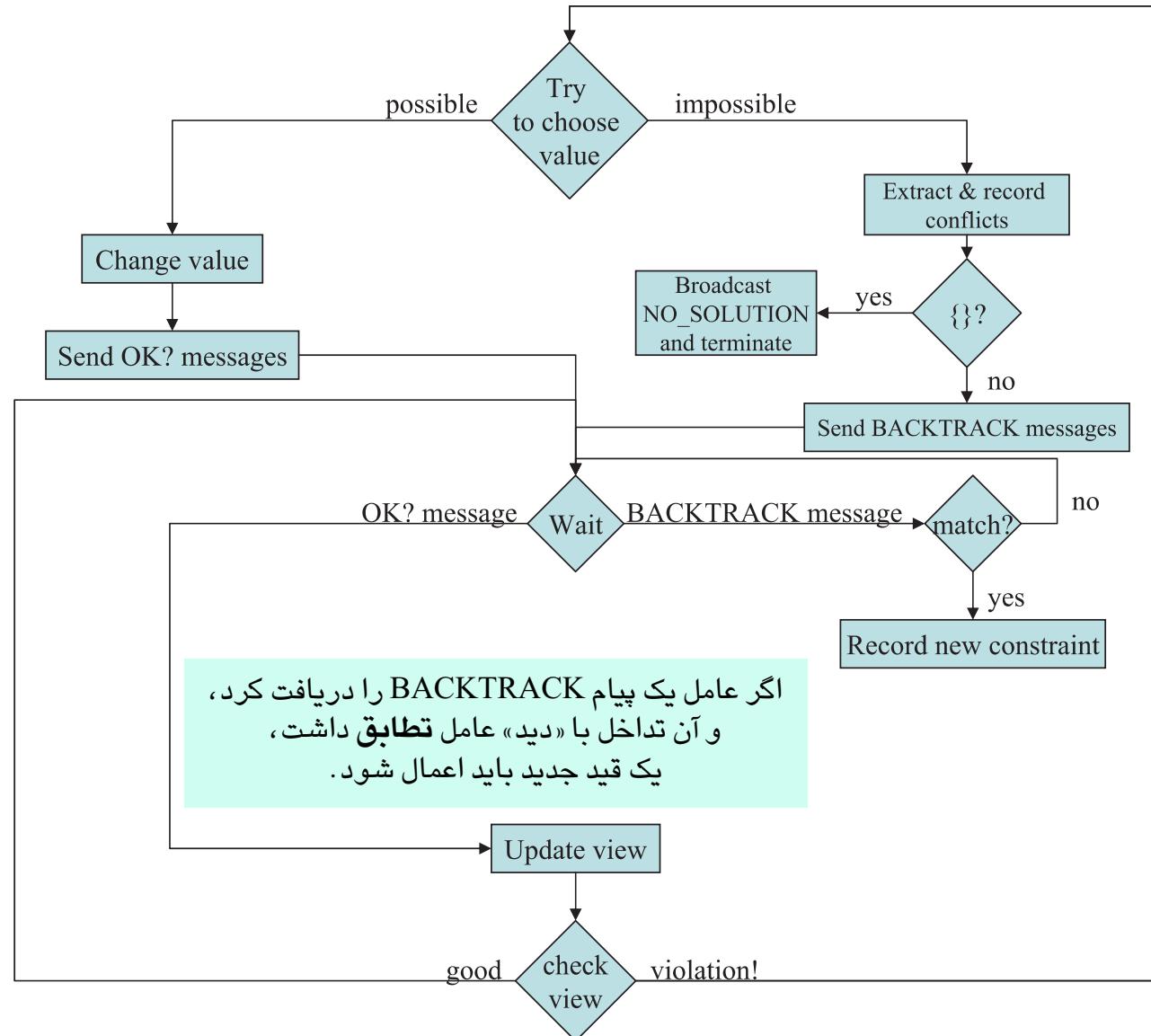
## THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از ۱۳

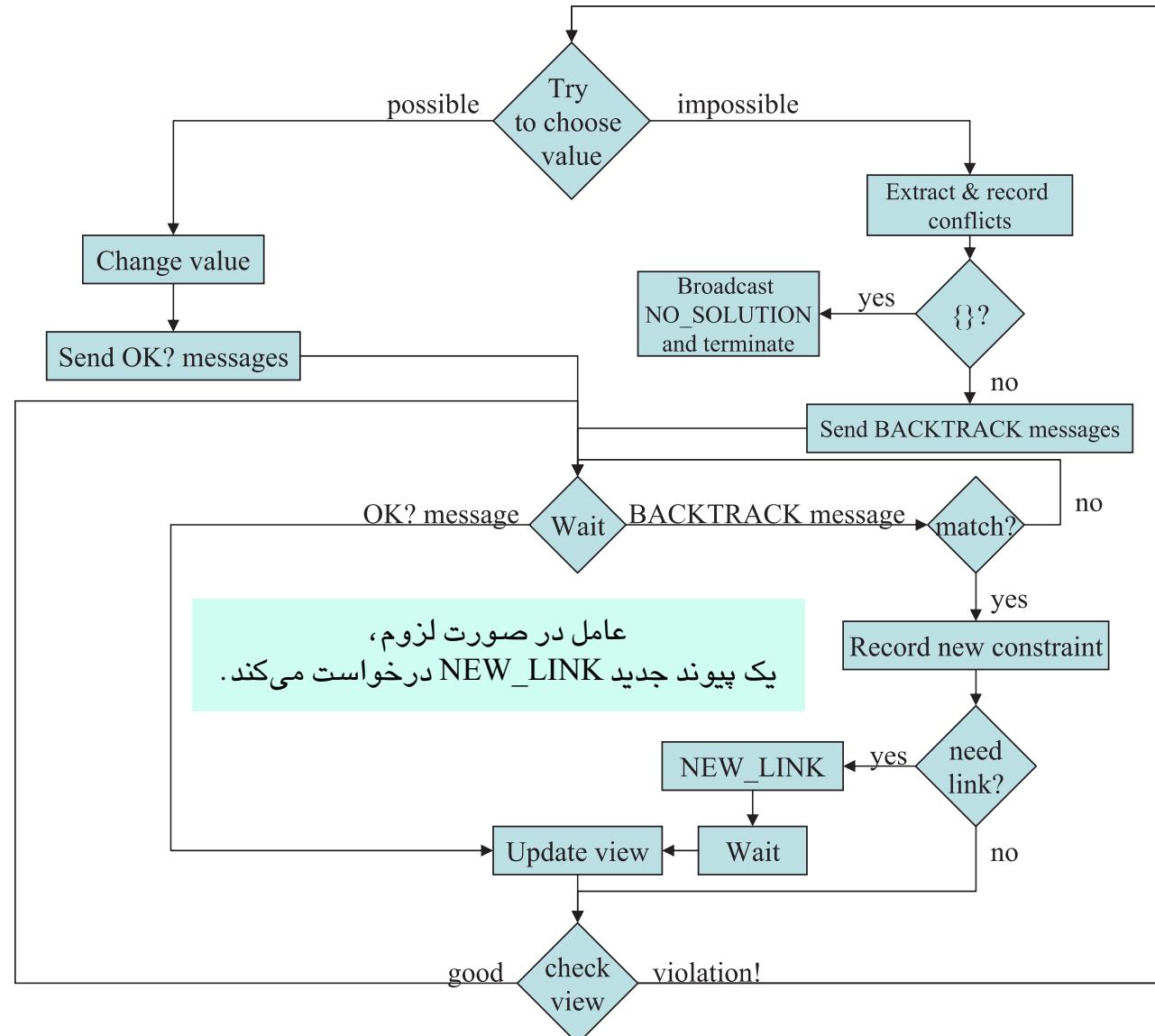
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



# الگوریتم عقب‌گرد ناهمکام

۱۶ از ۱۴

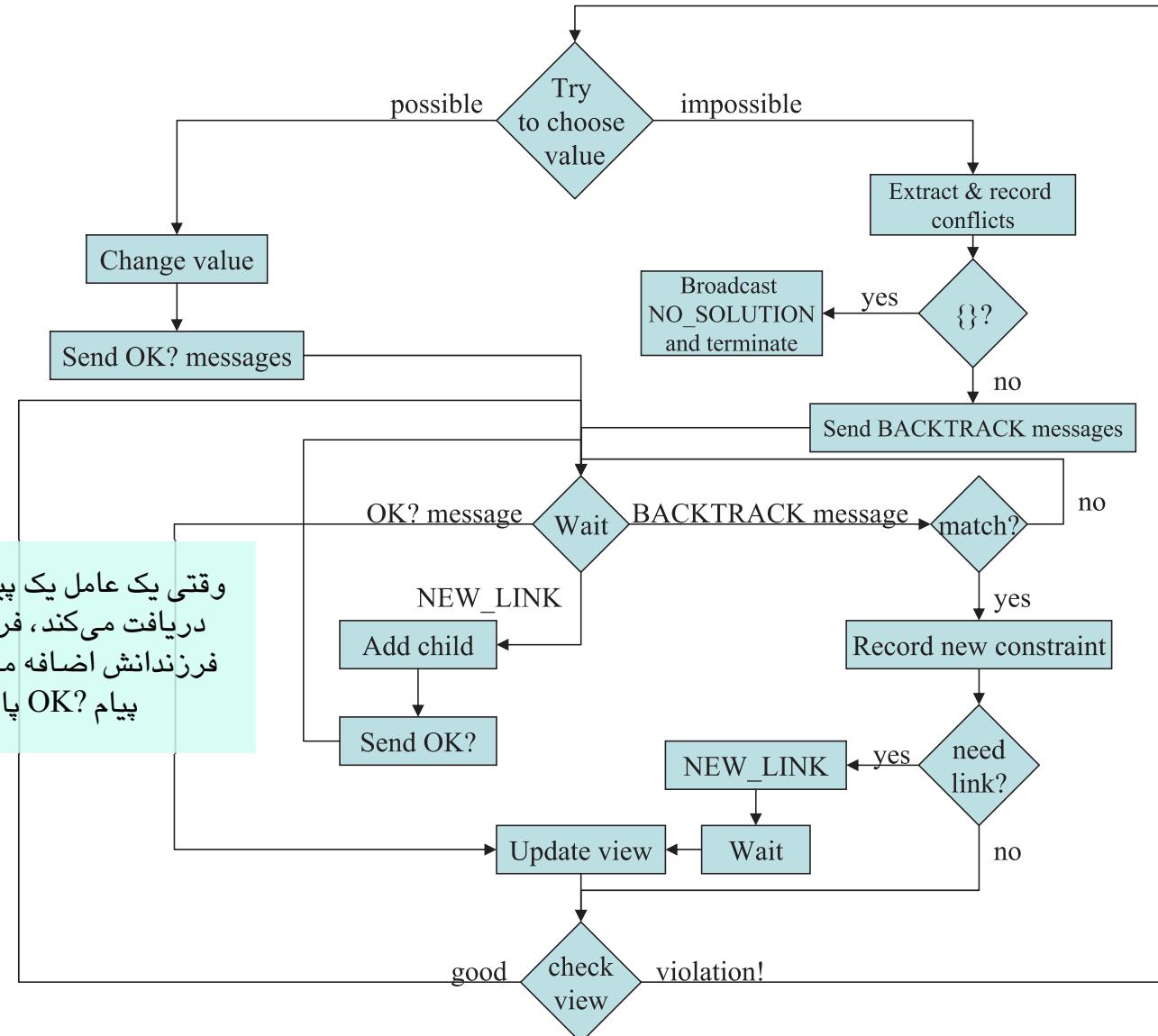
## THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۵ از ۱۶

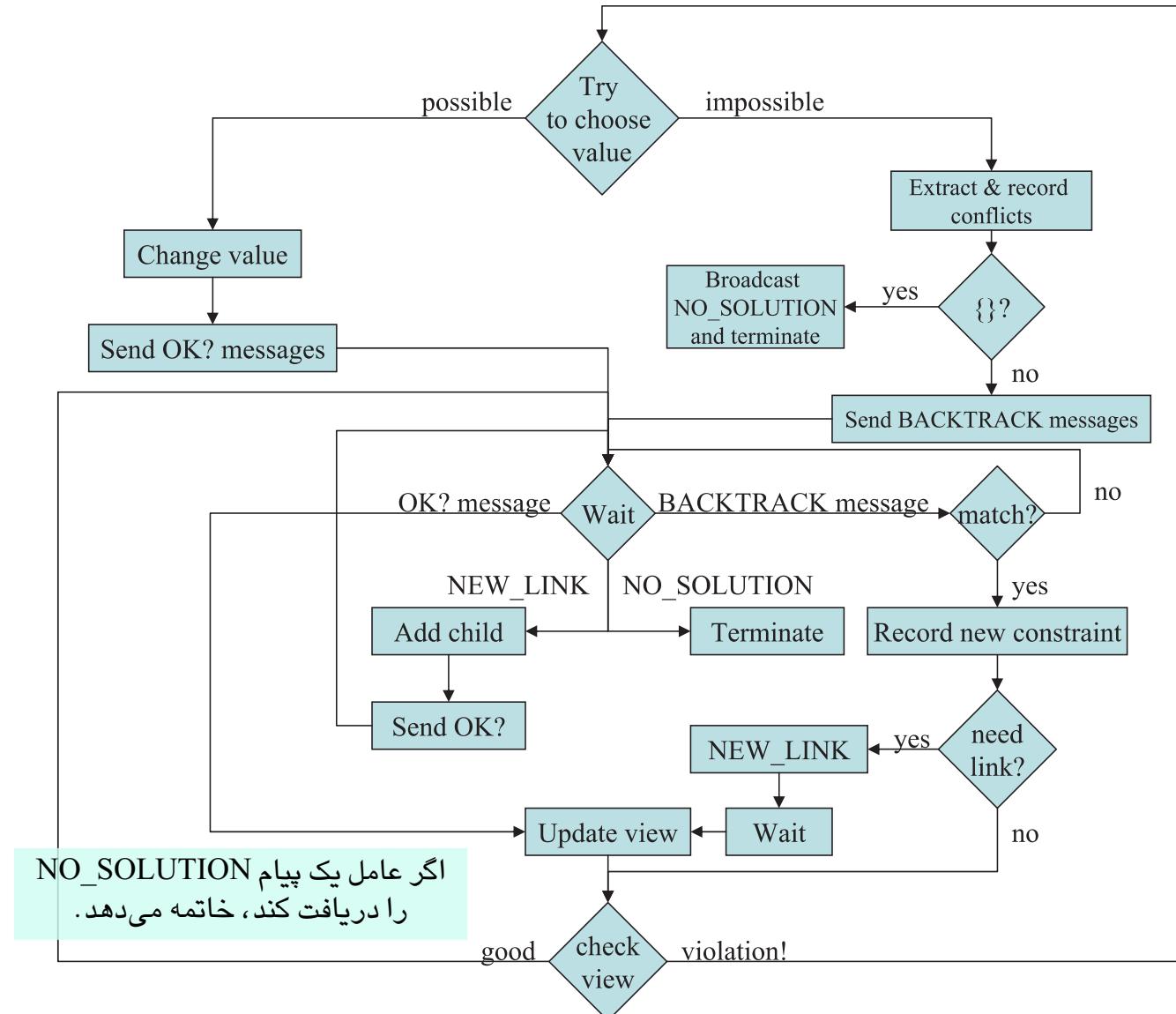
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

۱۶ از ۱۶

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

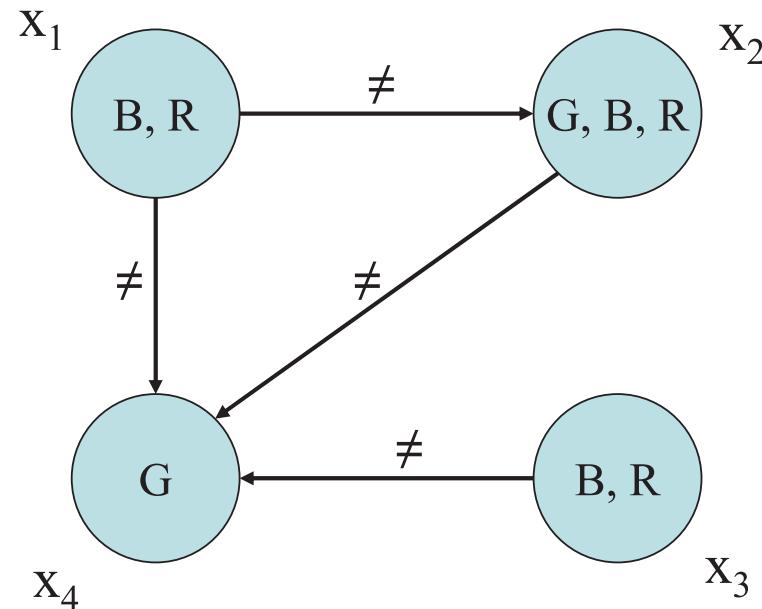


اگر عامل یک پیام NO\_SOLUTION را دریافت کند، خاتمه می‌دهد.

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱ از ۴۰)

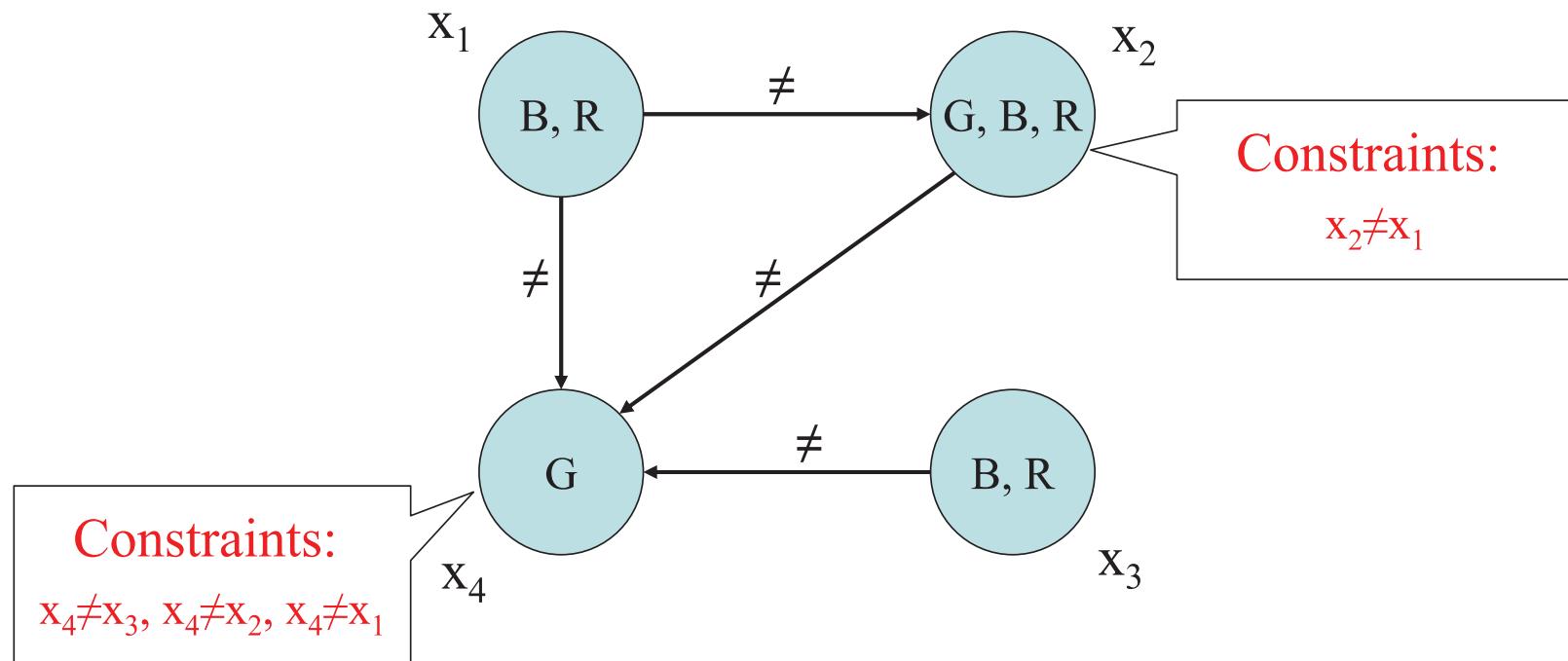
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۲)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمکام

مثال: رنگ‌آمیزی گراف (۳ از ۴۰) ساختمان داده‌ی لازم برای هر عامل/گره

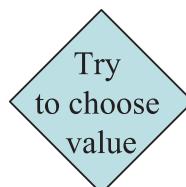
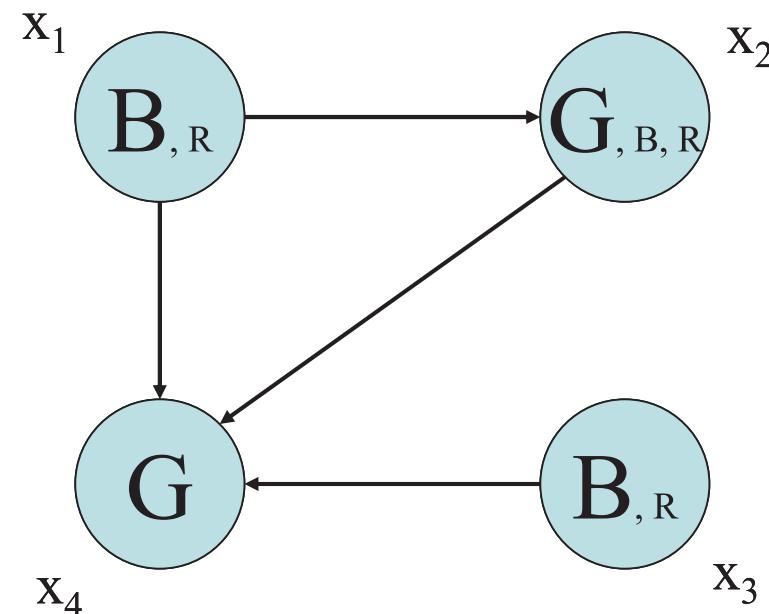
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

NAME	VALUE	DOMAIN
VIEW	CHILDREN	PARENTS
	KNOWN CONFLICTS	
	CONSTRAINTS TO ENFORCE	

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴ از ۴)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

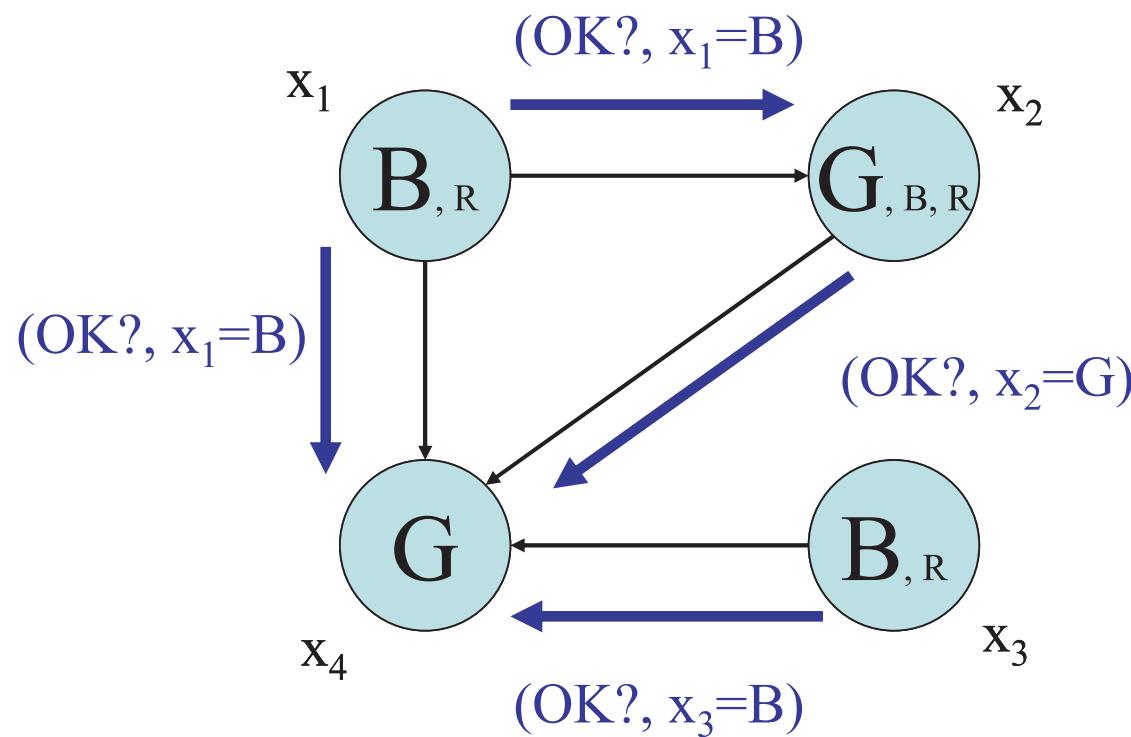


Each agent chooses an assignment to its variable

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۵ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



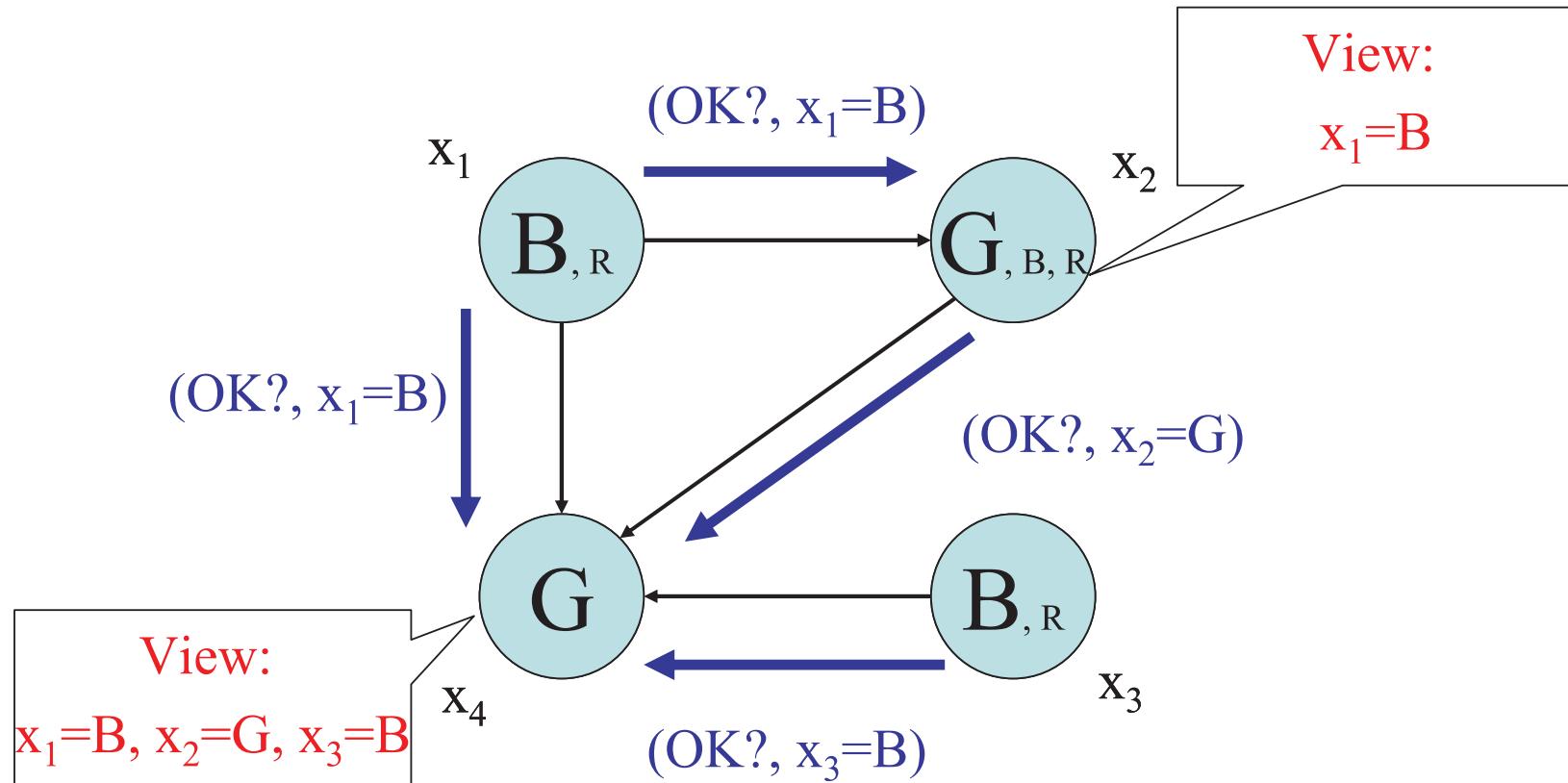
Send OK? messages

Each agent sends OK? messages to its children

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۶۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



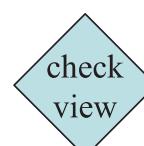
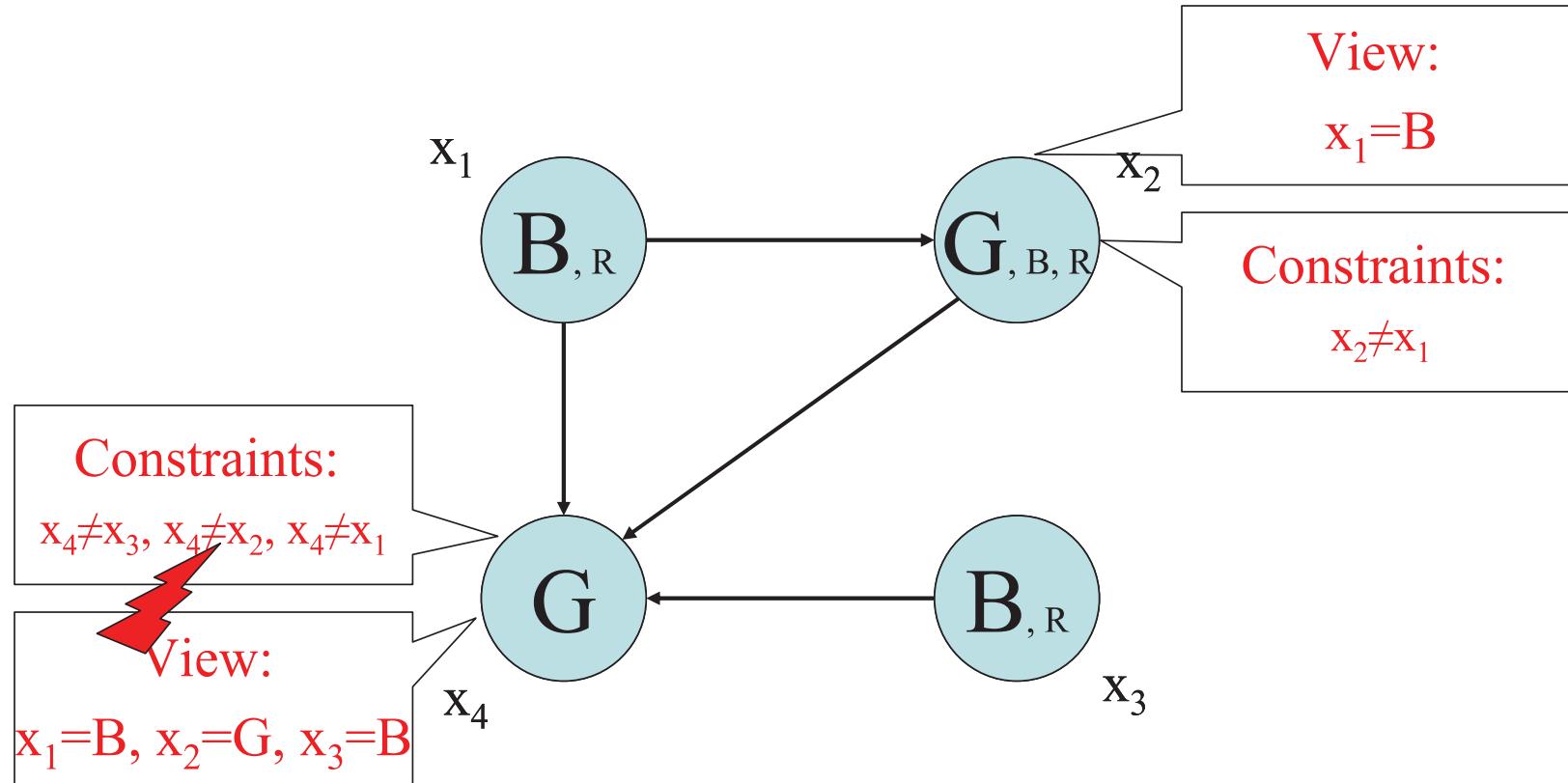
Update view

Agent x<sub>2</sub> and Agent x<sub>4</sub> update their view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۷ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

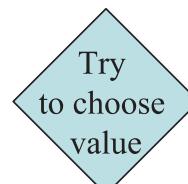
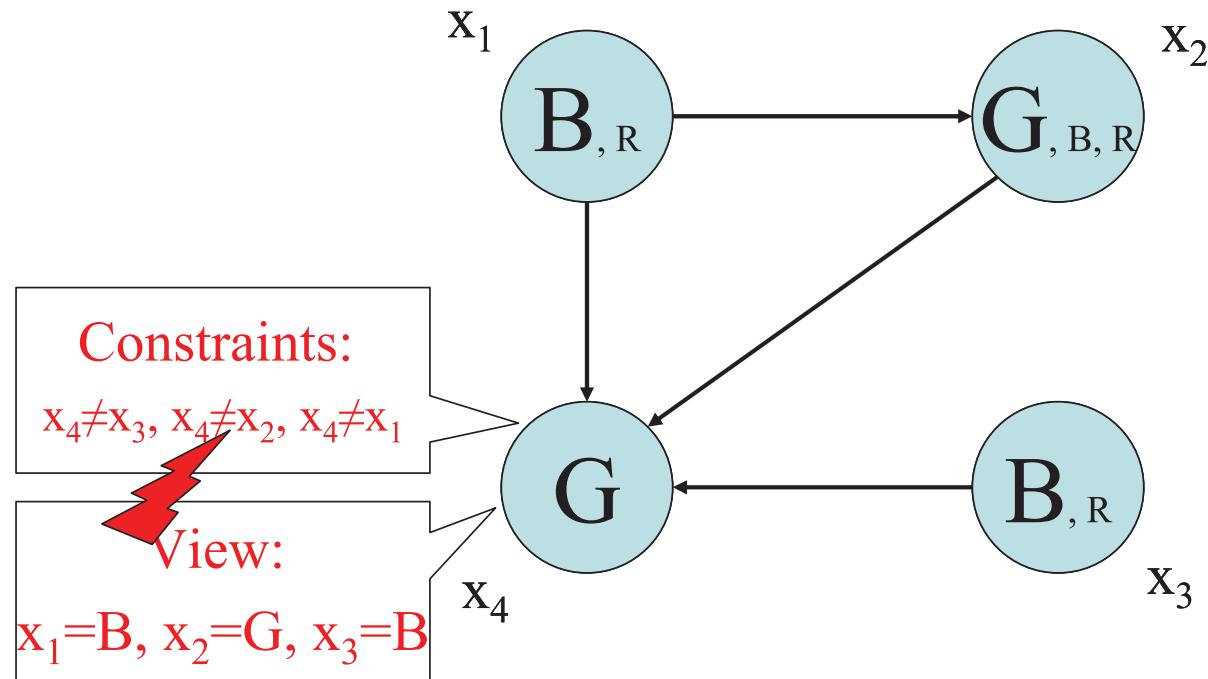


Agent  $x_2$  and Agent  $x_4$  check their view against their constraints, and Agent  $x_4$  discovers a violation

## الگوریتم عقب‌گرد ناهمکام

مثال: رنگ‌آمیزی گراف (۴۰ از ۸۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

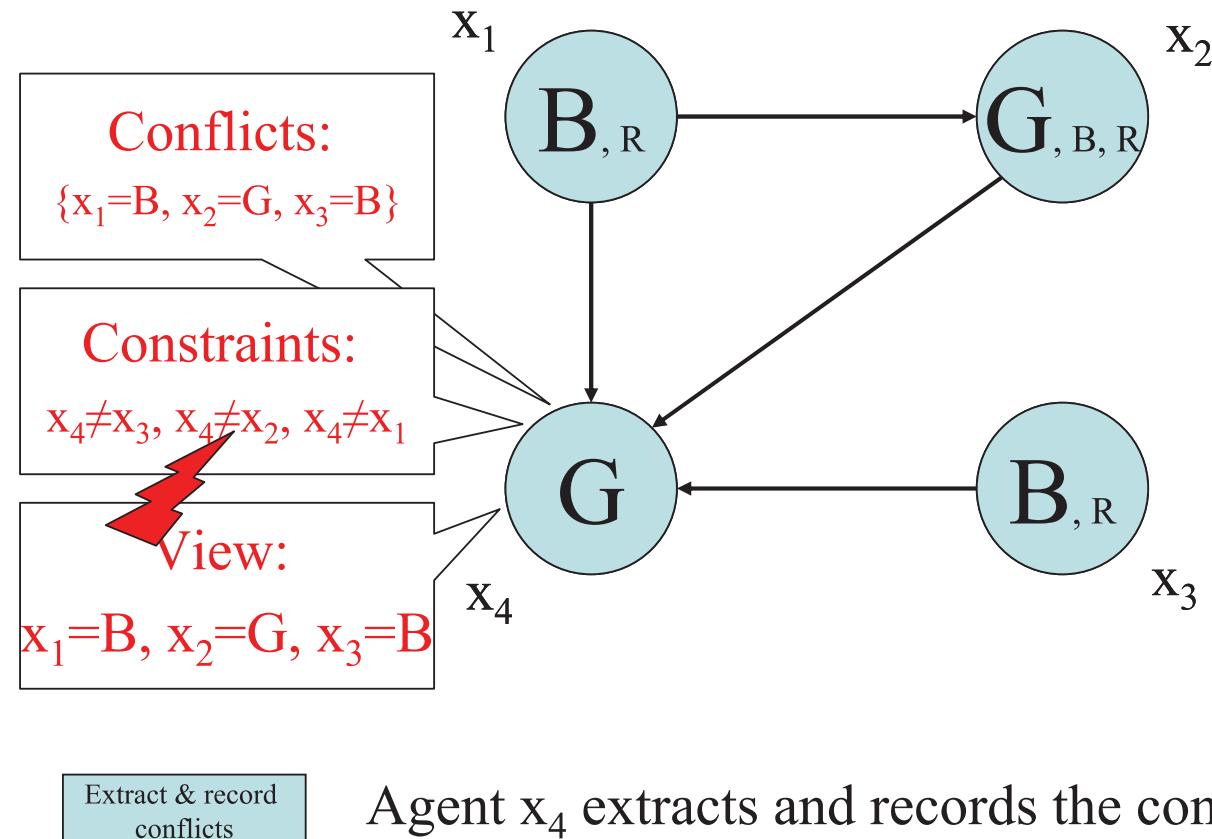


Agent  $x_4$  tries to change its assignment,  
which is impossible

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۹)

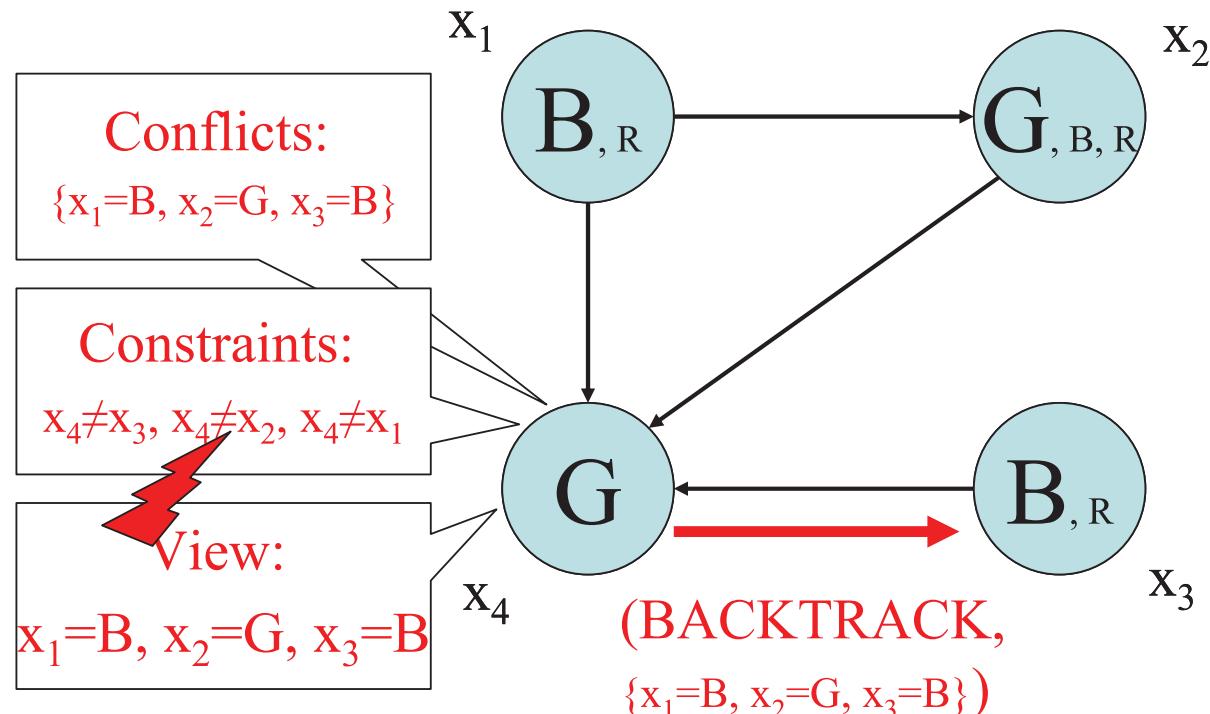
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۰ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

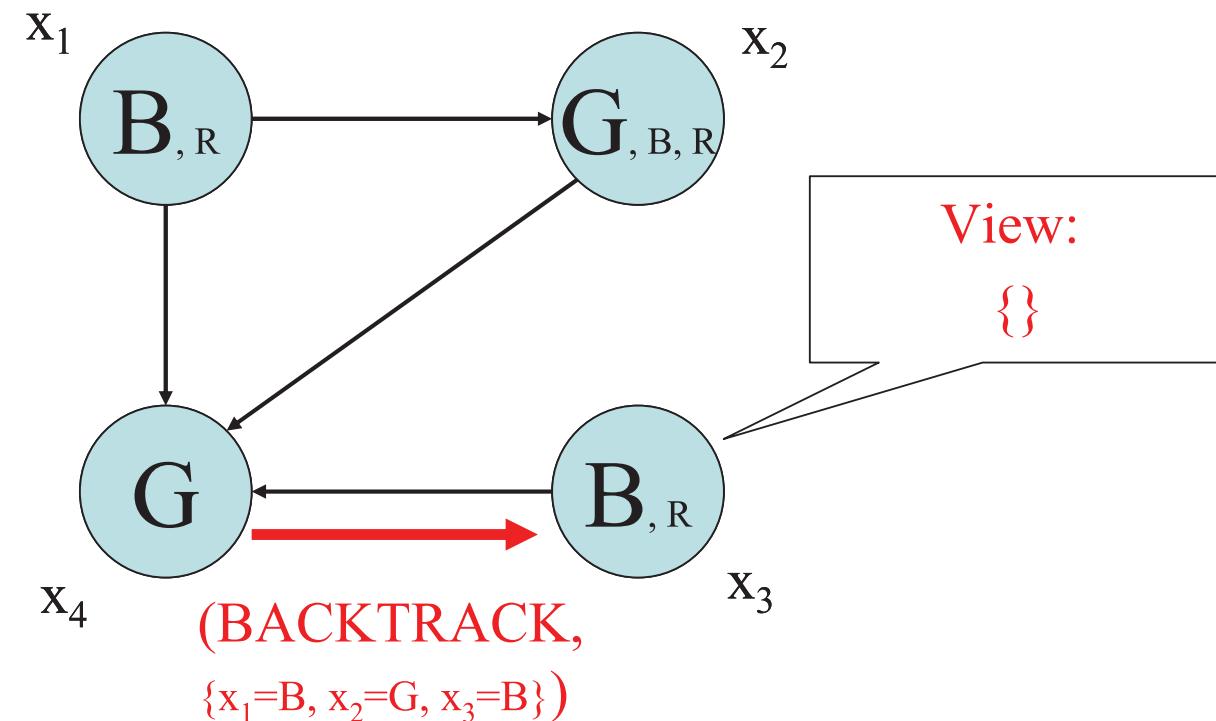


{ } is not among the new conflicts, so Agent  $x_4$  sends BACKTRACK messages

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۱ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

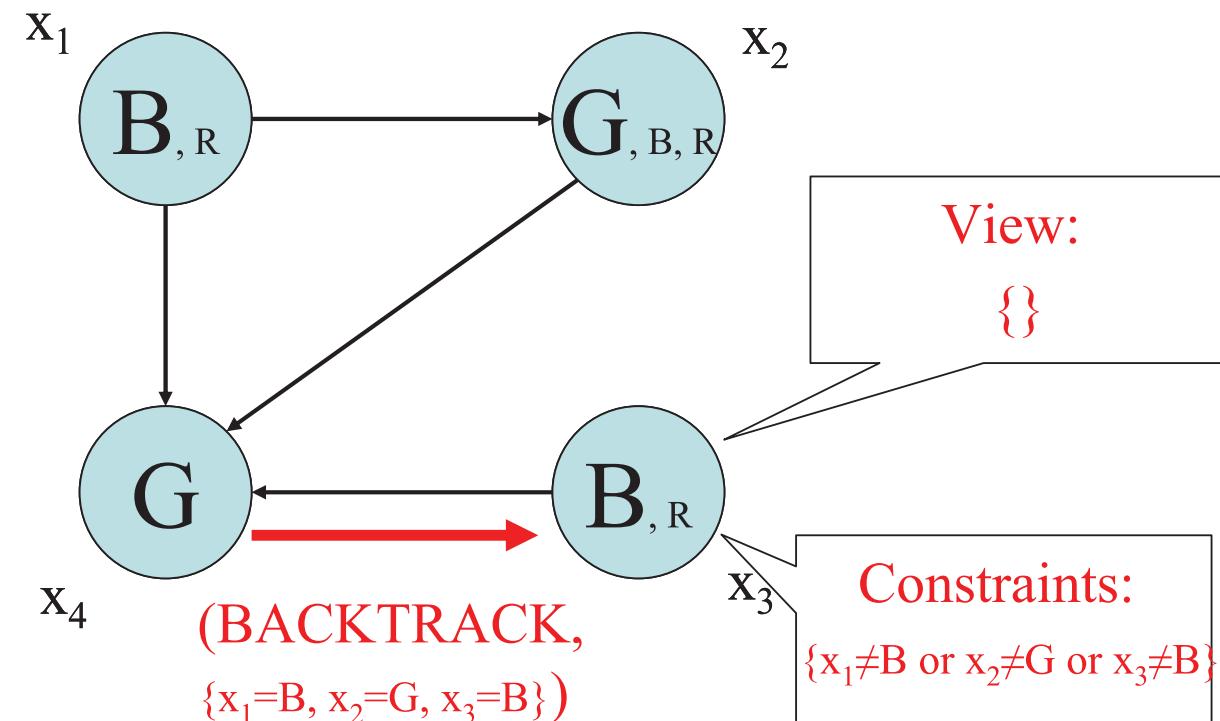


Agent  $x_3$  receives the message and checks the conflict against its view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۲ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



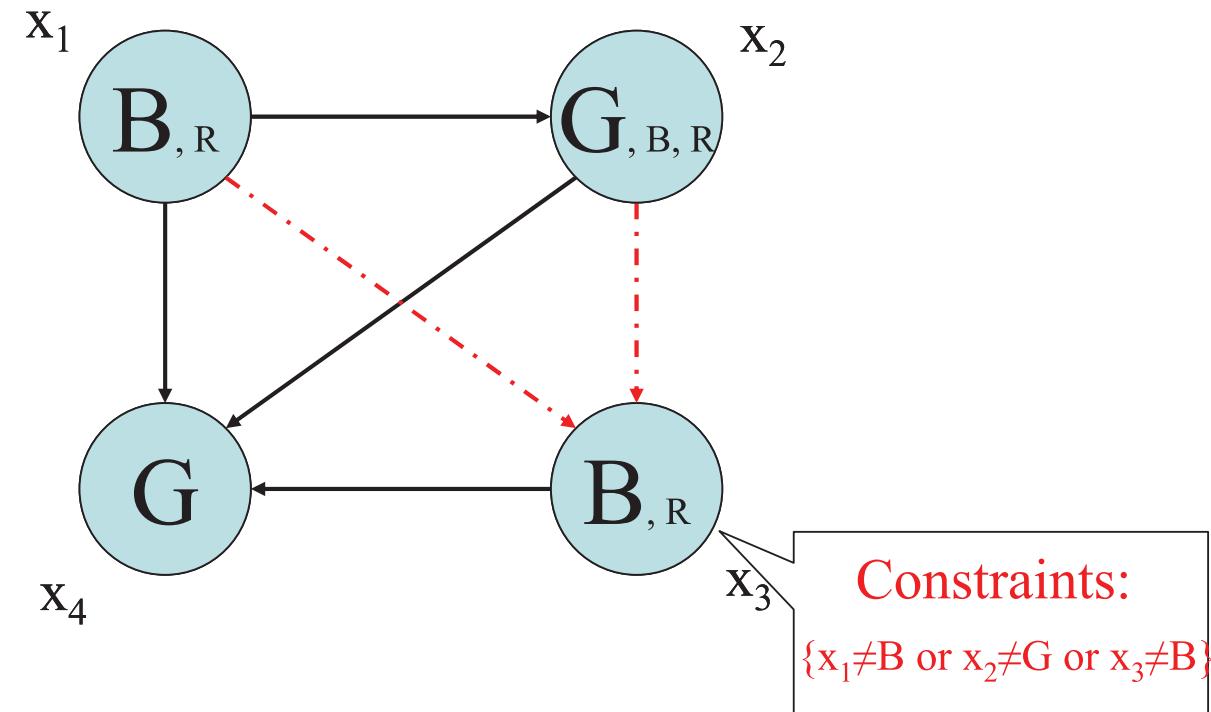
Record new constraint

Agent  $x_3$  records the conflict as a new constraint

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۳ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

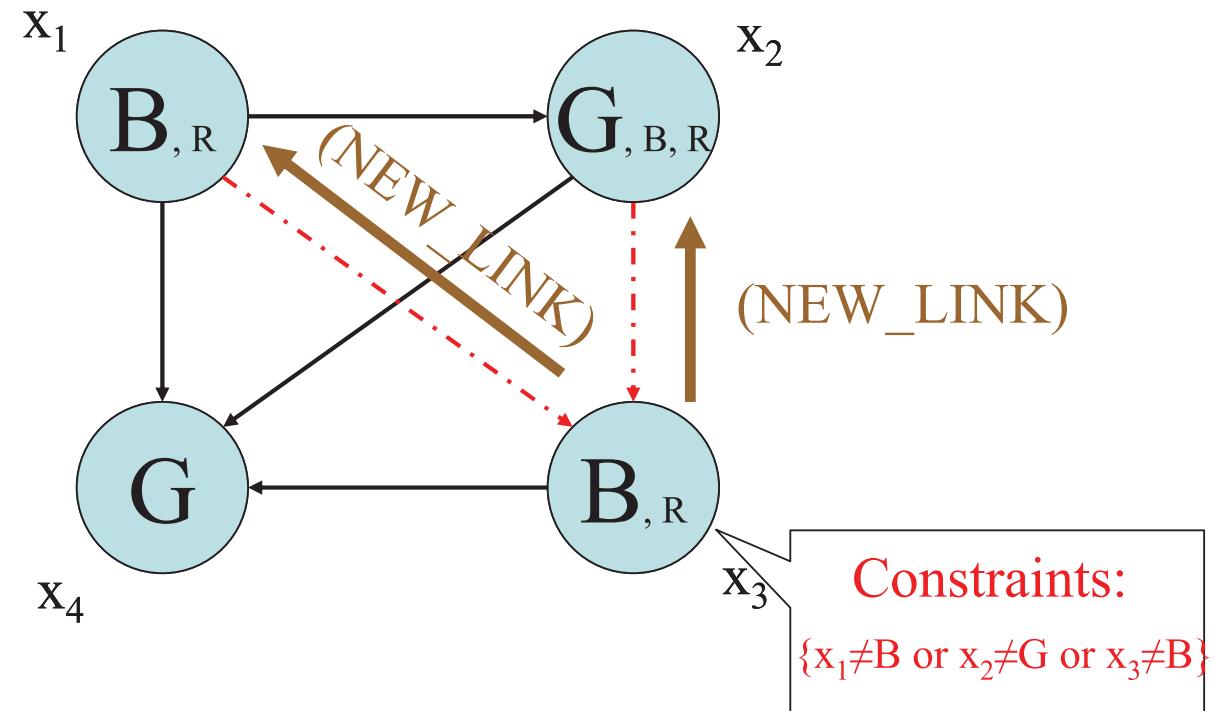


Agent  $x_3$  checks if it needs new links to enforce it

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۴ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

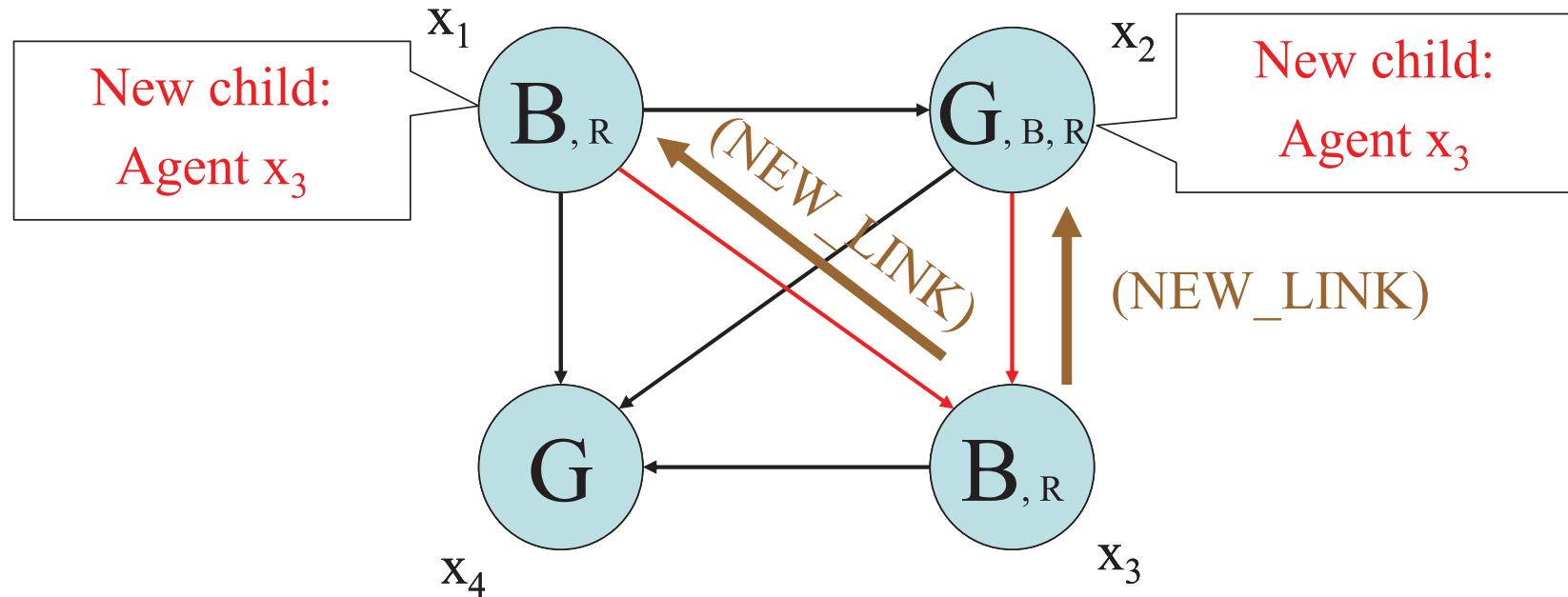


NEW\_LINK

Agent  $x_3$  sends NEW\_LINK requests

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۵ از ۴۰)

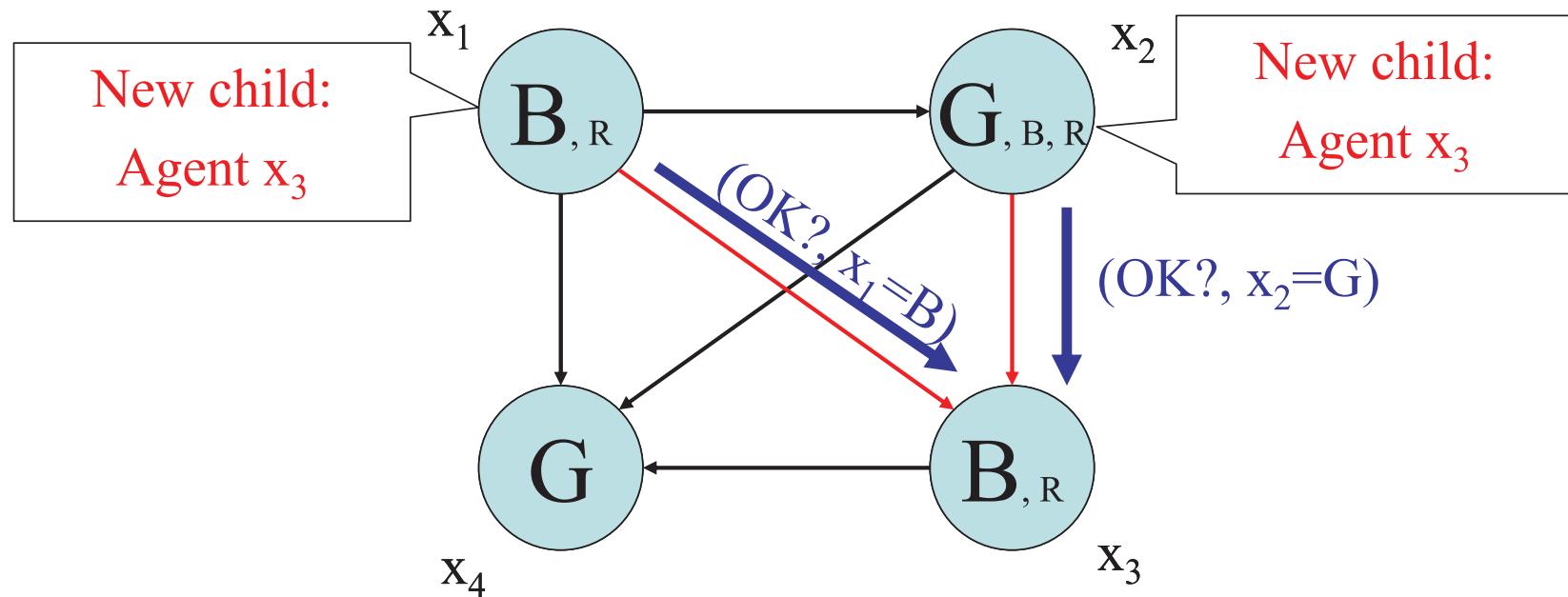
THE ASYNCHRONOUS BACKTRACKING ALGORITHM

Add child

Agents  $x_1$  and  $x_2$  receive the  $NEW\_LINK$  requests  
and add Agent  $x_3$  to their children list

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۶ از ۴۰)

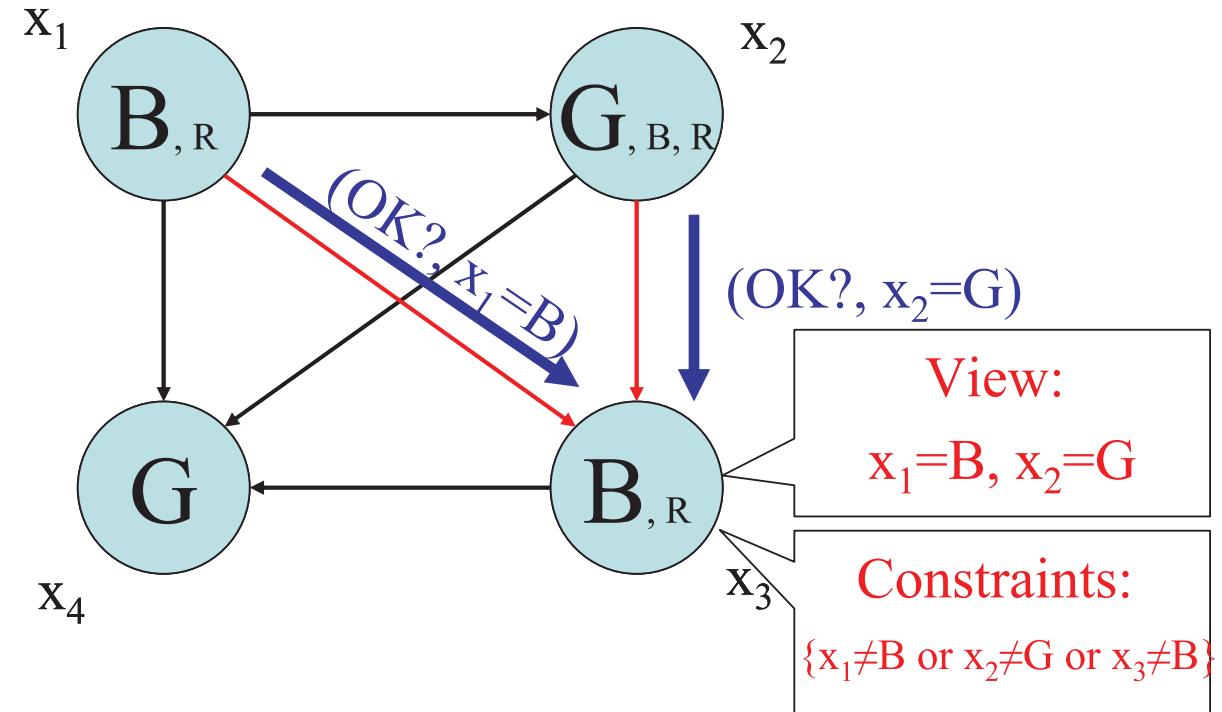
THE ASYNCHRONOUS BACKTRACKING ALGORITHM

Send OK?

Agents  $x_1$  and  $x_2$  send OK? to confirm new links

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۱۷)

THE ASYNCHRONOUS BACKTRACKING ALGORITHM

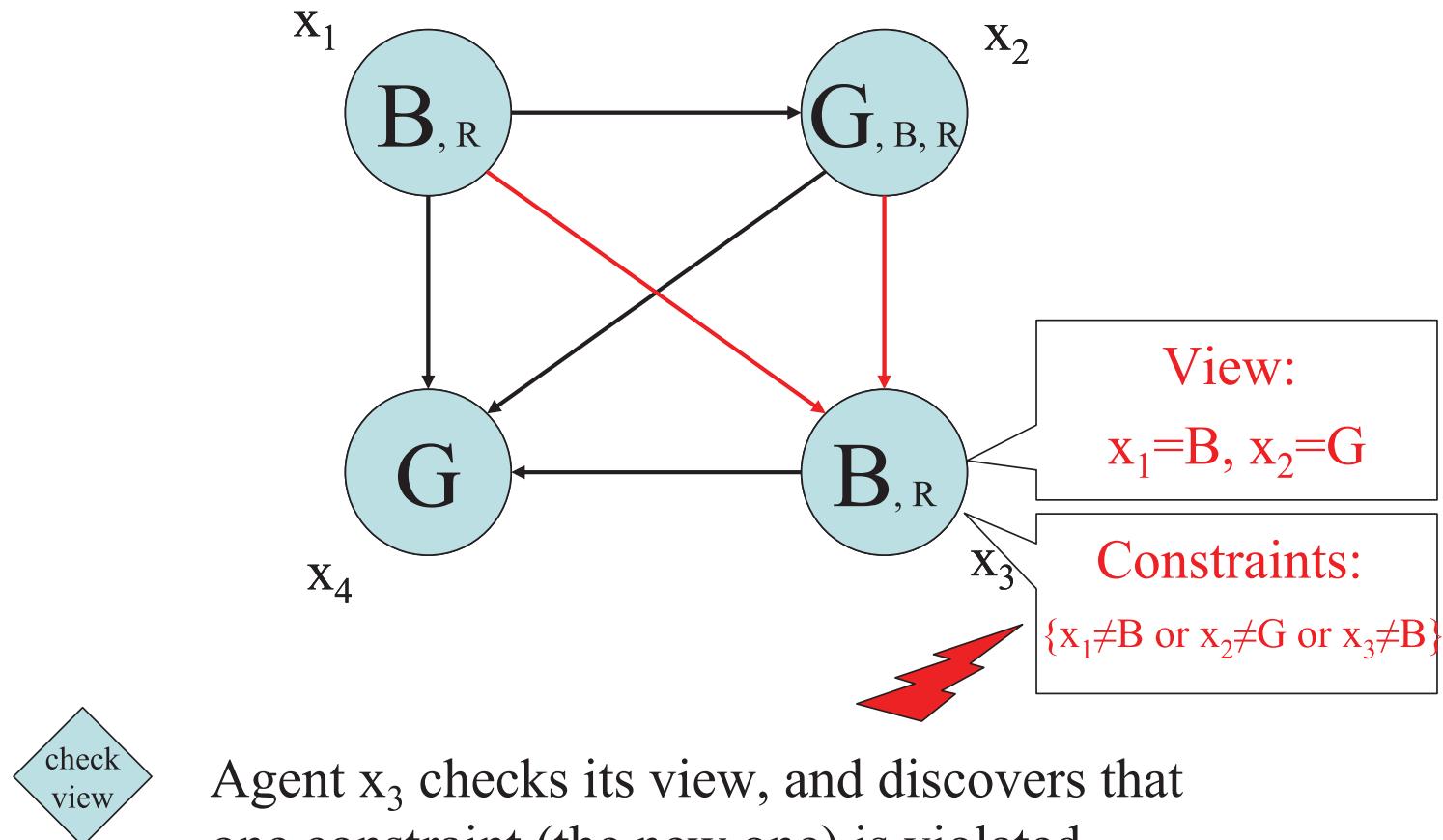
Update view

Agent  $x_3$  receives messages and updates its view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۸ از ۴۰)

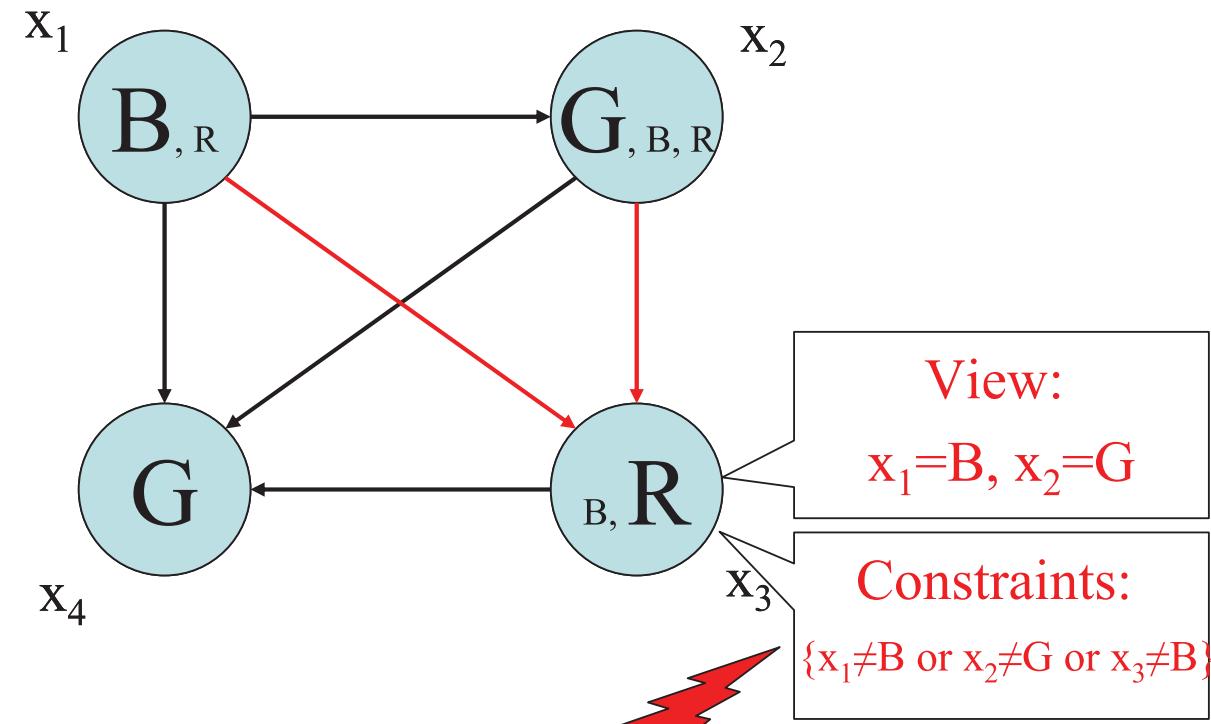
### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۱۹ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

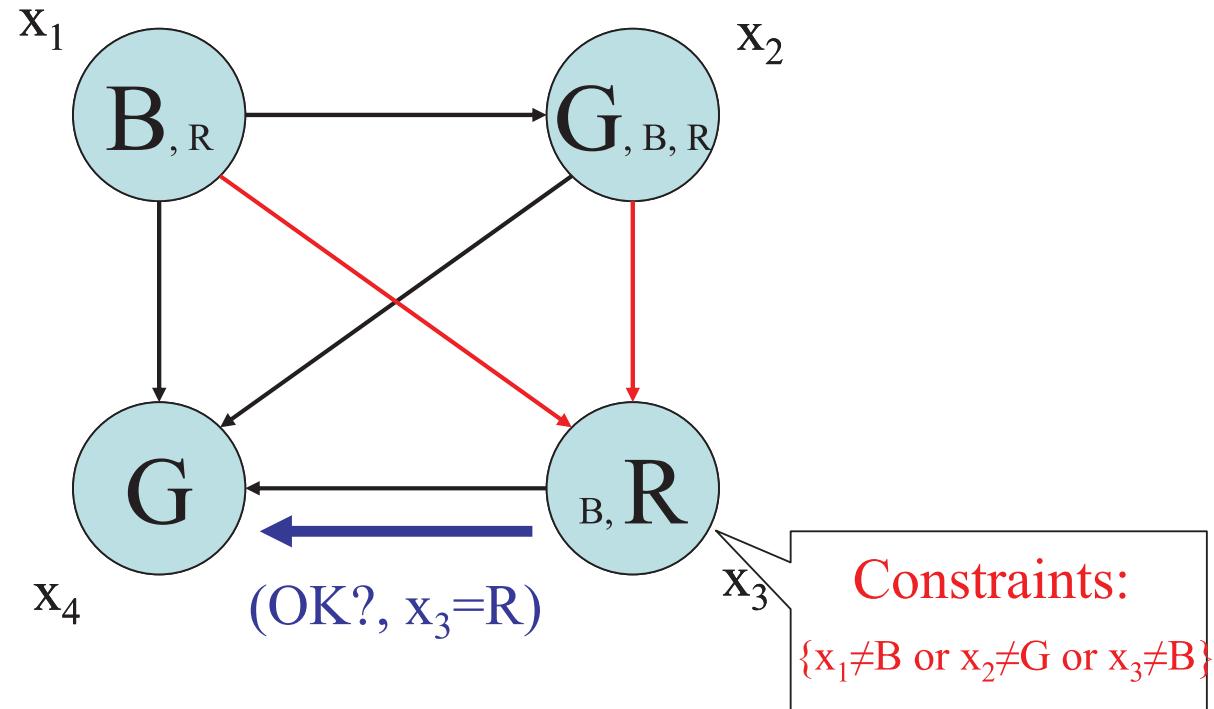


Agent  $x_3$  tries to change its value to R

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۰ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

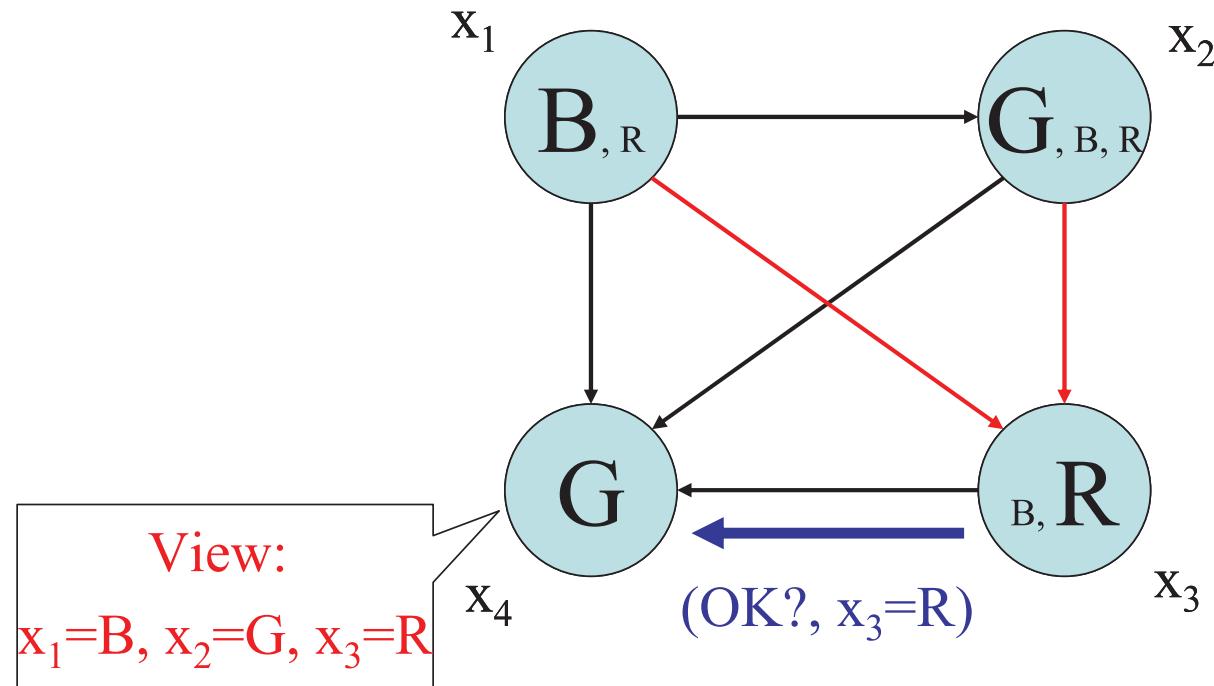


There is no more violation, so Agent  $x_3$  communicates its new value to its children

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۱ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



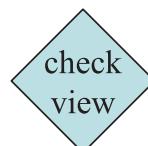
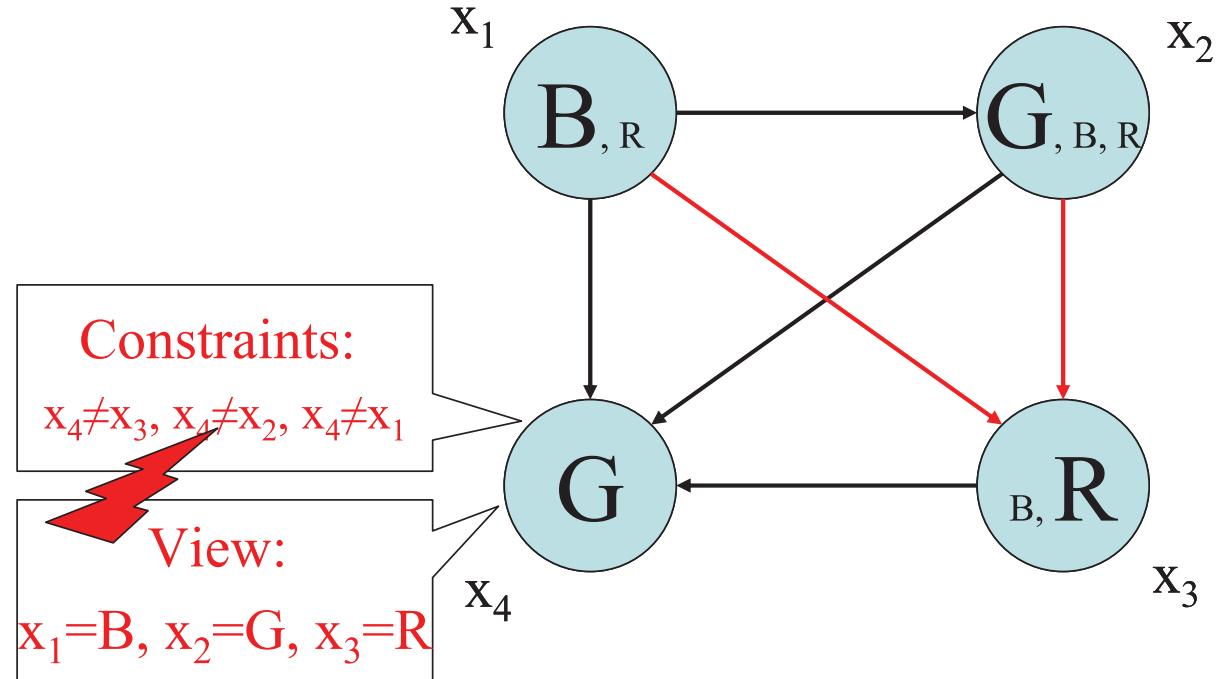
Update view

Agent  $x_4$  receives the OK? message  
and updates its view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۰ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

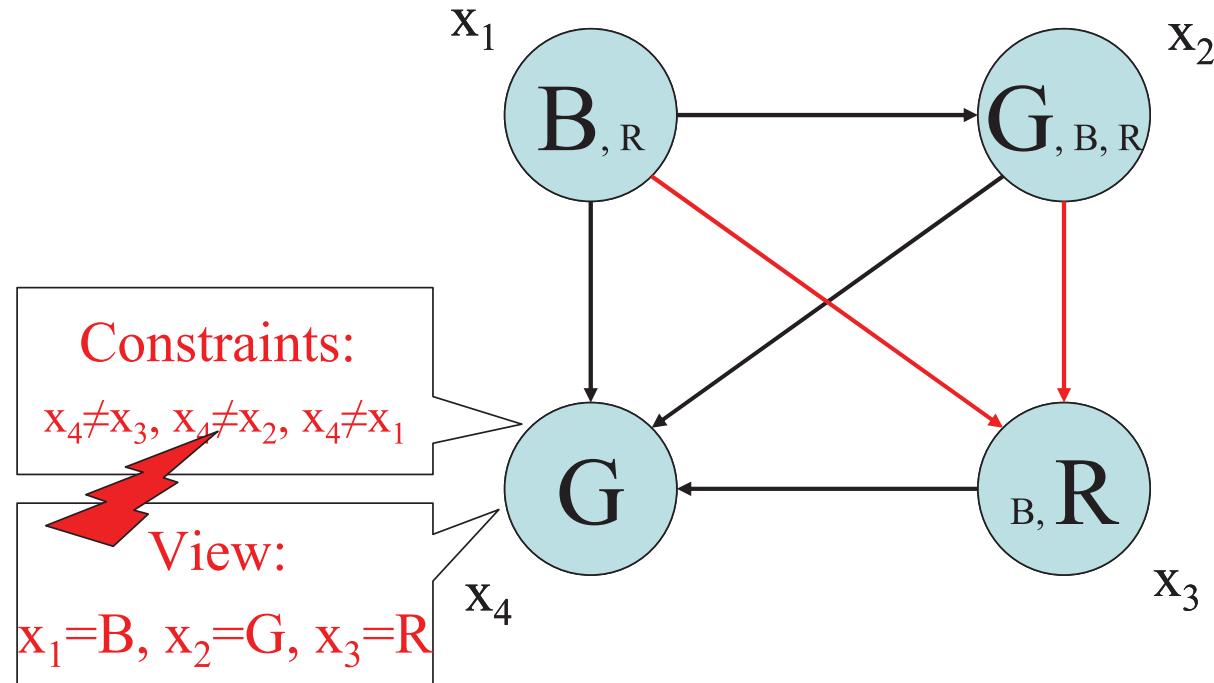


Agent  $x_4$  checks its view against its constraints and sees the violation has not been resolved...

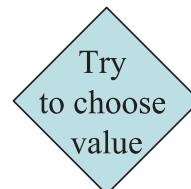
## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۳ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

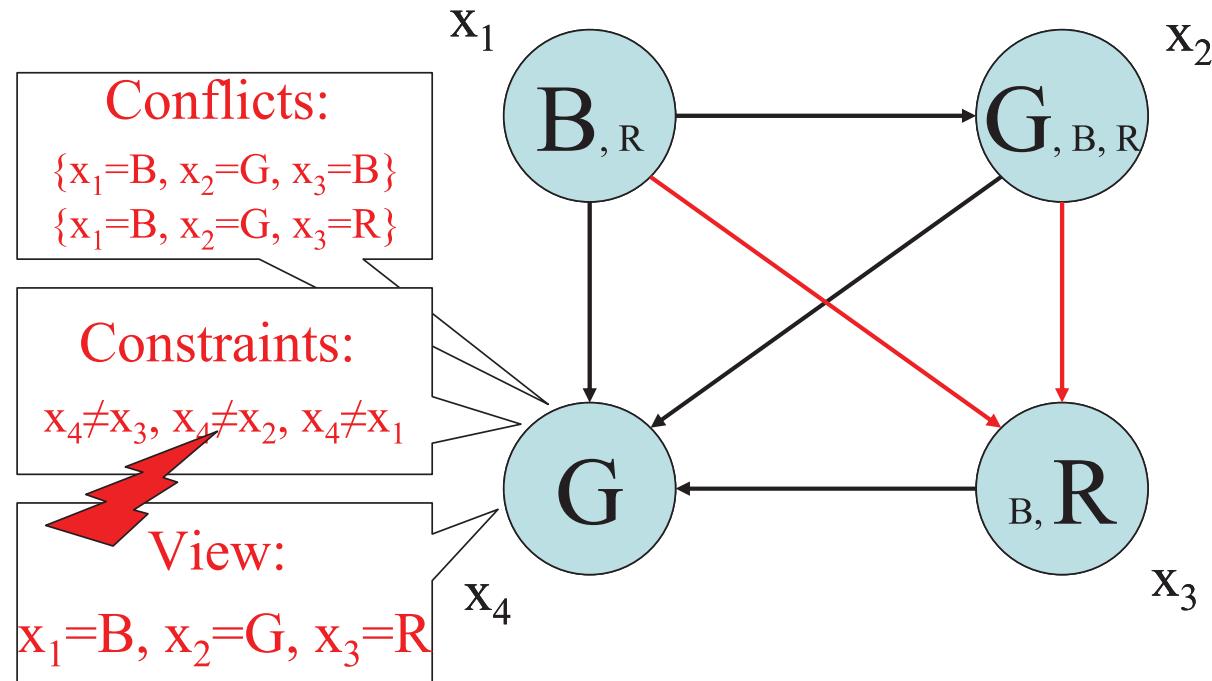


Agent  $x_4$  tries to change its value, but this is impossible



## الگوریتم عقب‌گرد ناهمکام

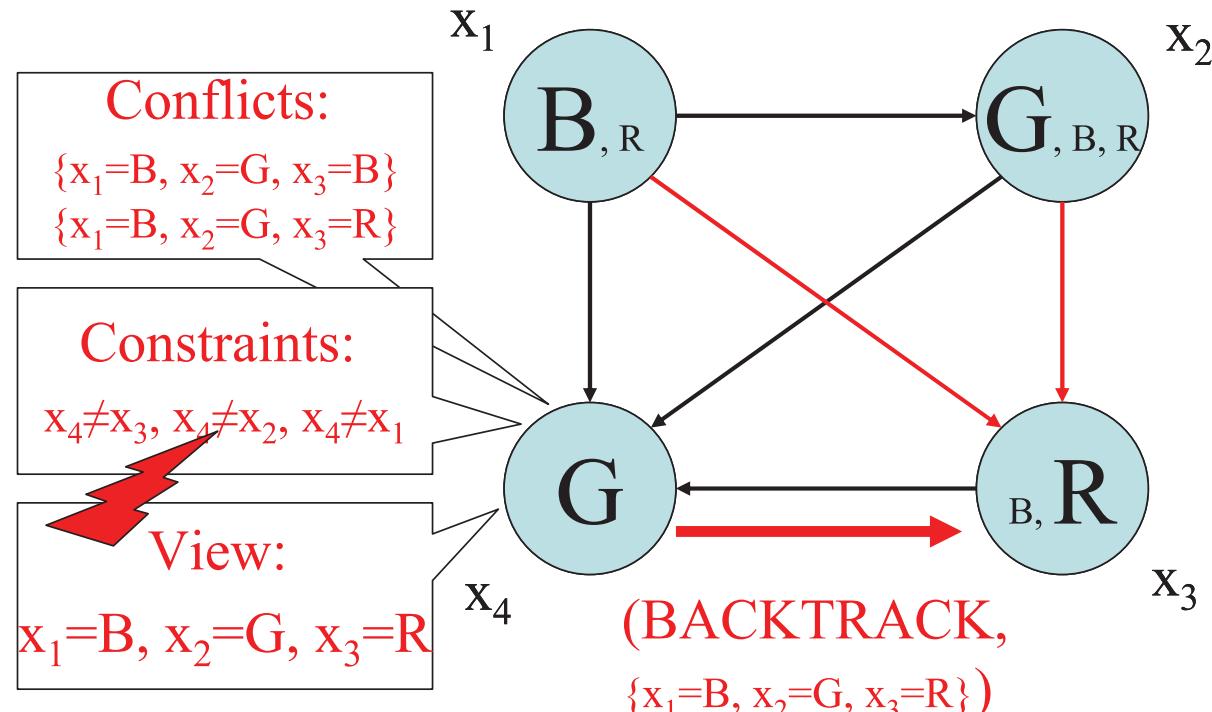
مثال: رنگ‌آمیزی گراف (۲۰ از ۴۰)

THE ASYNCHRONOUS BACKTRACKING ALGORITHMExtract & record  
conflictsAgent  $x_4$  extracts and records the conflicts

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۵ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

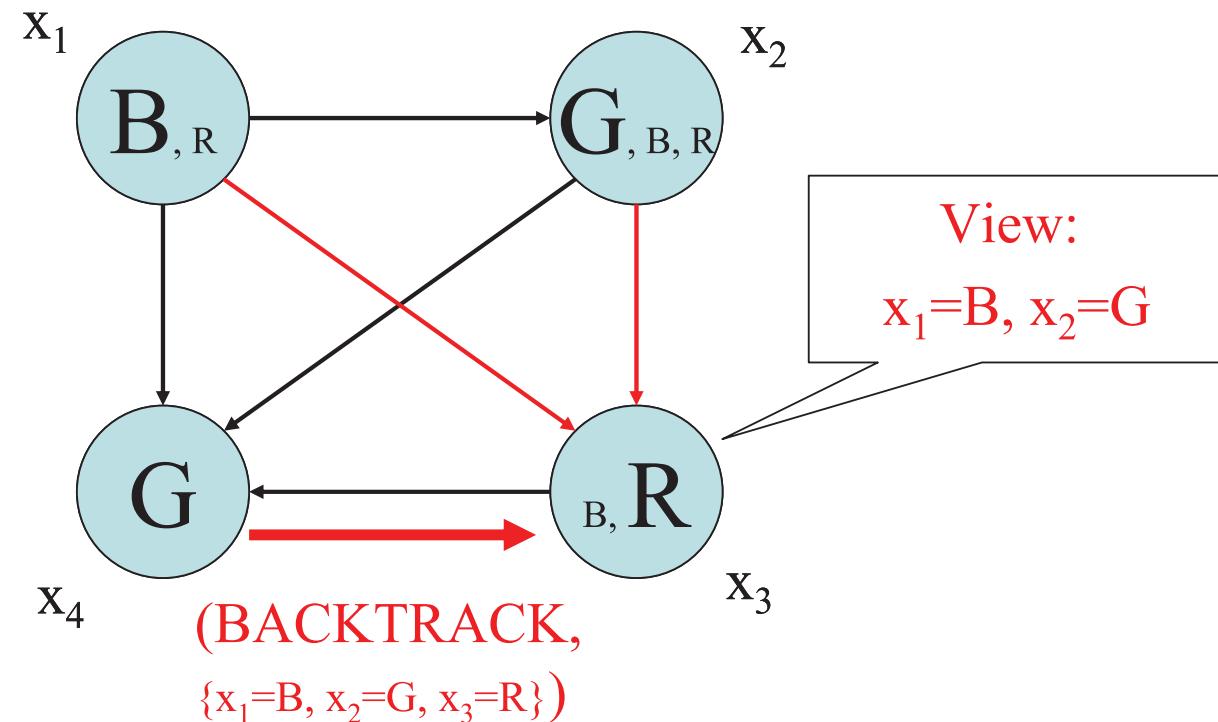


$\{\}$  is not among the new conflicts, so  
Agent  $x_4$  sends BACKTRACK messages

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۶ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

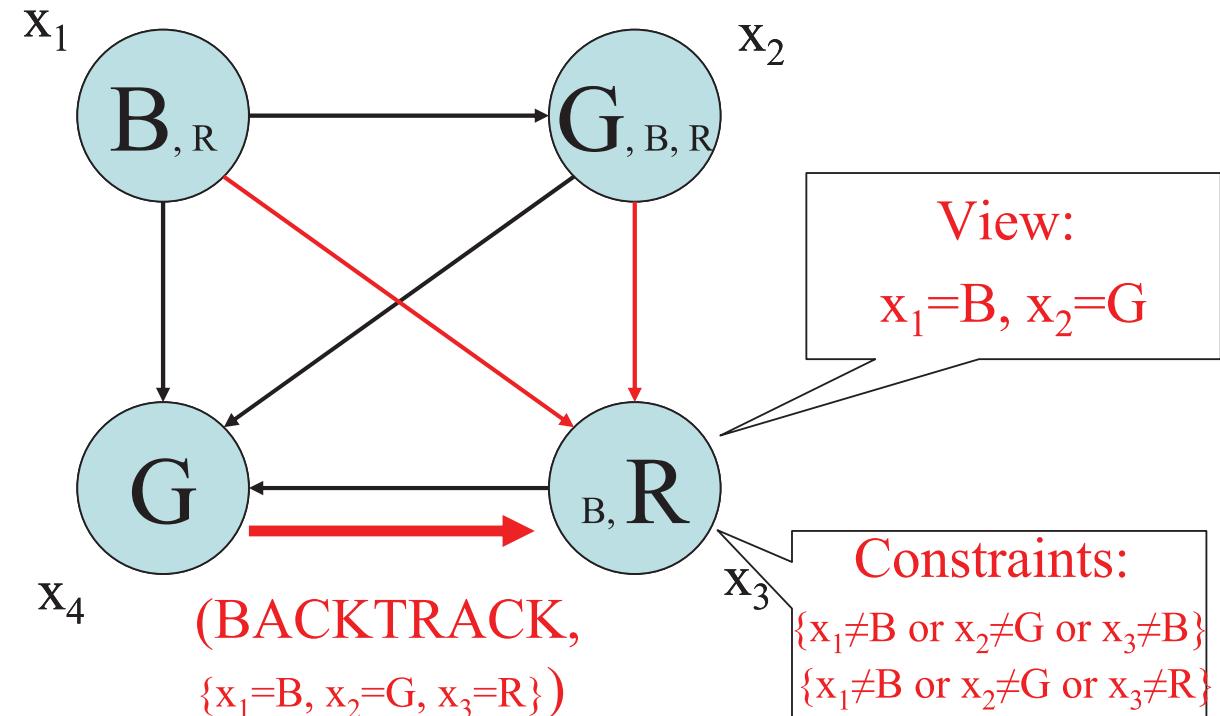


Agent  $x_3$  receives the message and checks the conflict against its view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۲۷ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

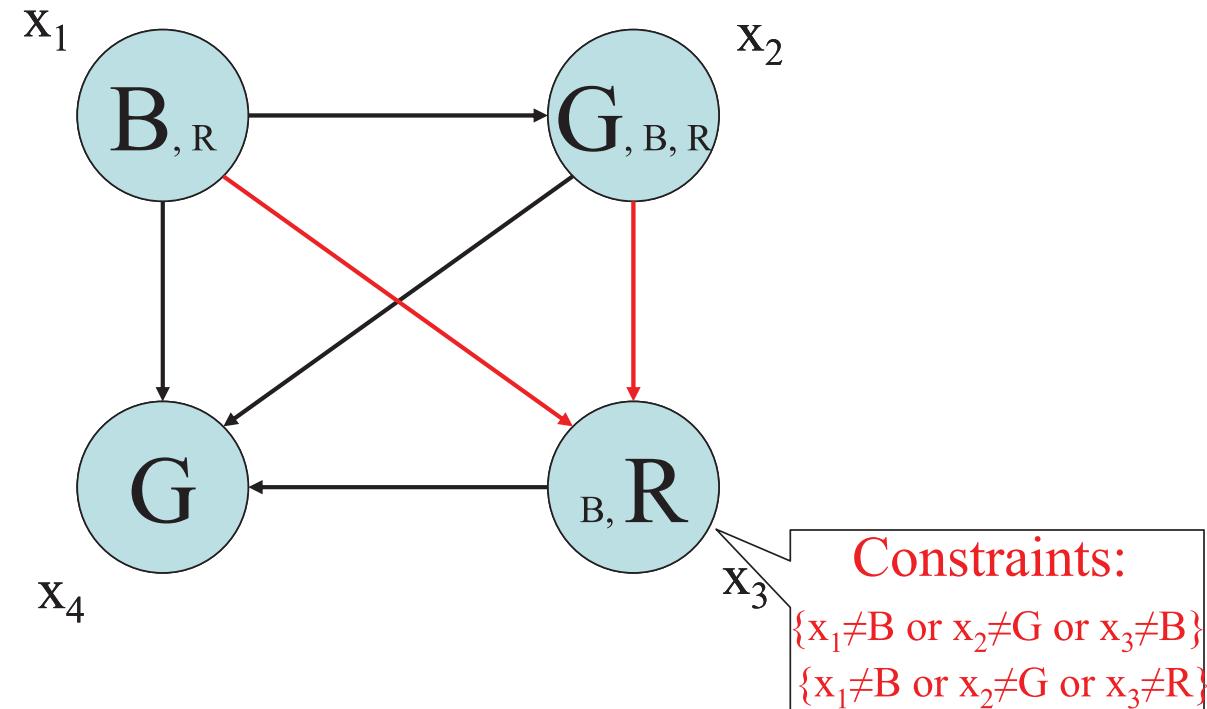


Agent  $x_3$  records the conflict as a new constraint

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۲۸)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

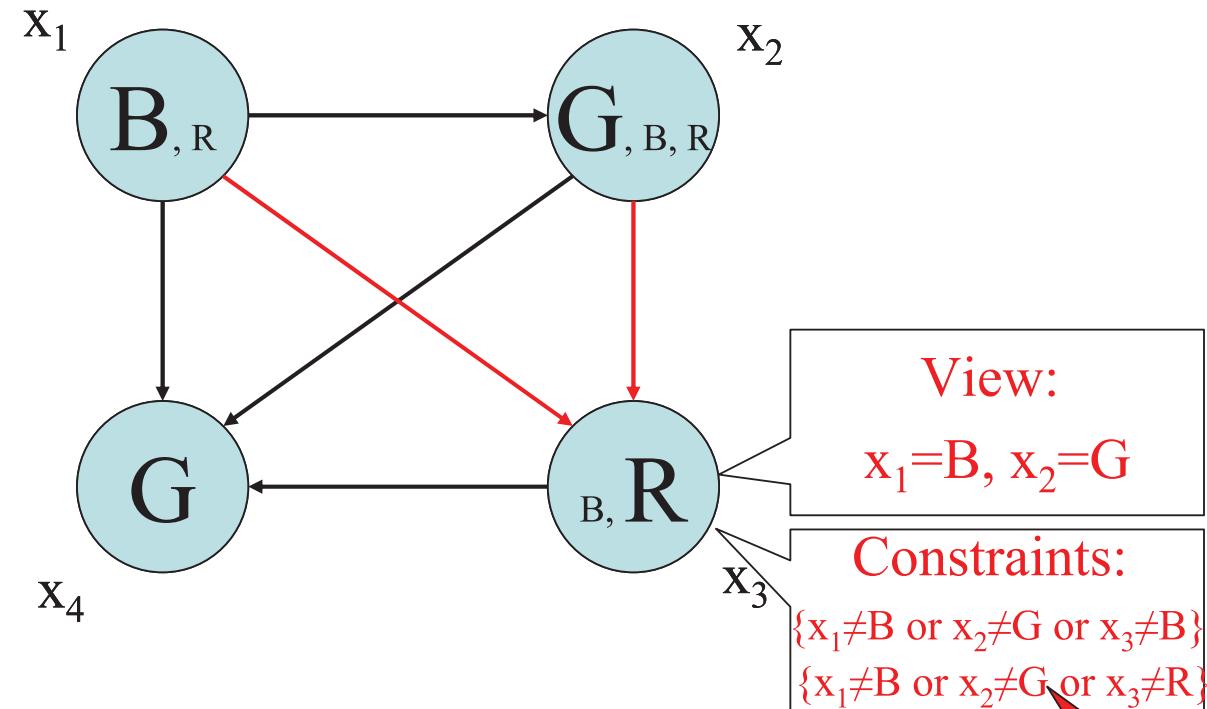


Agent  $x_3$  needs no new link to enforce it

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۲۹)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



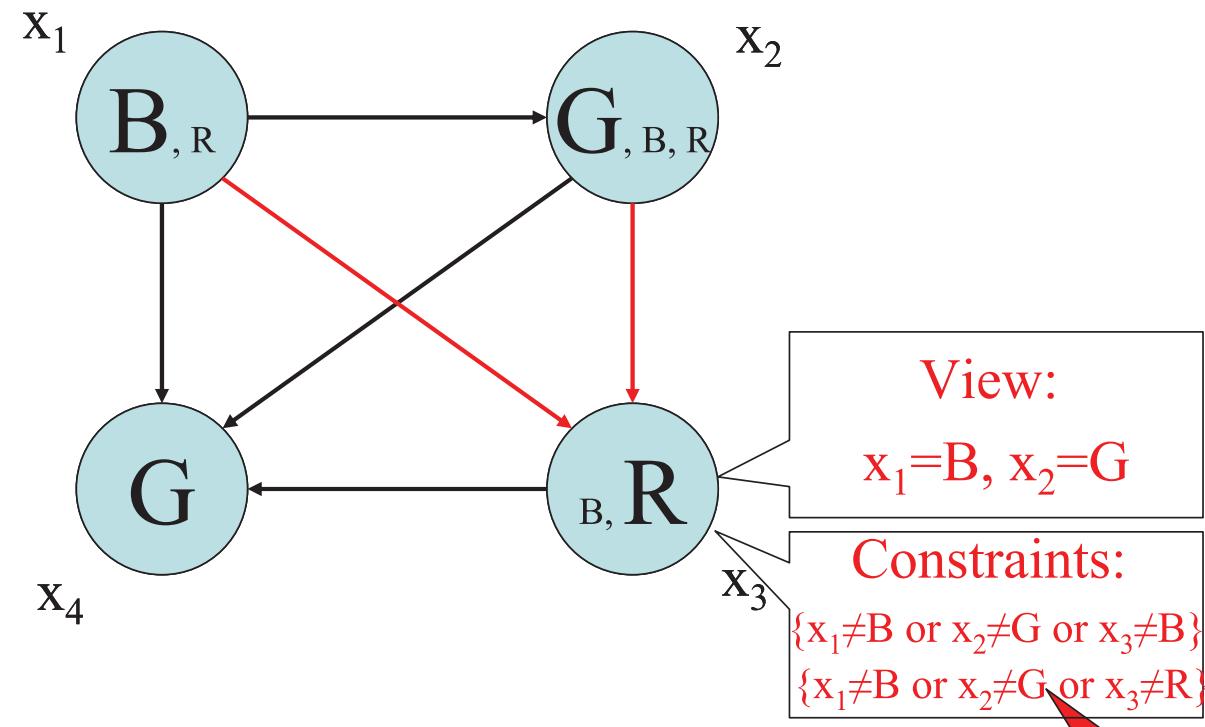
Agent  $x_3$  checks its view, and discovers that one constraint (the new one) is violated

check  
view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۰ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



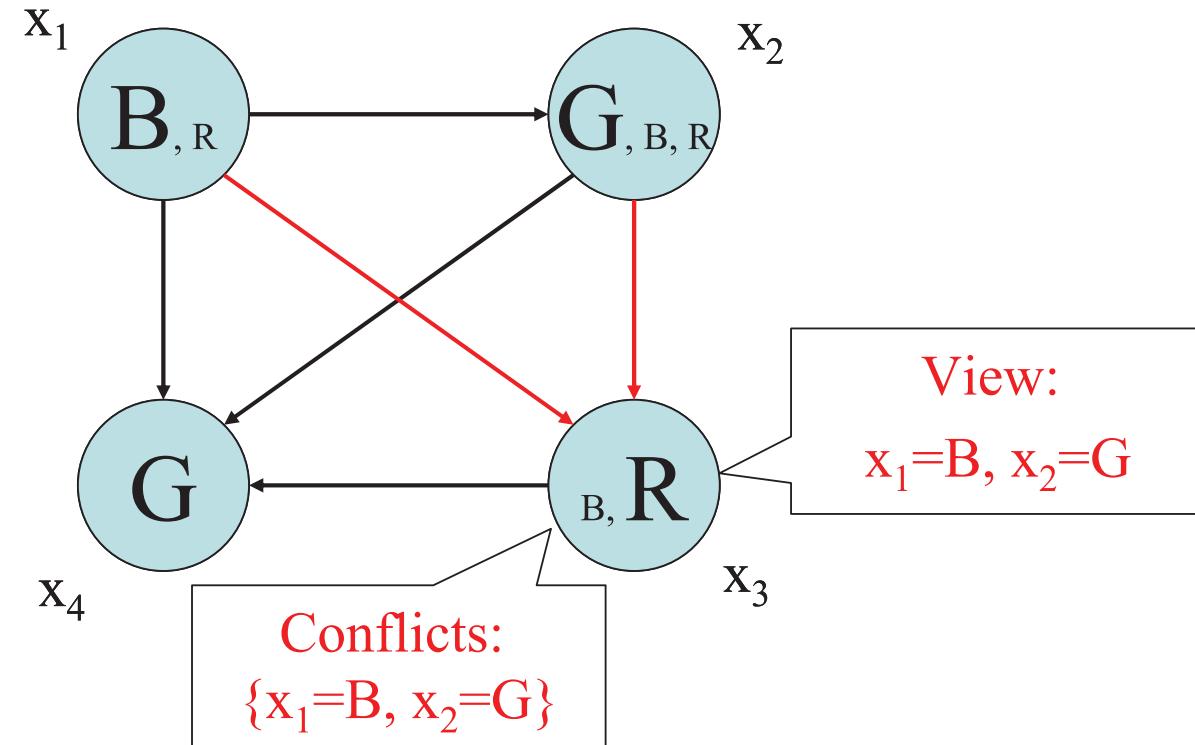
Try  
to choose  
value

Agent  $x_3$  tries to change its value, but no value satisfies all the constraints

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۱ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



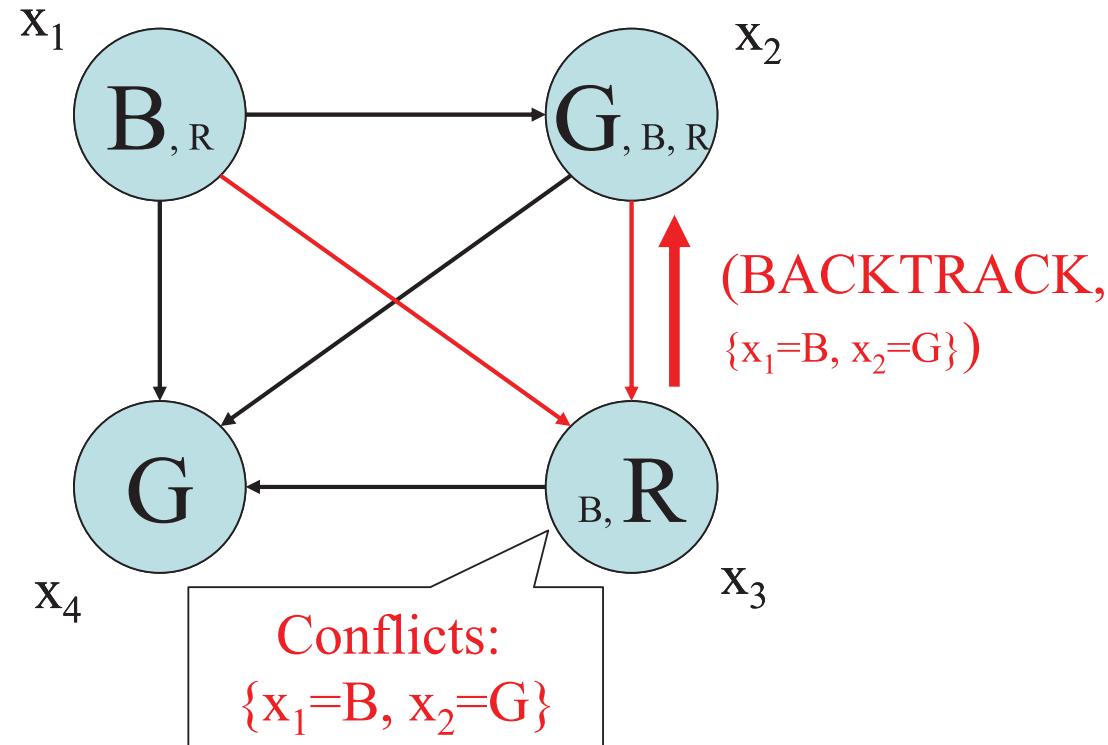
Agent  $x_3$  extracts and records new conflicts

Extract & record  
conflicts

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۲ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

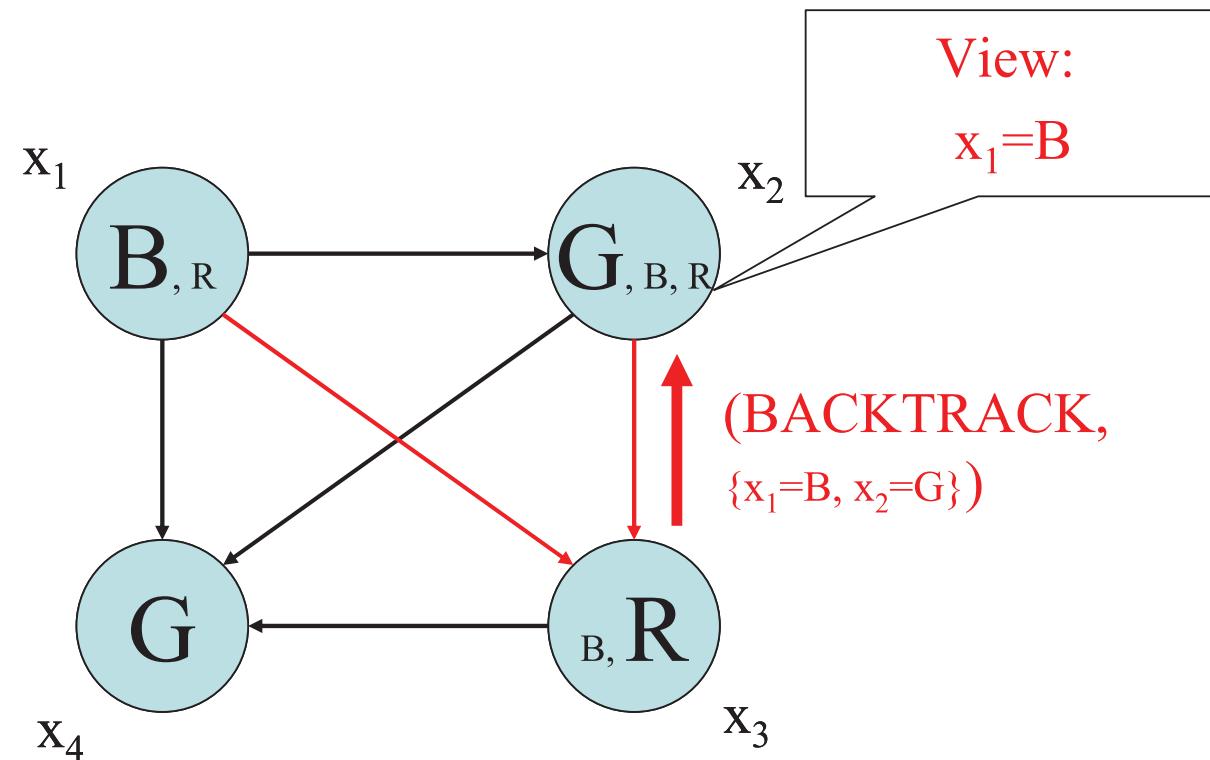


$\{\}$  is not a new conflict, so Agent  $x_3$  sends BACKTRACK messages

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۰ از ۳۳)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

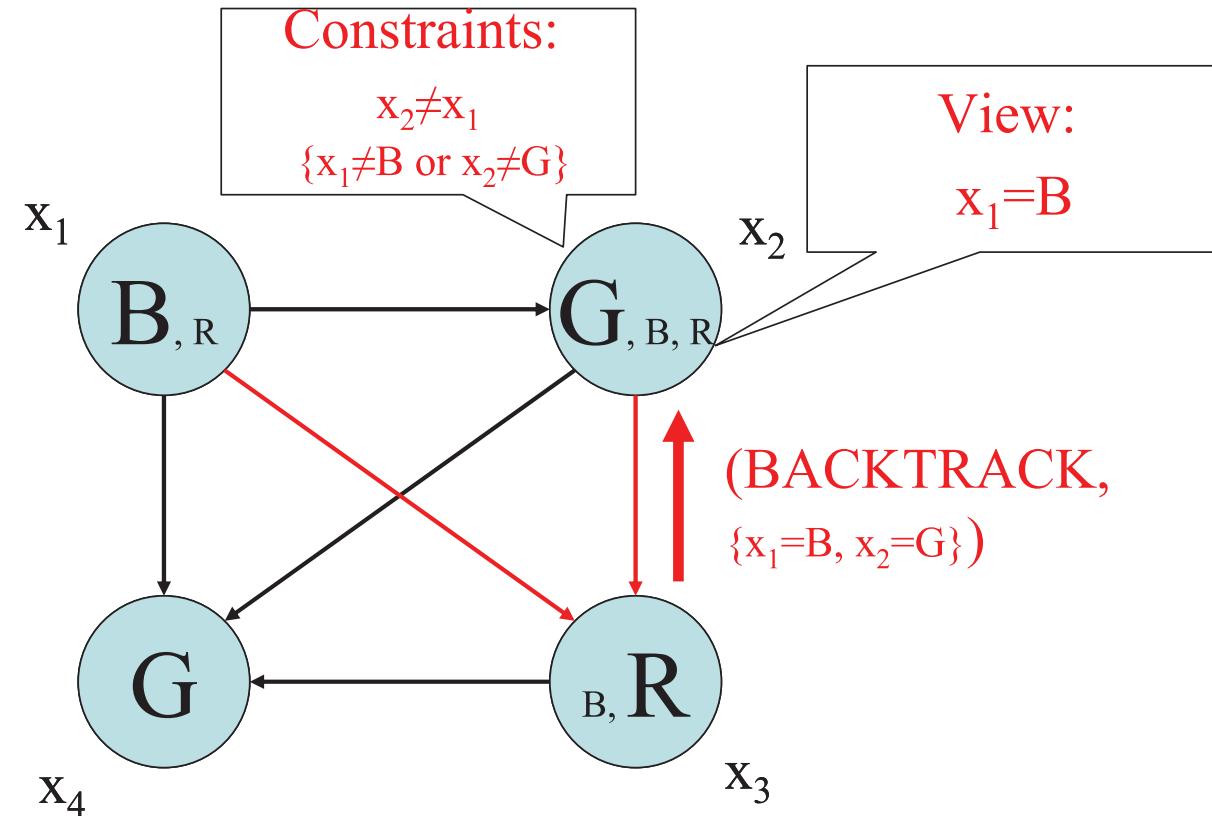


Agent  $x_2$  receives the message and checks the conflict against its view

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۴ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



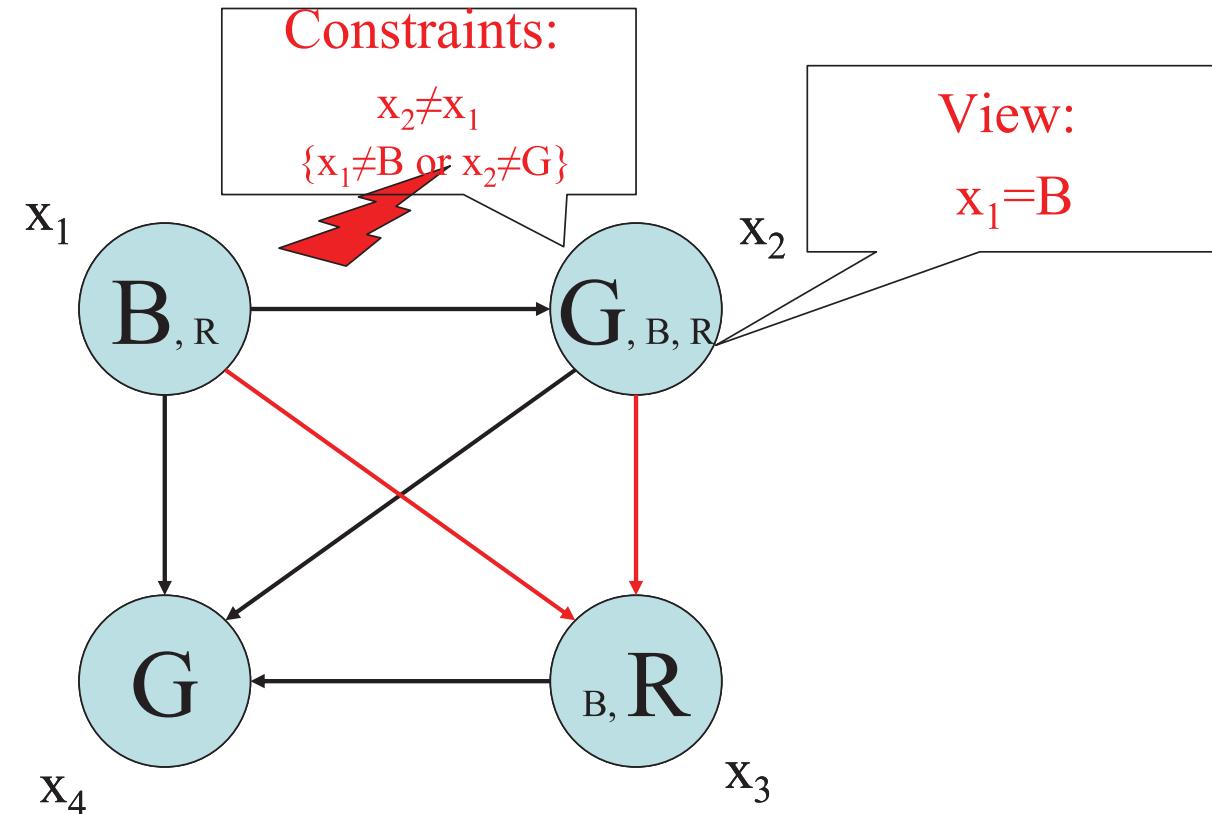
Record new constraint

Agent  $x_2$  records the conflict as a new constraint

## الگوریتم عقب‌گرد ناهمکام

مثال: رنگ‌آمیزی گراف (۳۵ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

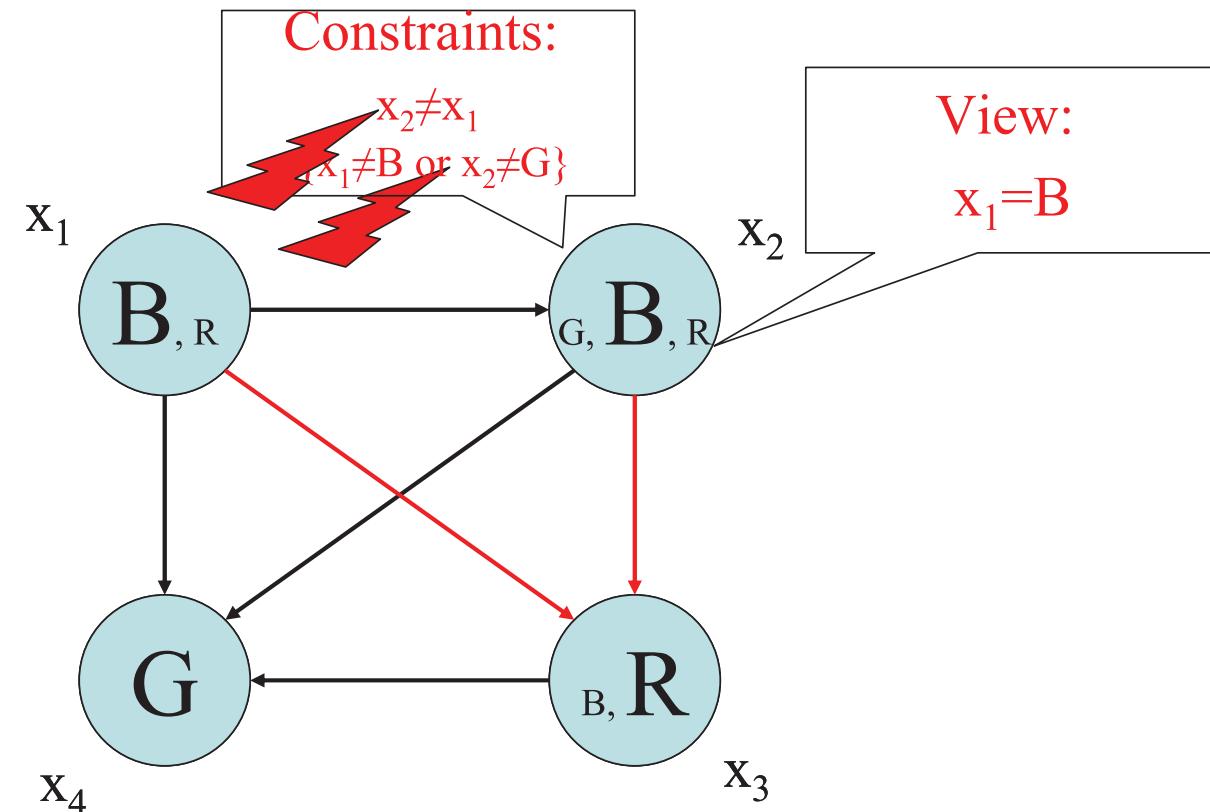


Agent  $x_3$  checks its view, and discovers that one constraint (the new one) is violated

## الگوریتم عقب‌گرد ناهمکام

مثال: رنگ‌آمیزی گراف (۳۶ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



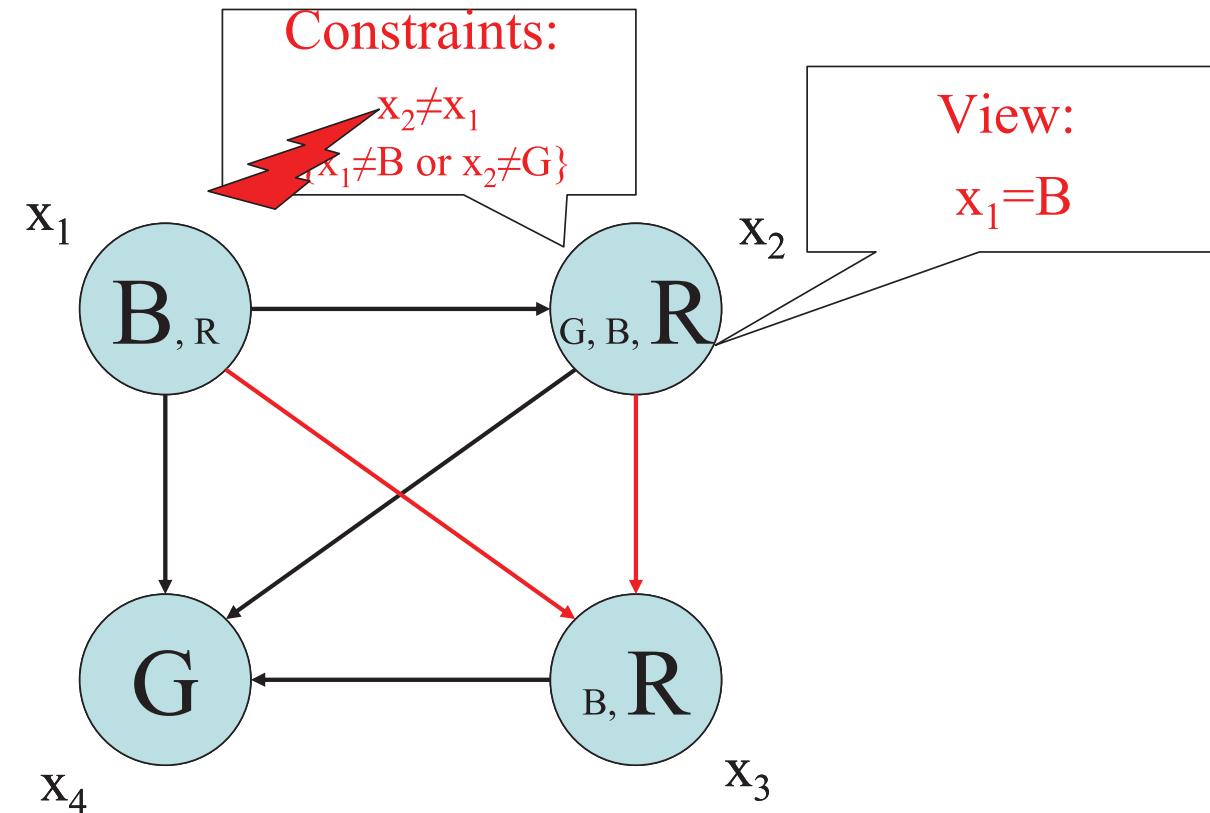
Try  
to choose  
value

Agent  $x_2$  tries to change its value to  $B$ ,  
but it would violate the first constraint

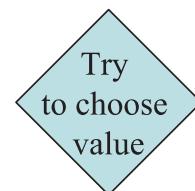
## الگوریتم عقب‌گرد ناهمکام

مثال: رنگ‌آمیزی گراف (۳۷ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



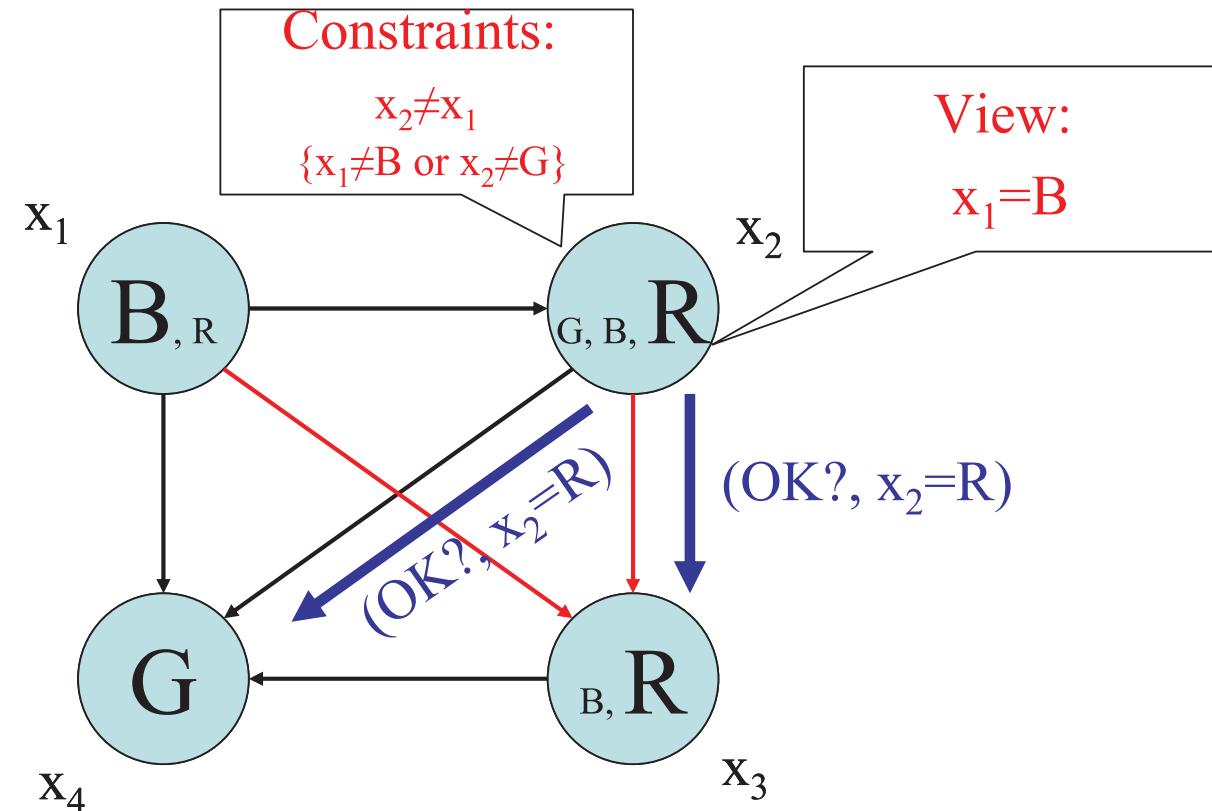
Agent  $x_2$  tries to change its value to R



## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۳۸ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



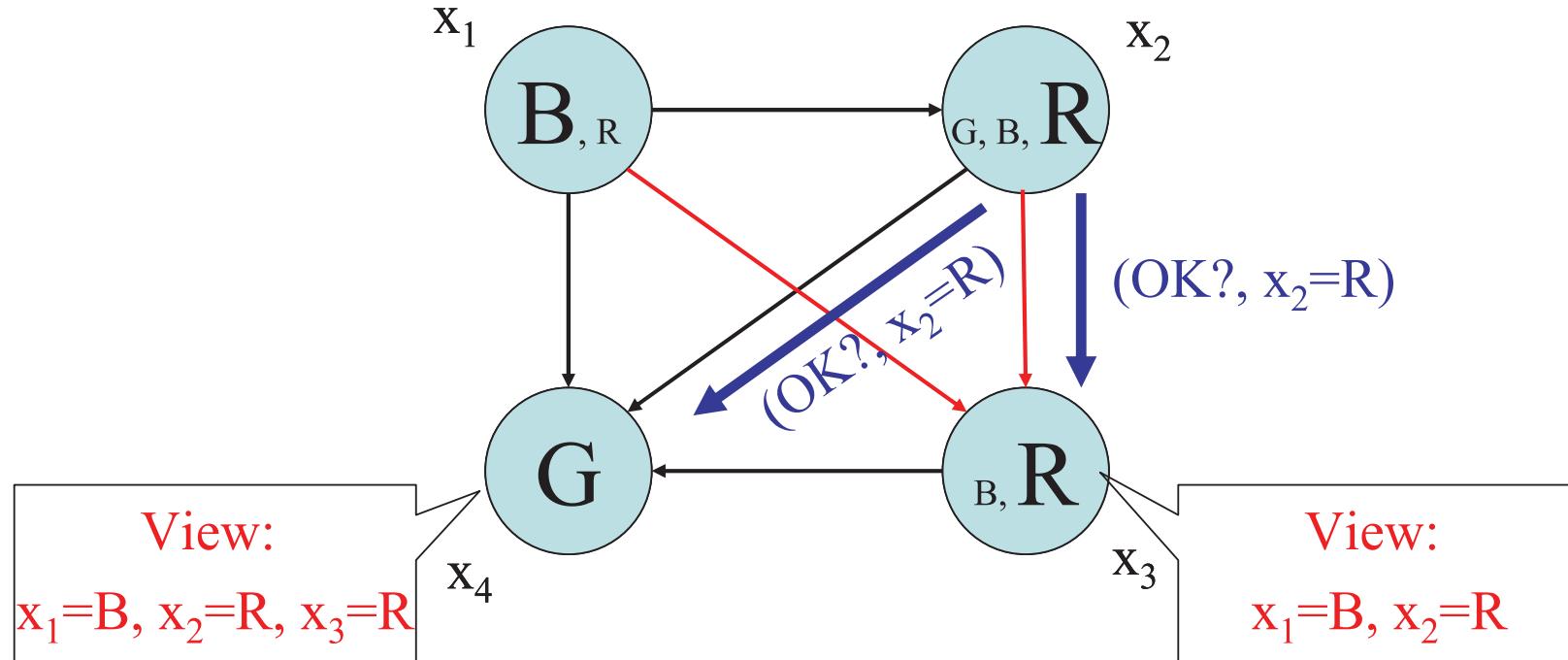
Send OK? messages

The constraint is no longer violated, so Agent  $x_2$  chooses value  $R$  and communicates it to its children.

## الگوریتم عقب‌گرد ناهمگام

مثال: رنگ‌آمیزی گراف (۴۰ از ۳۹)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM



Agent  $x_3$  and Agent  $x_4$  receive the messages and update their views

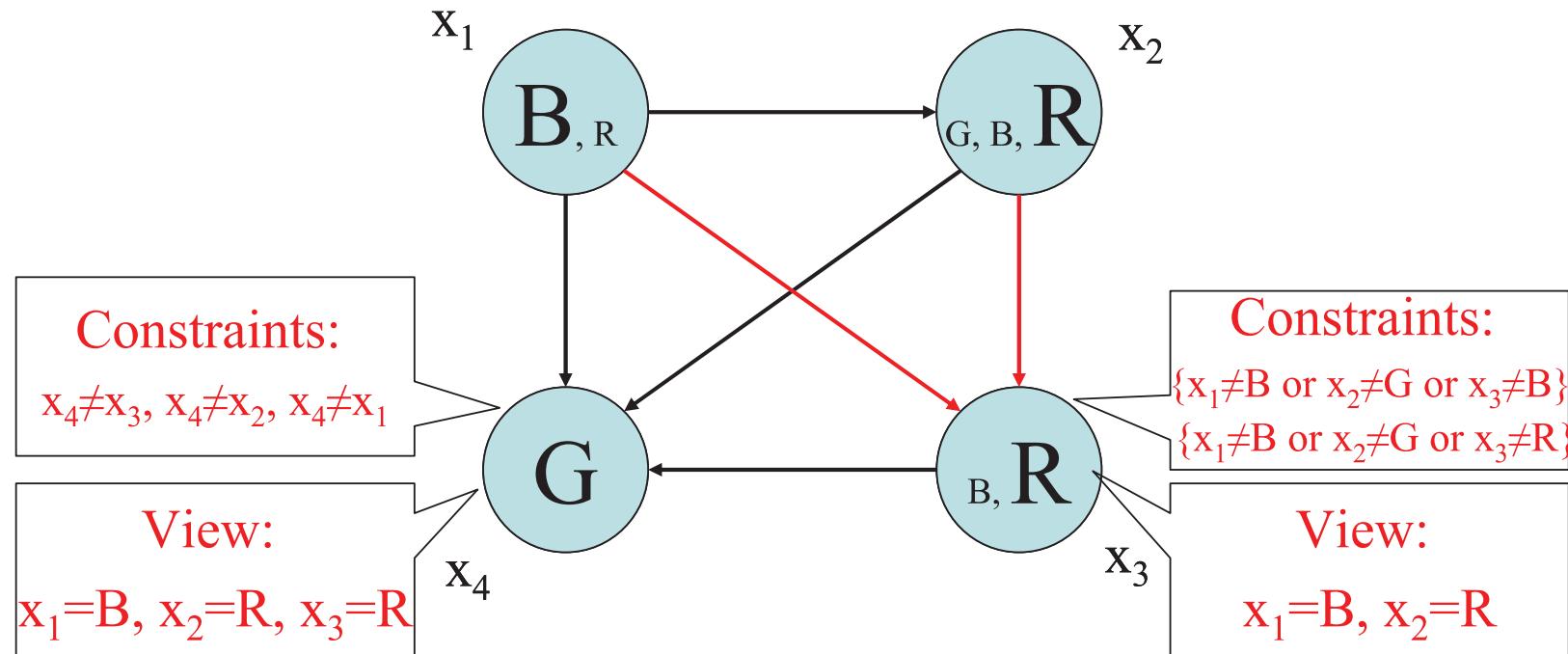
Update view

## الگوریتم عقب‌گرد ناهمگام

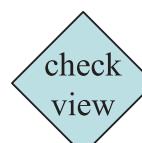
مثال: رنگ‌آمیزی گراف (۴۰ از ۴۰)

### THE ASYNCHRONOUS BACKTRACKING ALGORITHM

# SOLVED!



Agent  $x_3$  and Agent  $x_4$  check their view against their constraints, and no violation is discovered



## الگوریتم عقب‌گرد ناهمگام

نقاط ضعف

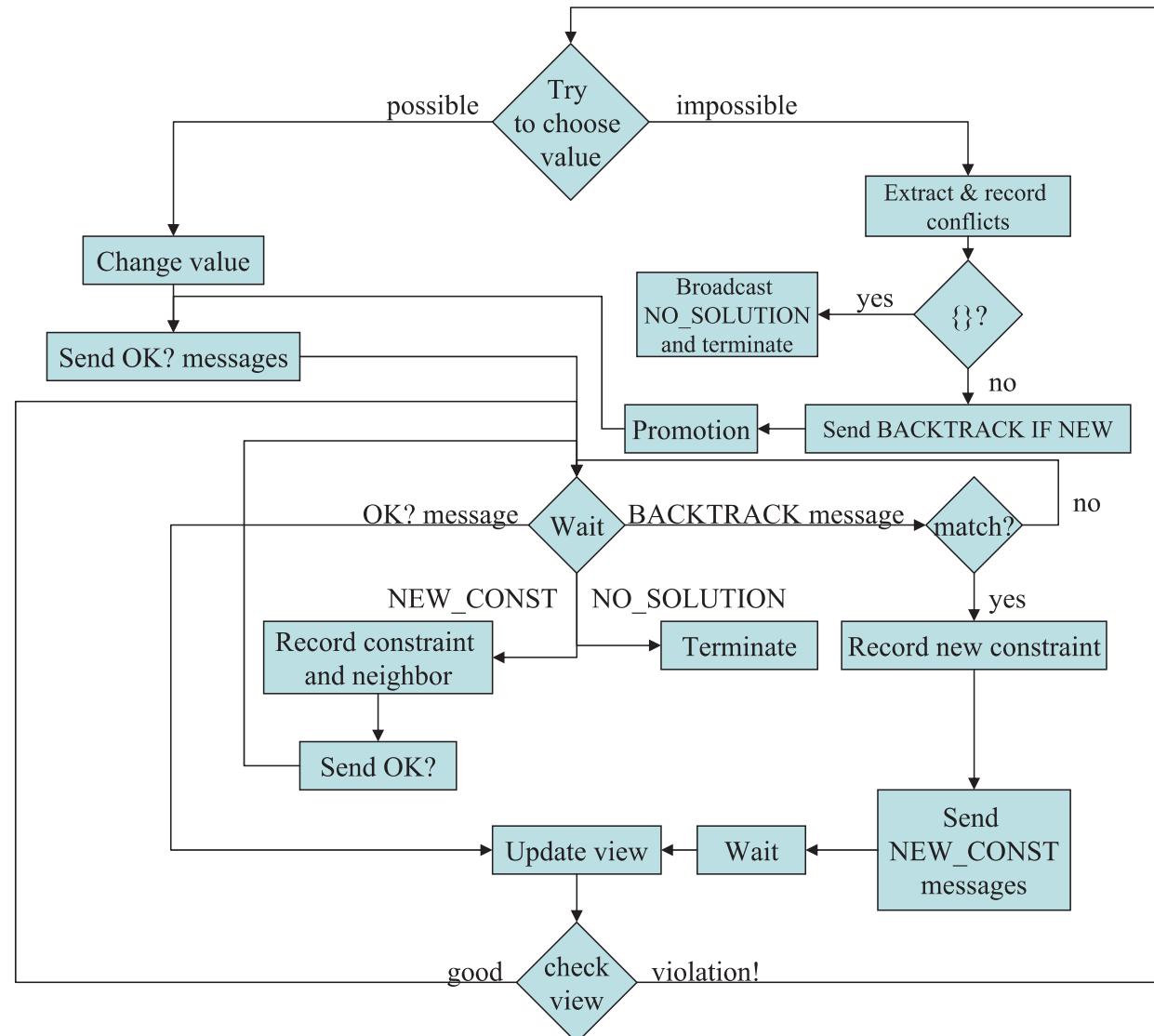
THE ASYNCHRONOUS BACKTRACKING ALGORITHM

## الگوریتم جستجوی تعهد ضعیف ناهمگام

THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

## الگوریتم جستجوی تعهد ضعیف ناهمگام

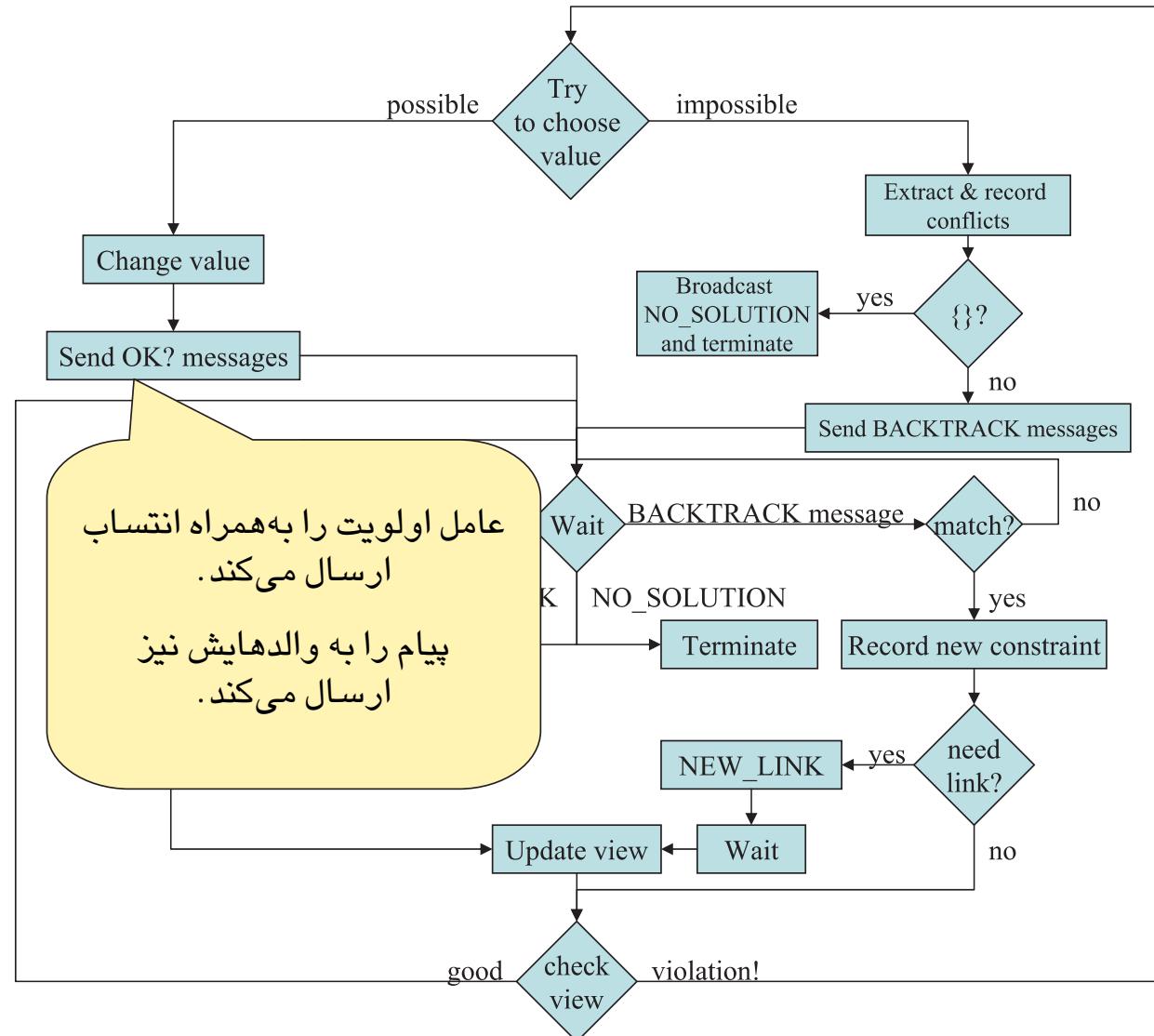
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۱

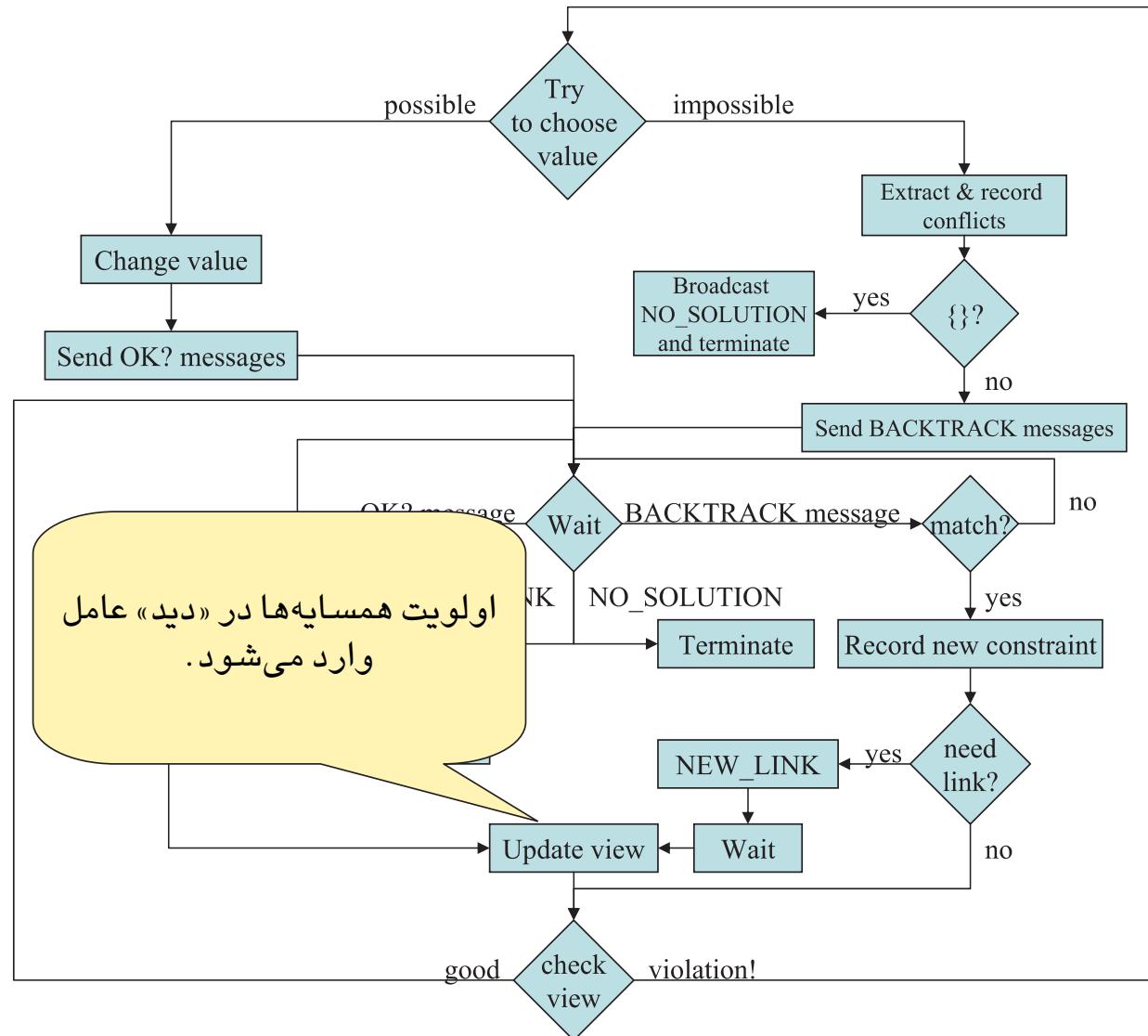
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۲

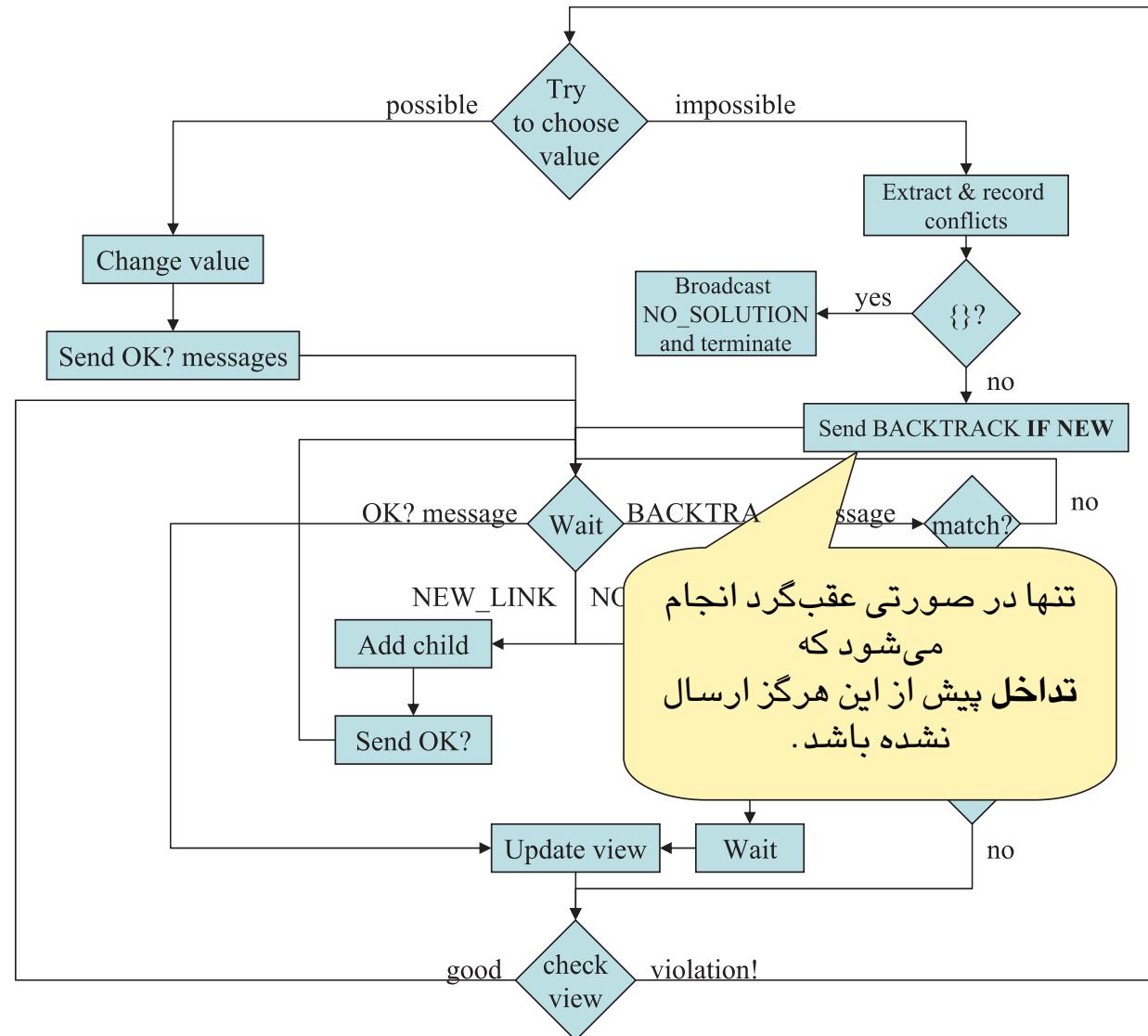
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوريتم جستجوی تعهد ضعیف ناهمگام

۳۱۹

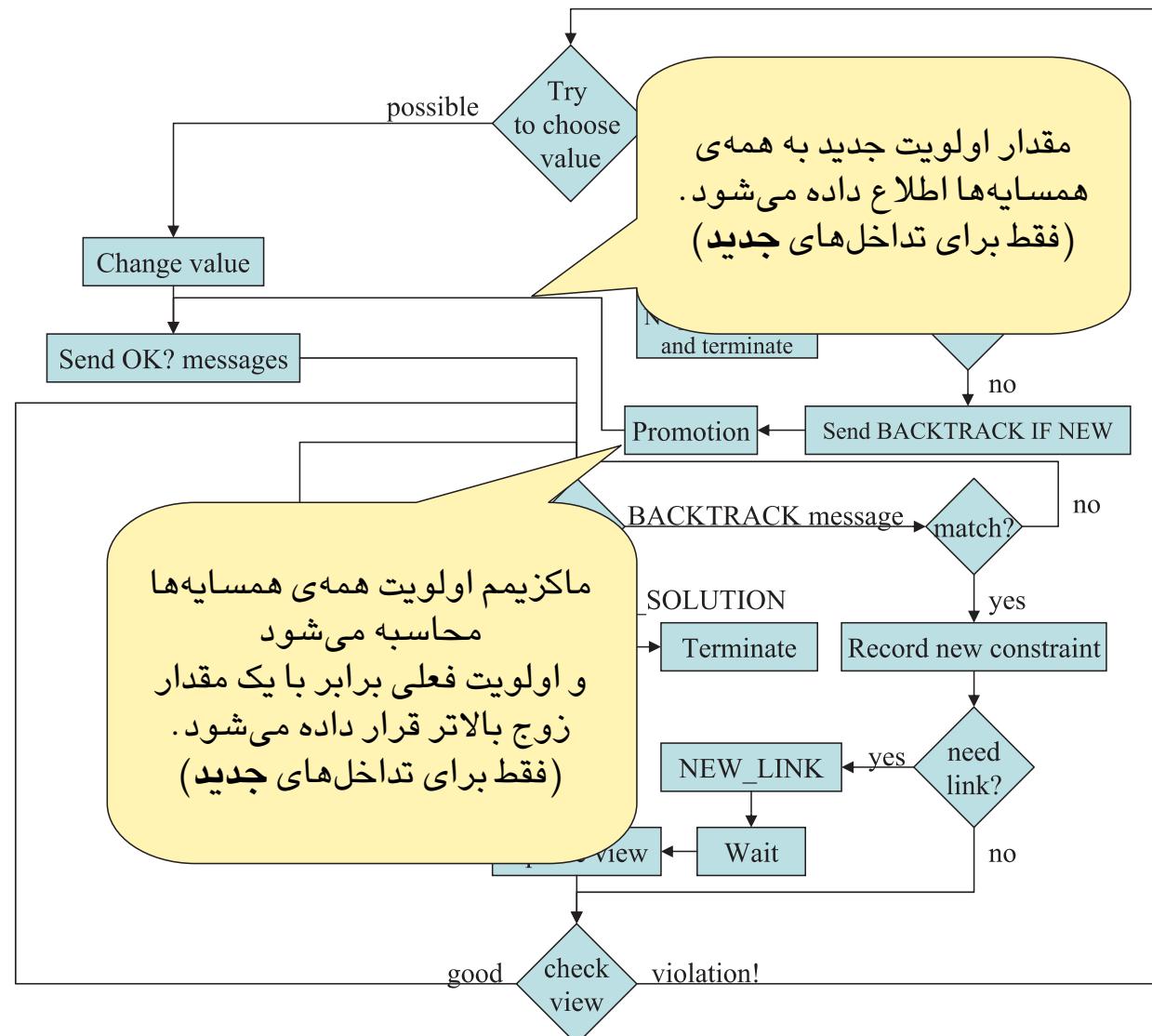
# THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۴

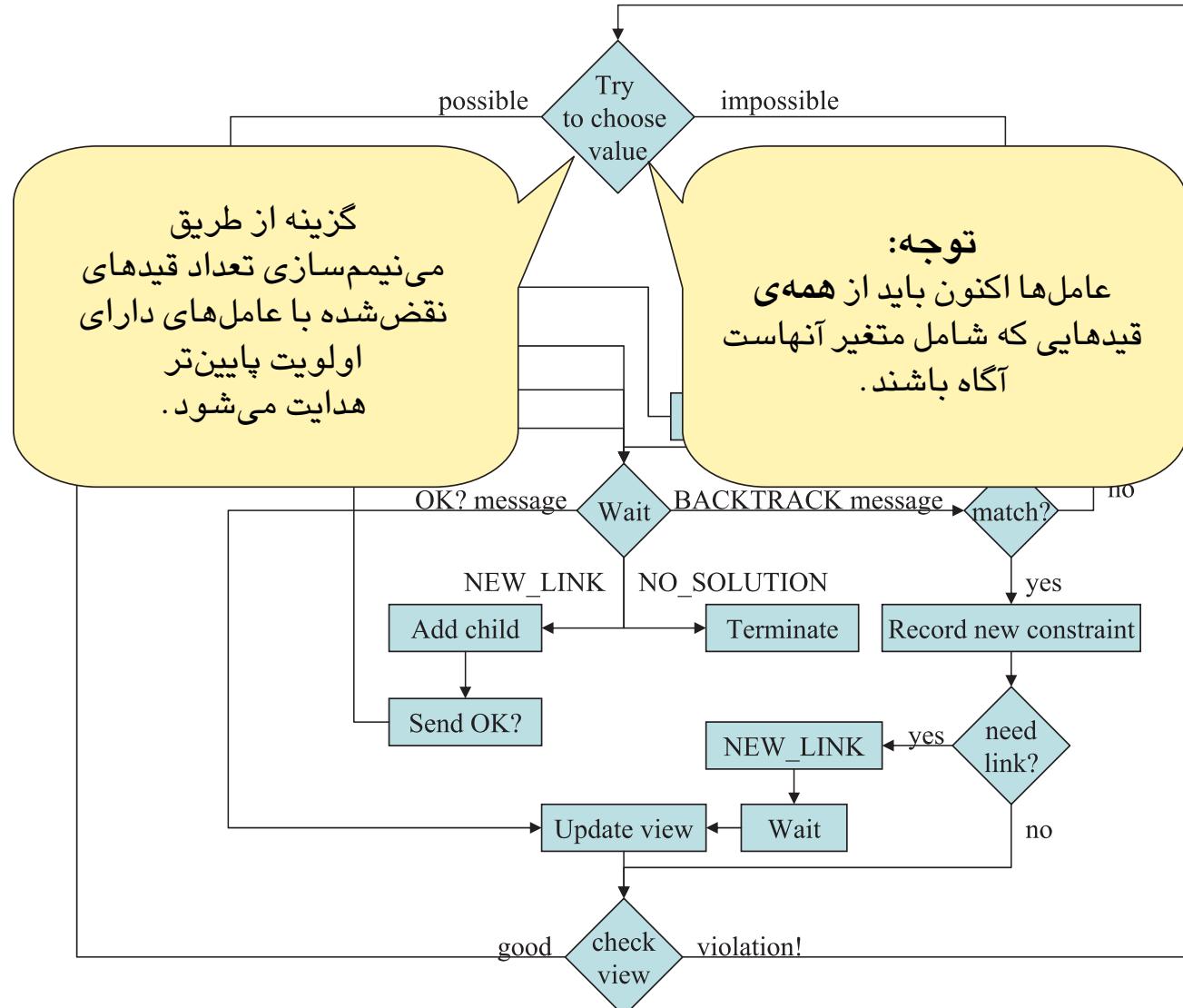
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۵

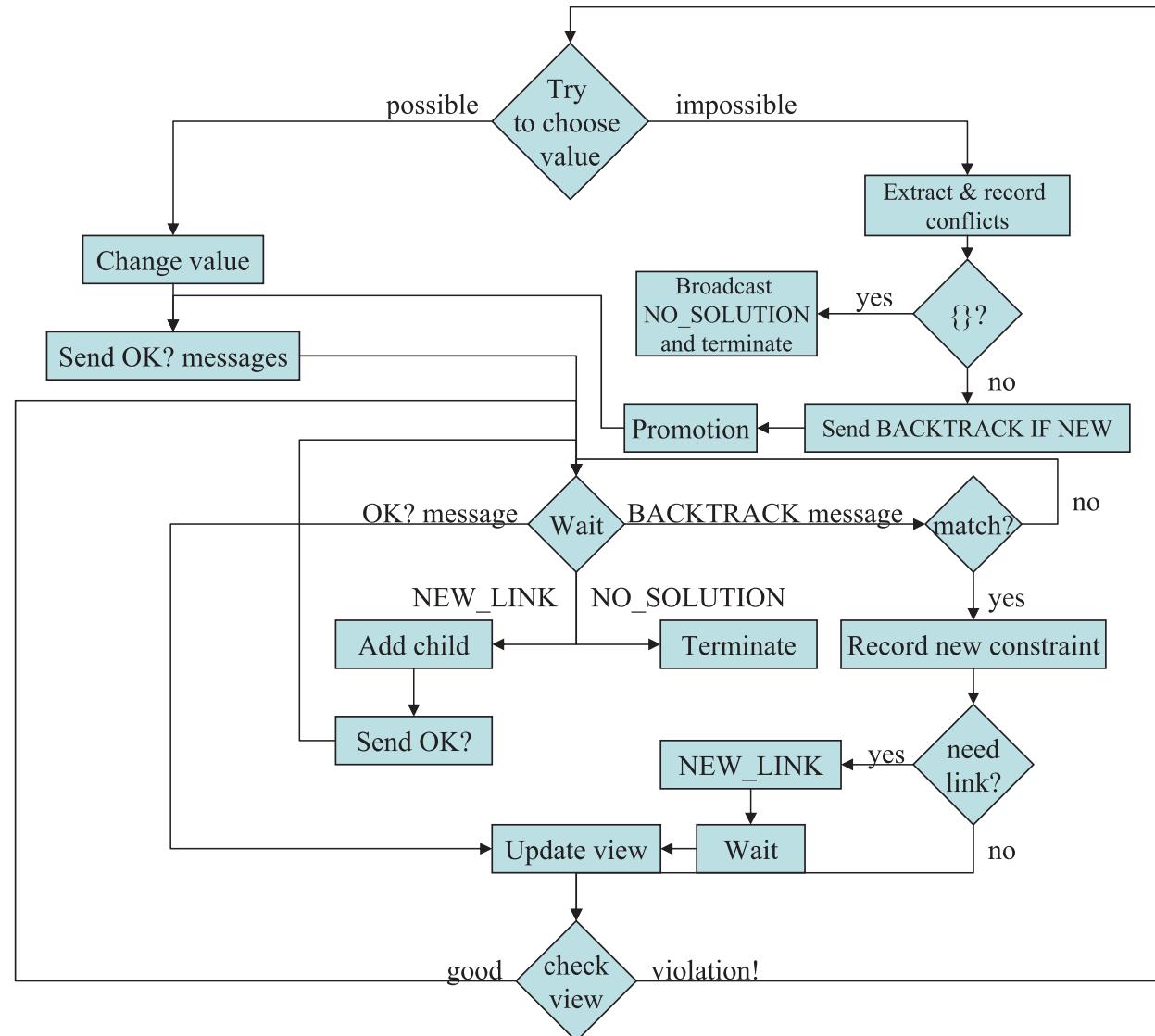
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۹

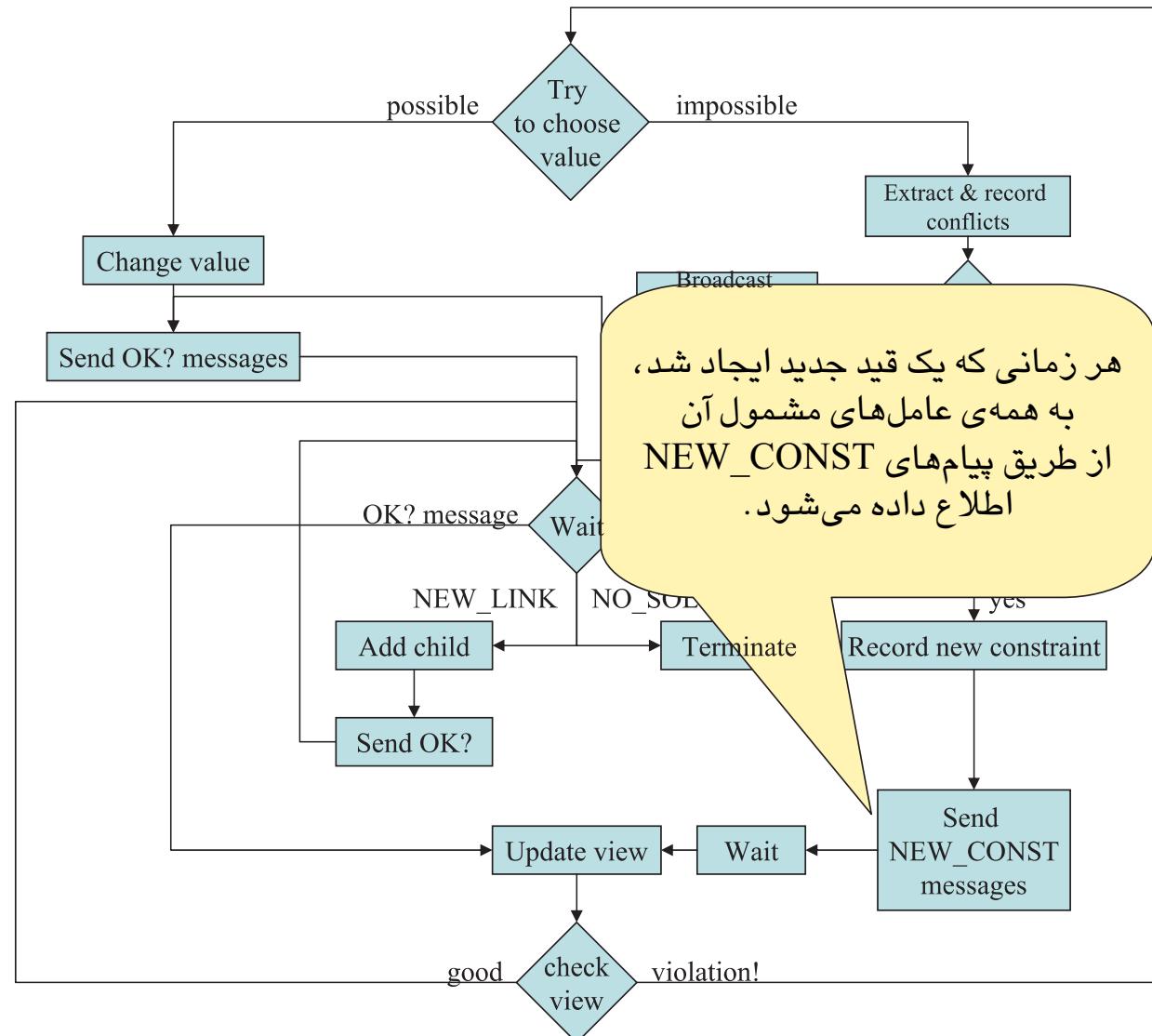
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۷

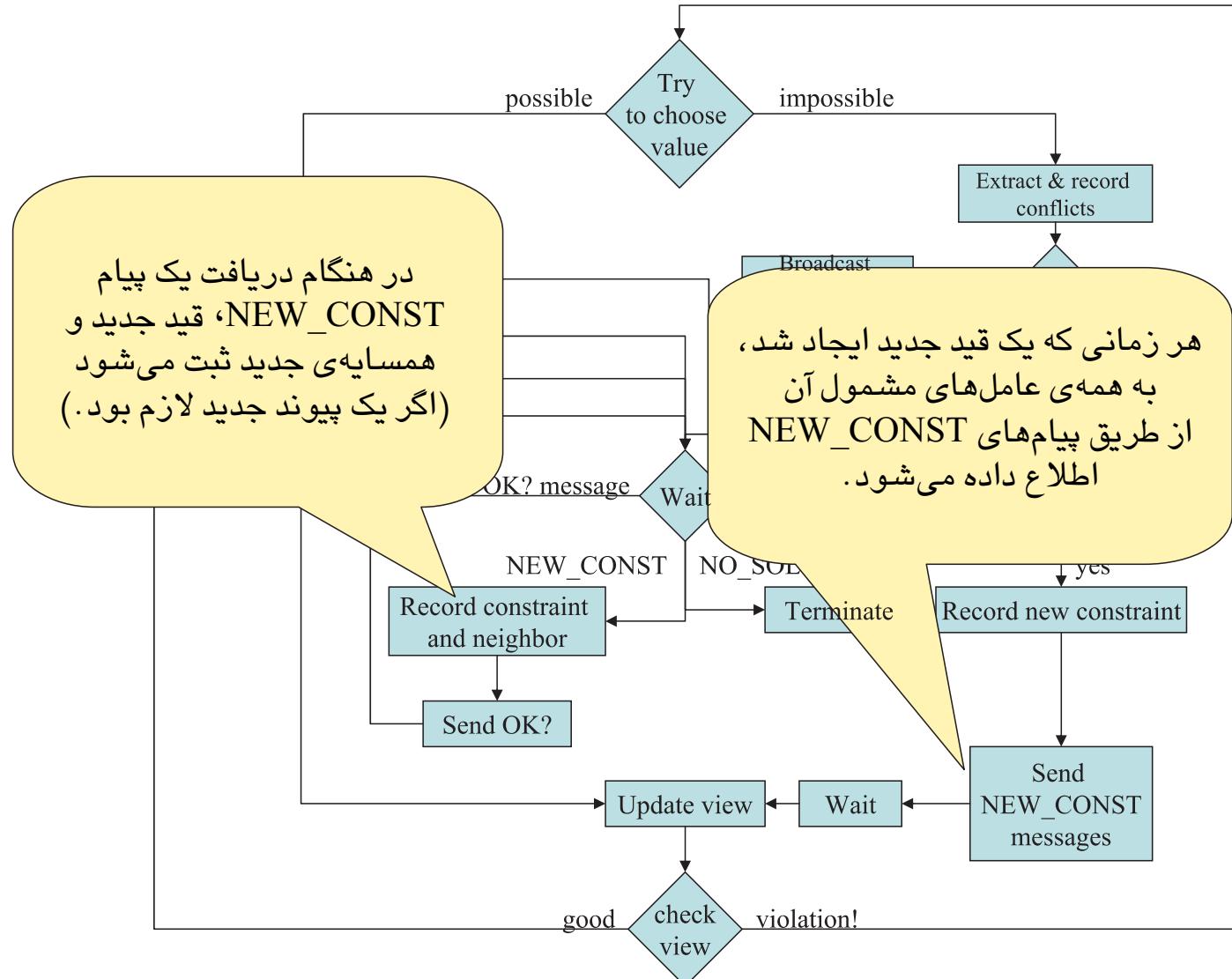
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۸

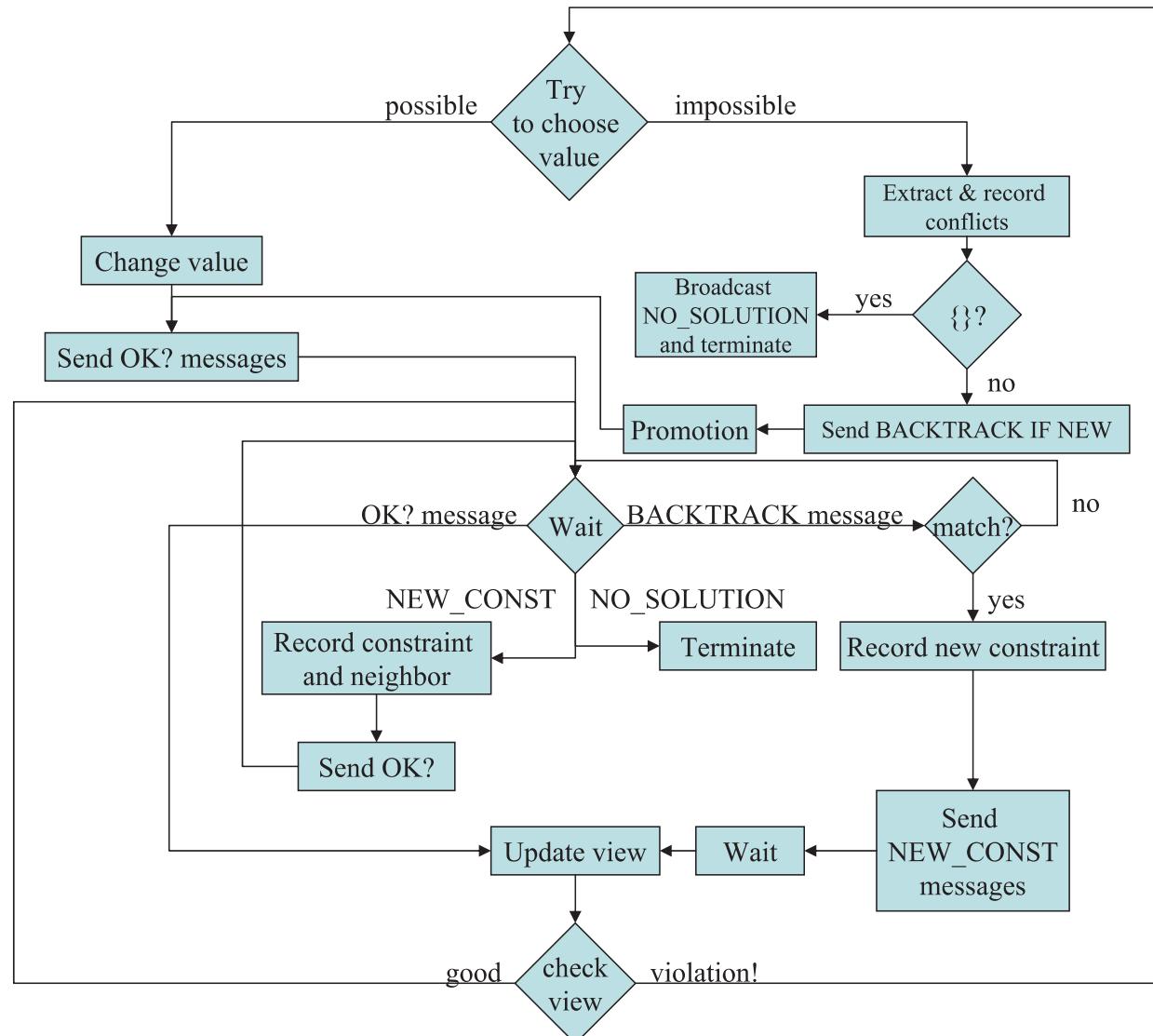
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

۹ از ۱۹

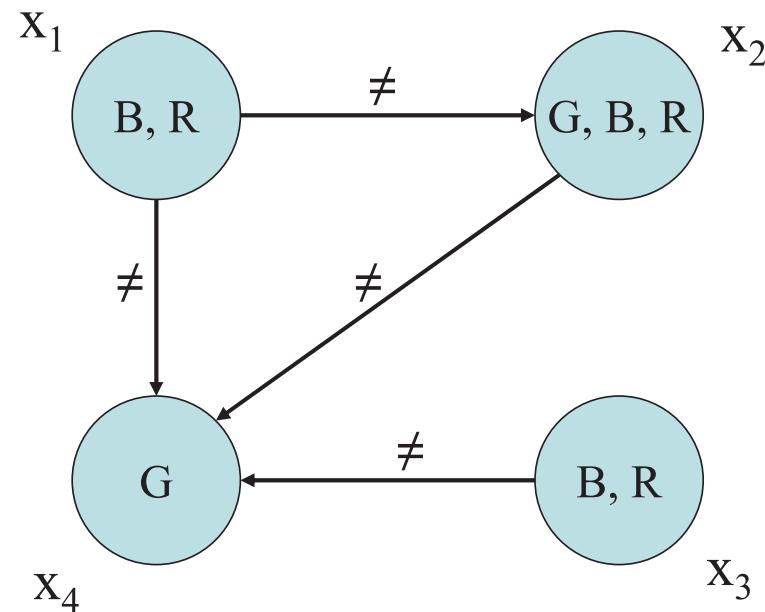
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱ از ۲۹)

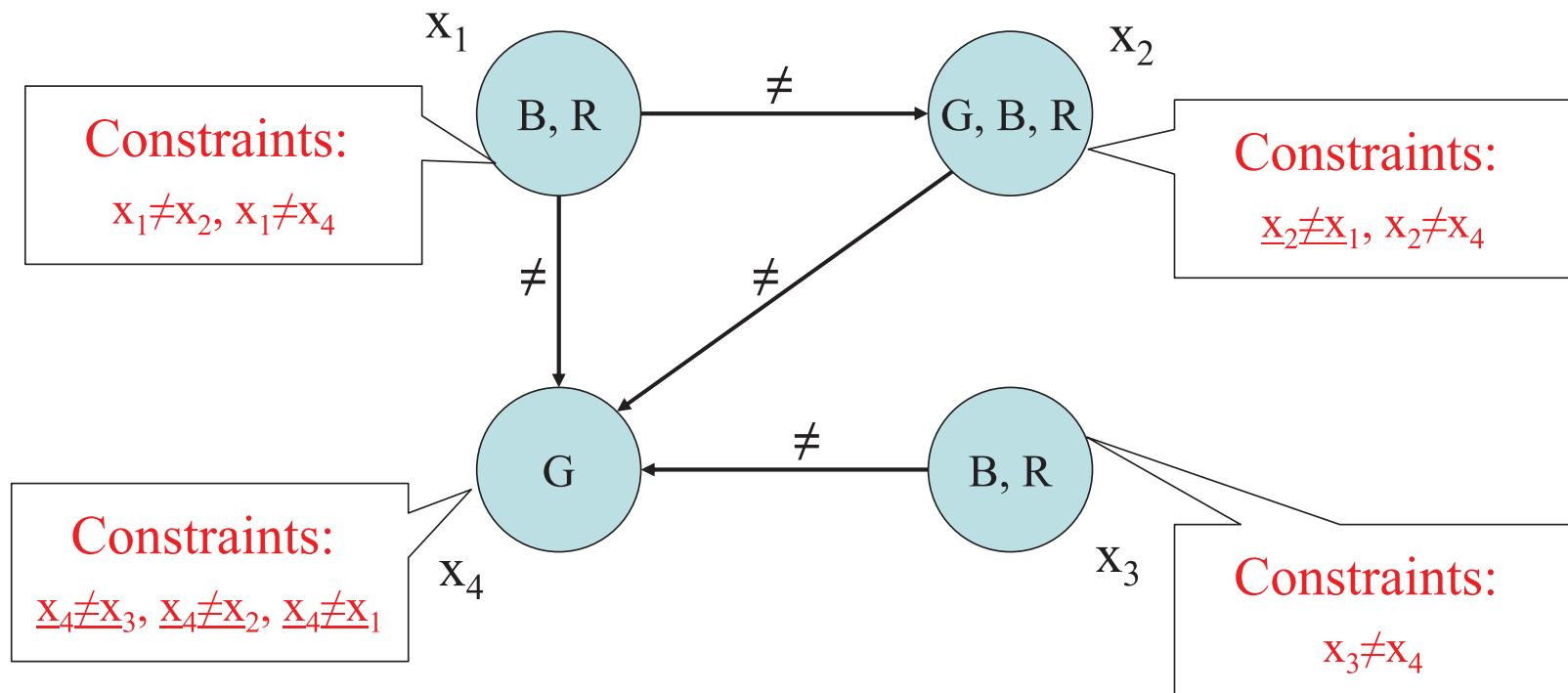
THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲)

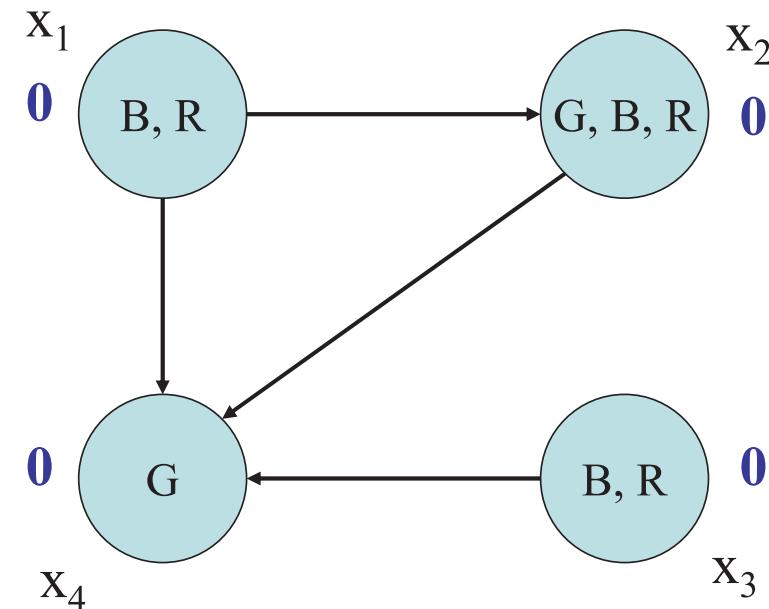
### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

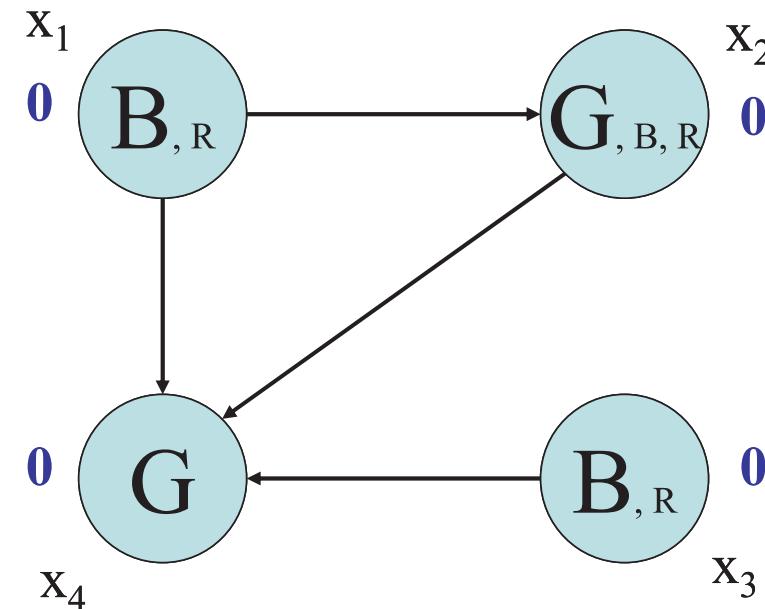


Initial priority values are all set to 0. Two agents with identical priorities are ordered with respect to their index

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۴)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



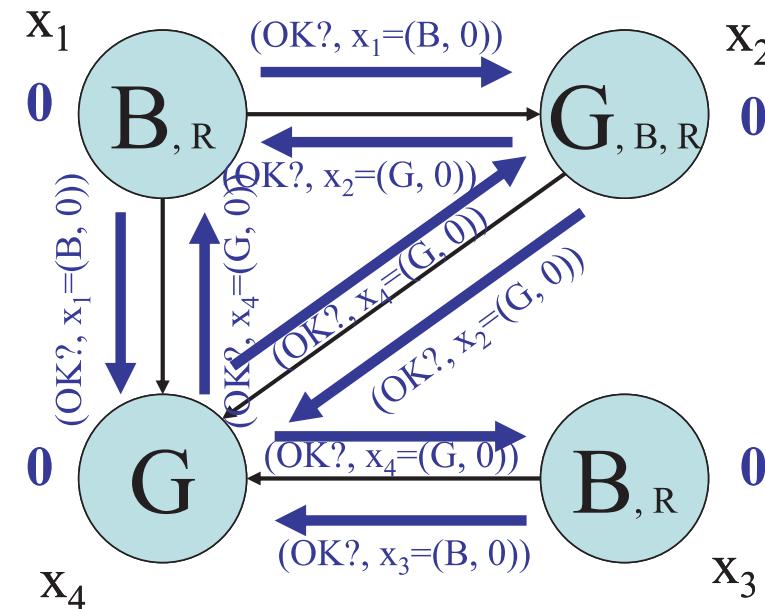
Try  
to choose  
value

Each agent chooses an assignment to its variable  
(at the first time step, we cannot use the heuristic  
because agents still have empty views)

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۵)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



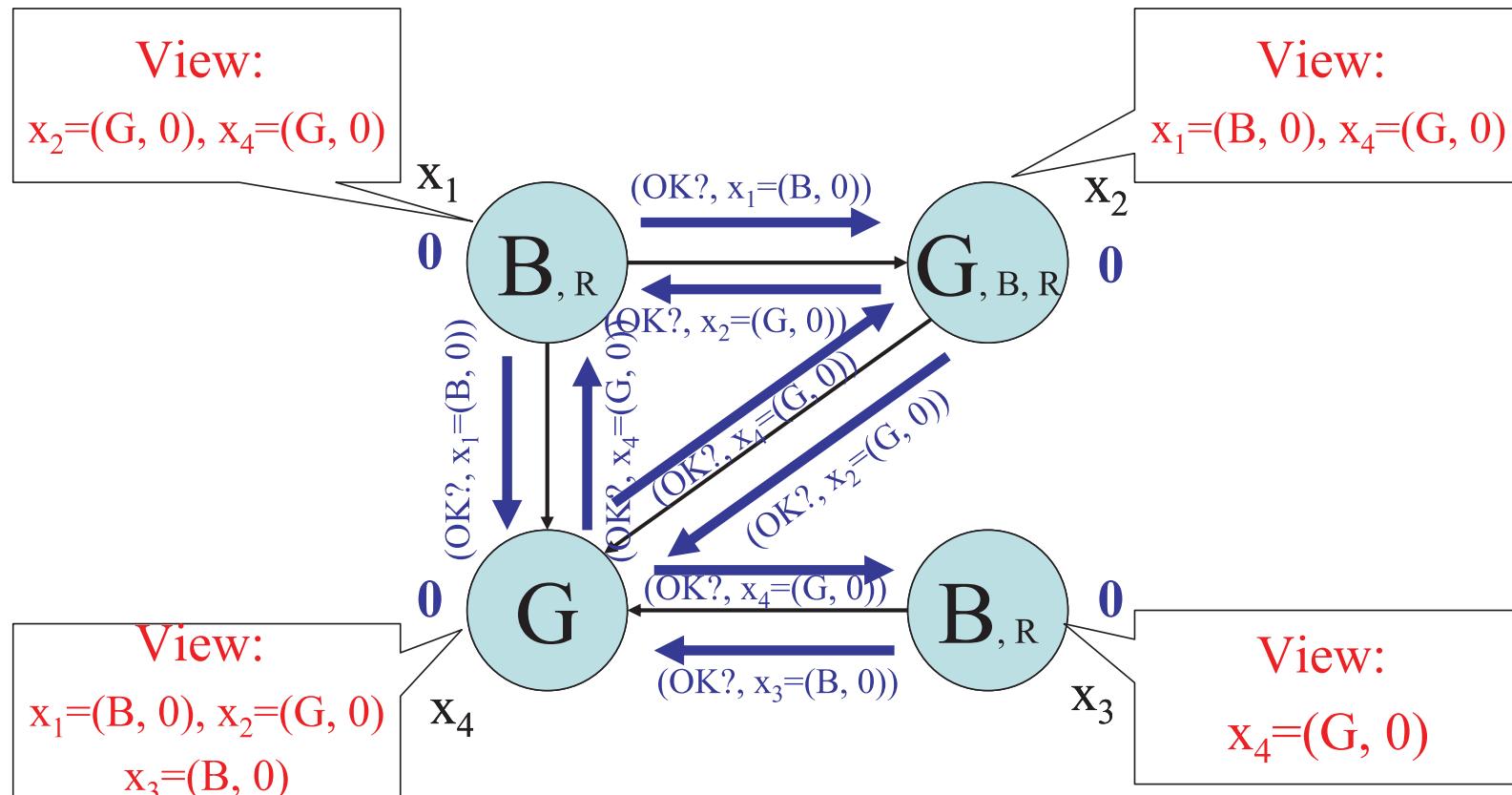
Send OK? messages

Each agent sends OK? messages to ALL of its neighbors

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۶)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



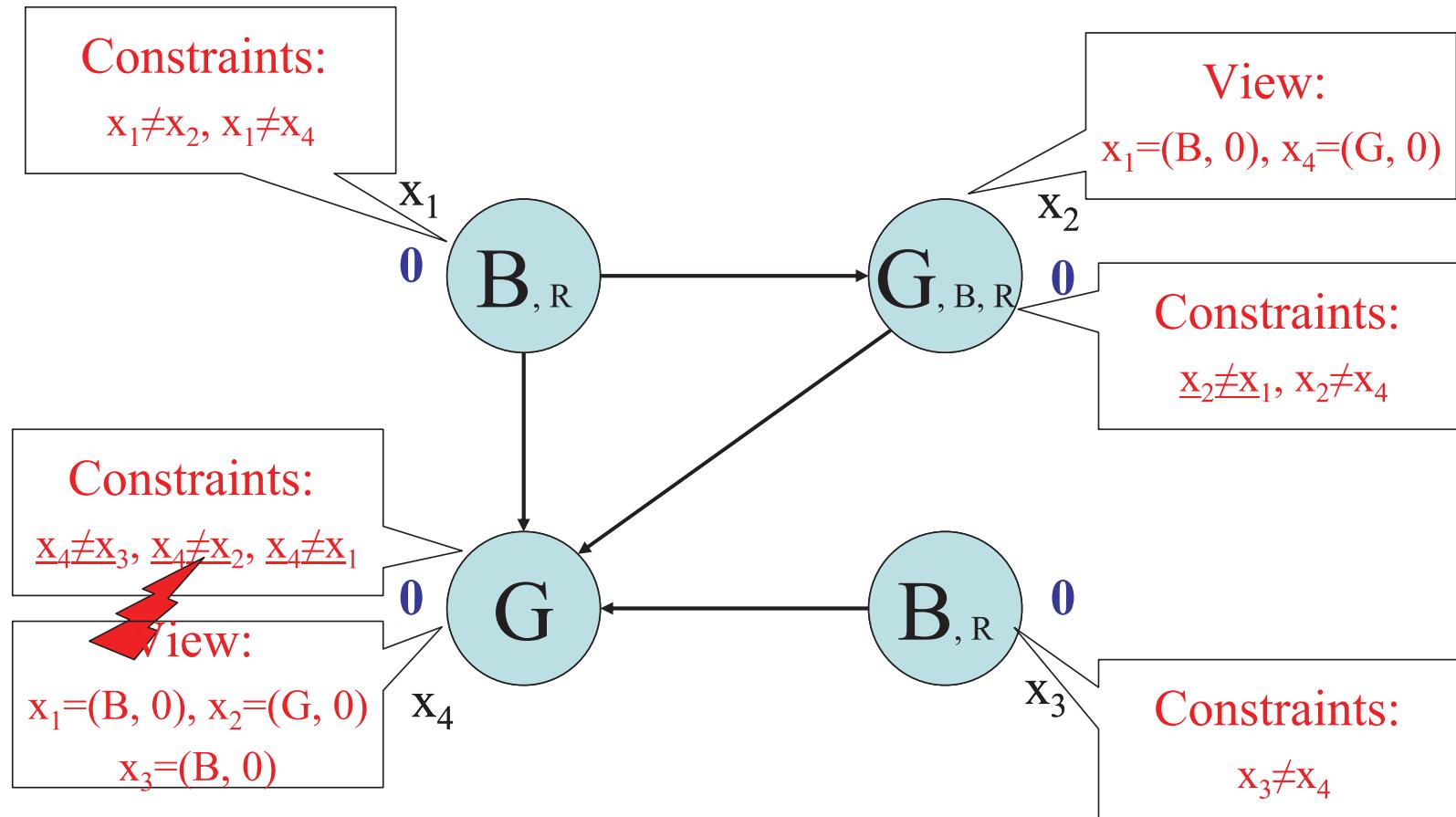
Update view

All agents update their view

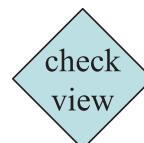
## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۷)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



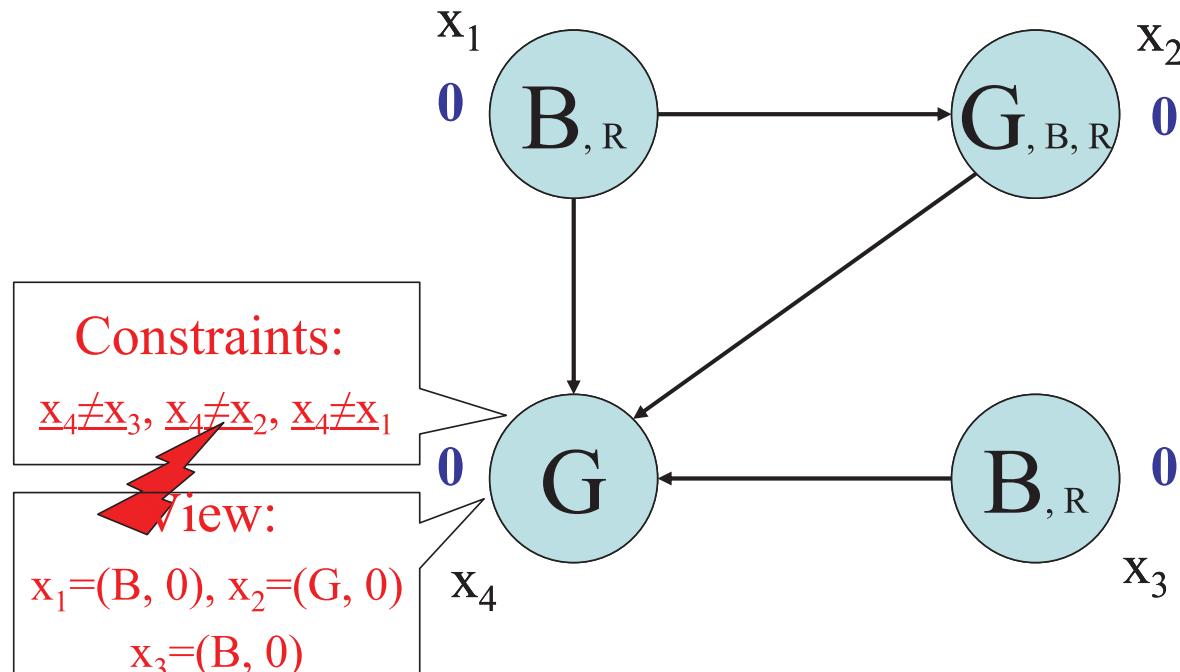
All agents check their view against the constraints they are responsible for, and Agent  $x_4$  discovers a violation



## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



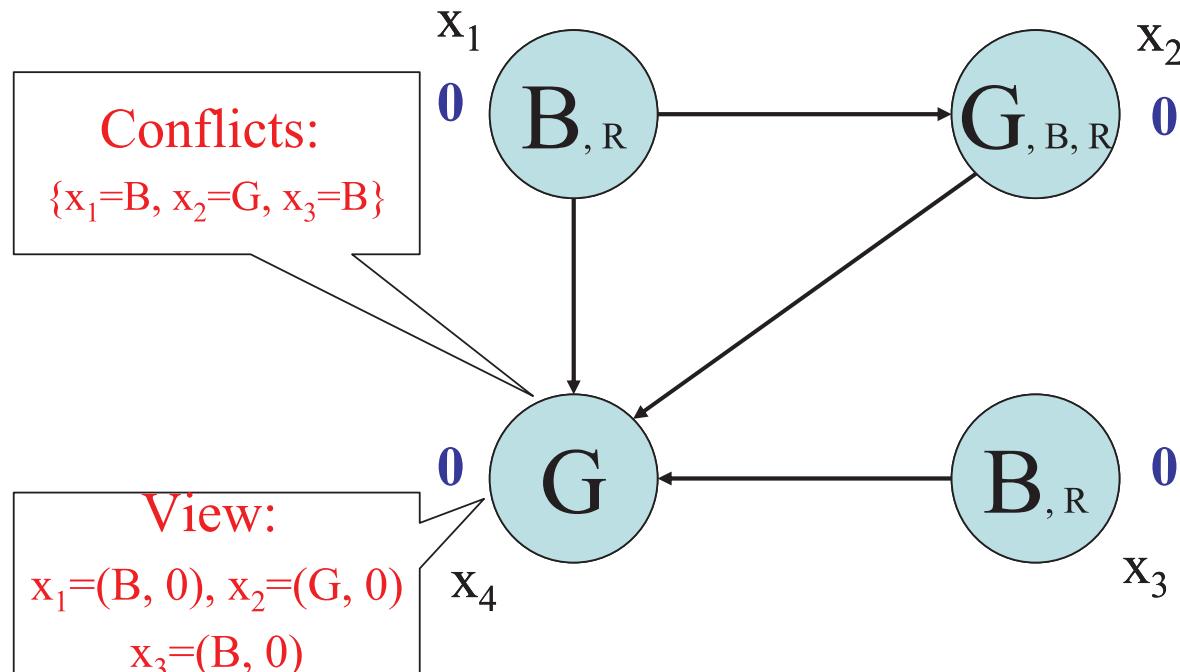
Try  
to choose  
value

Agent  $x_4$  tries to change its assignment,  
which is impossible

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



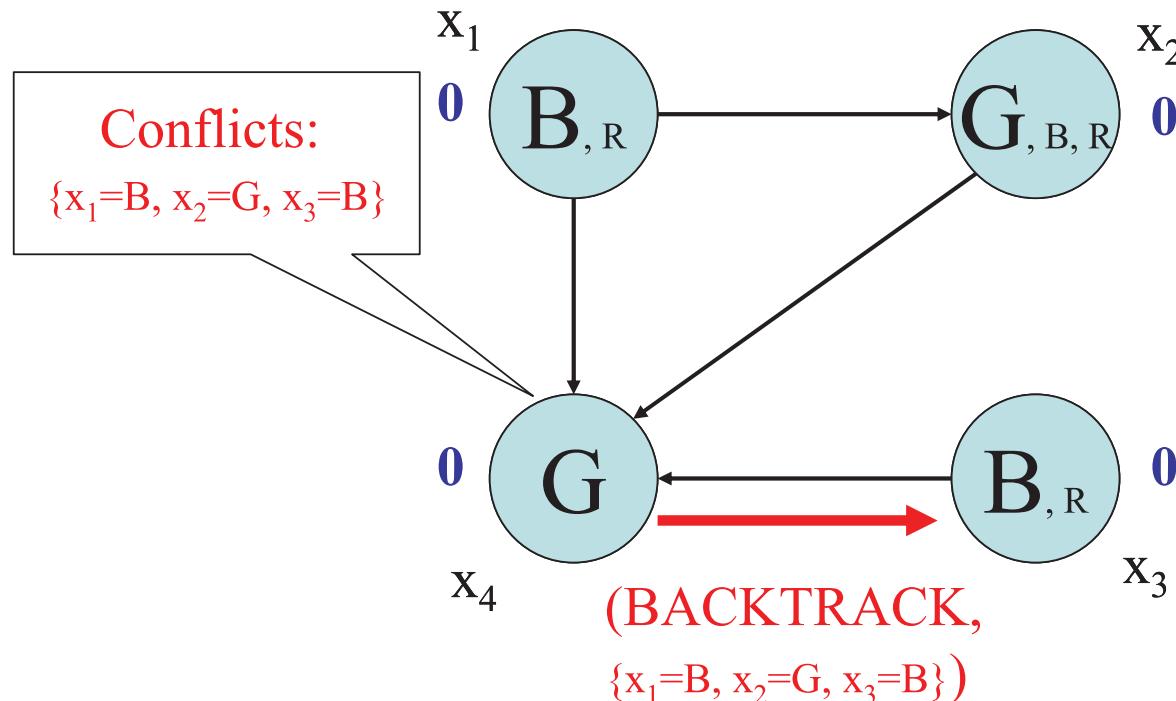
Extract & record  
conflicts

Agent  $x_4$  extracts and records the conflicts

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱۰ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



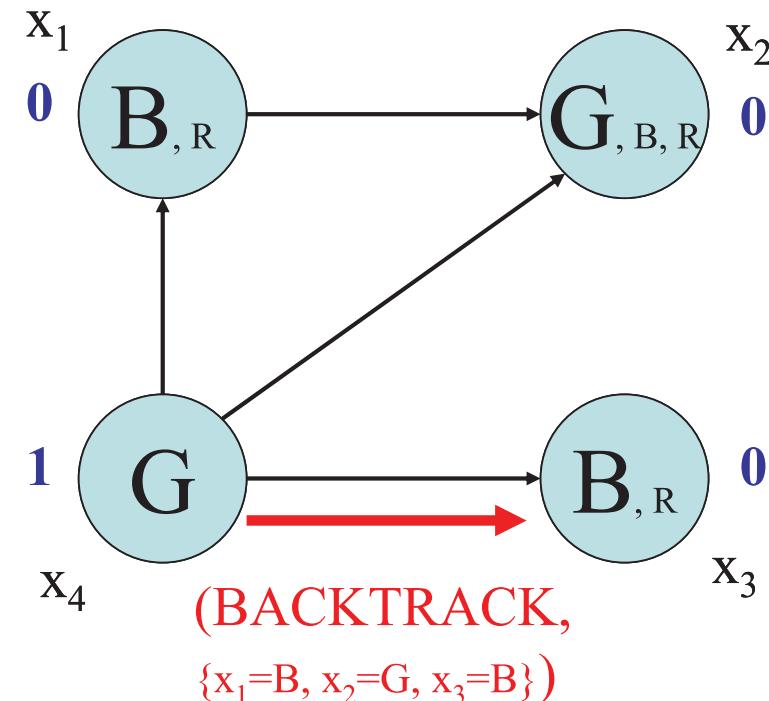
Send BACKTRACK messages

$\{\}$  is not among the new conflicts, and no new conflict has already been sent, so Agent  $x_4$  sends BACKTRACK messages

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱۱ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

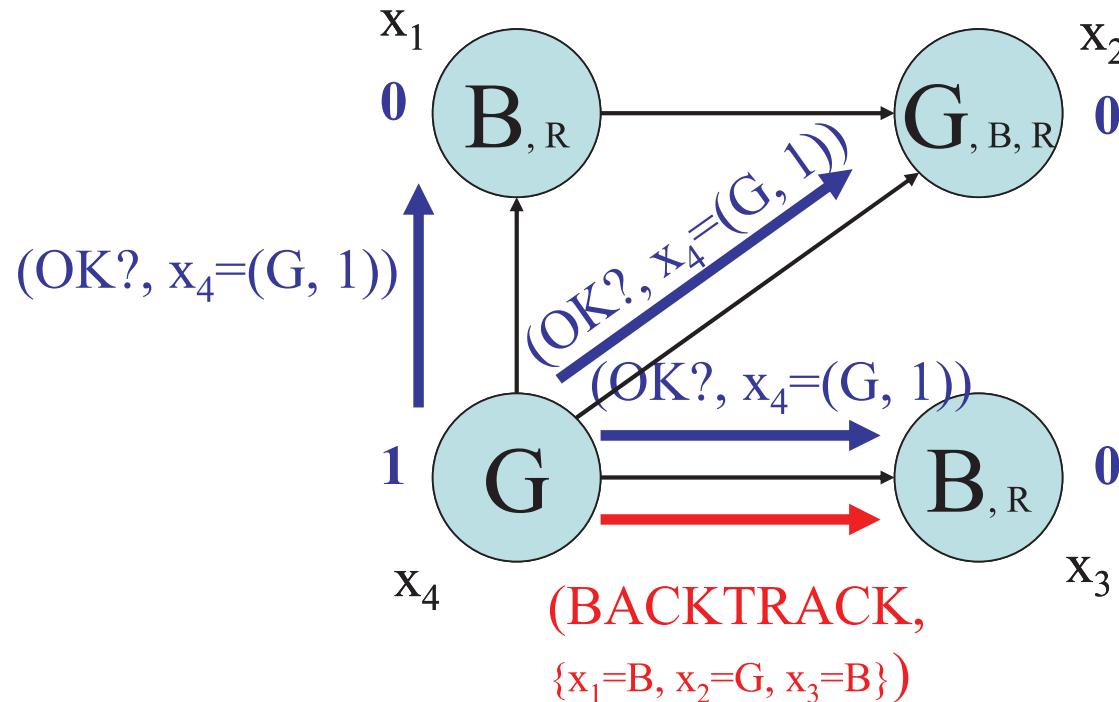


Promotion

Agent  $x_4$  promotes itself, changing its priority value from 0 to 1

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱۲ از ۲۹)

THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

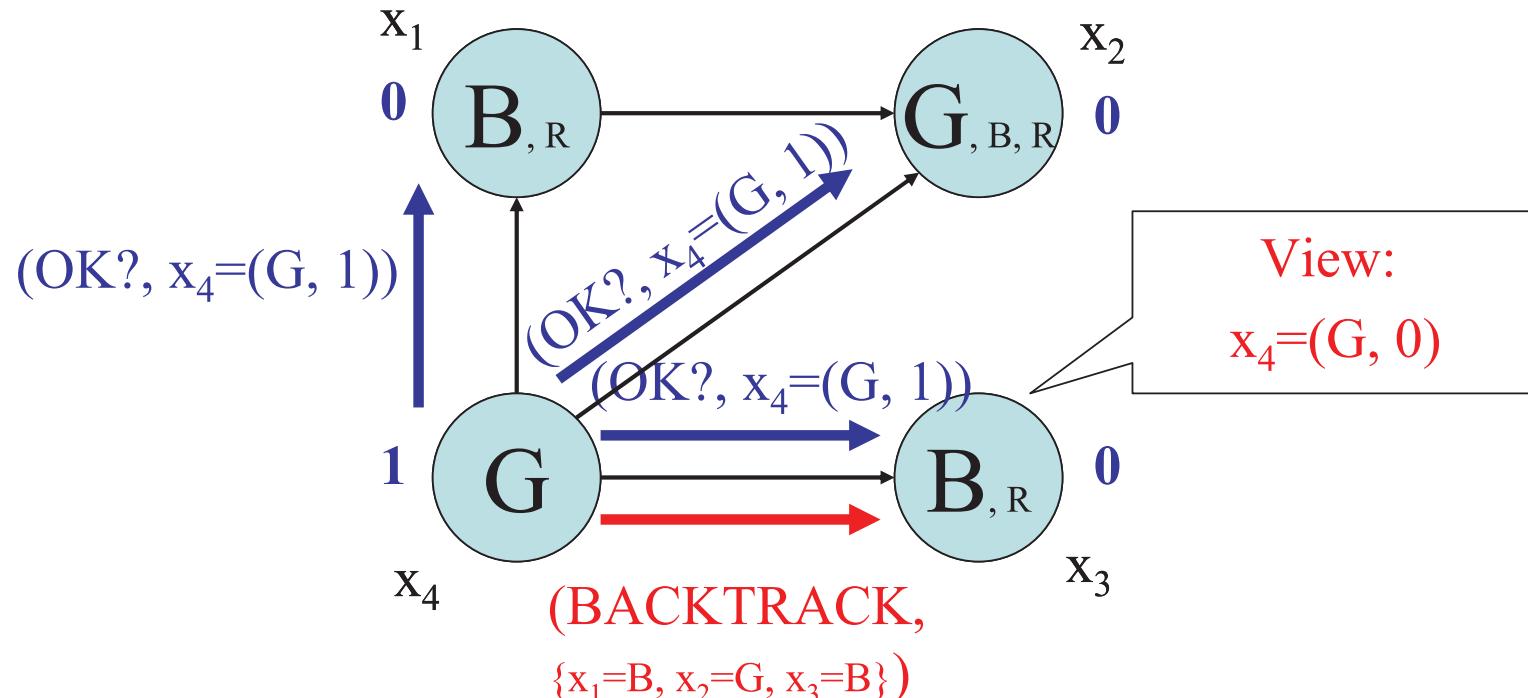
Send OK? messages

Agent  $x_4$  communicates its new priority value to ALL its neighbors

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱۳ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

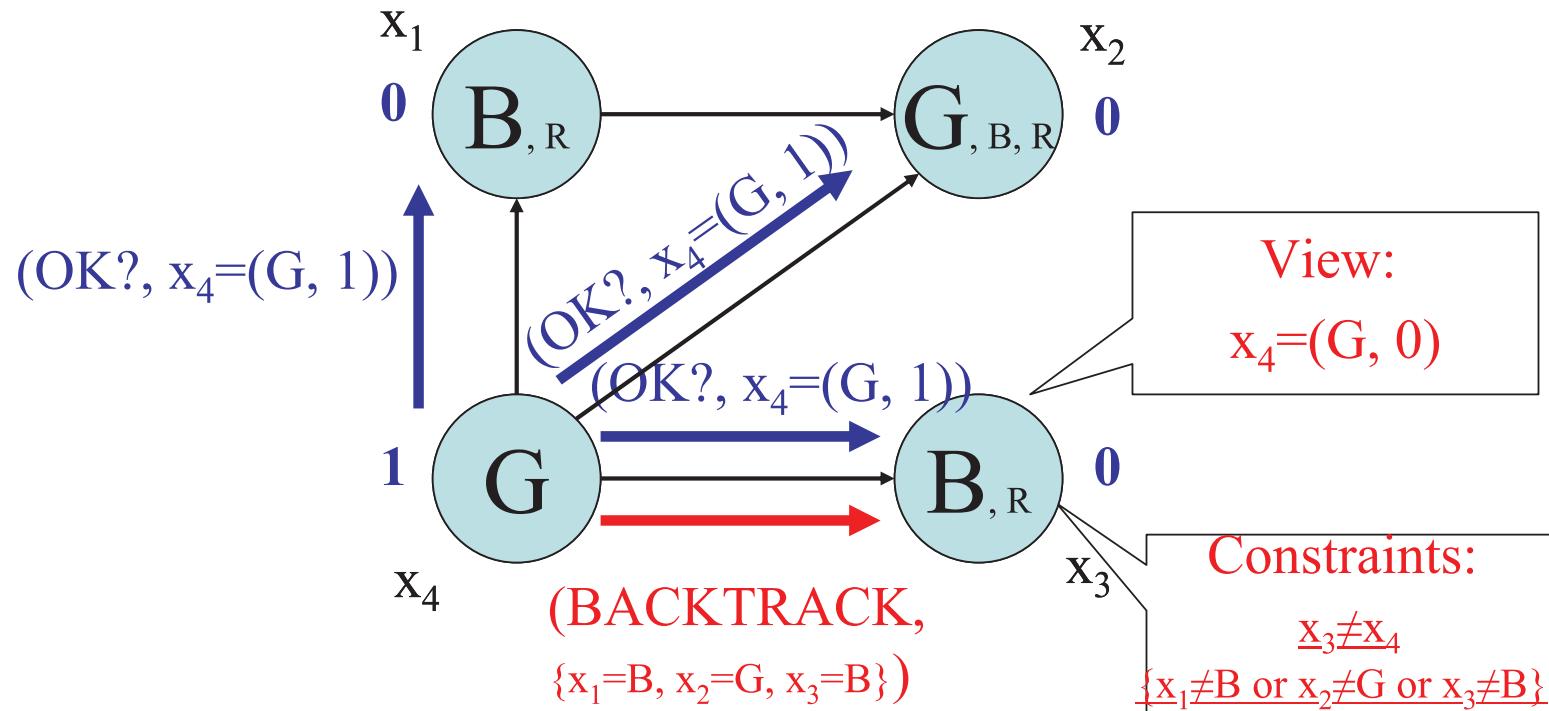


CONCURRENTLY, Agent  $x_3$  receives the BACKTRACK message and checks the conflict against its view

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۱۴)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



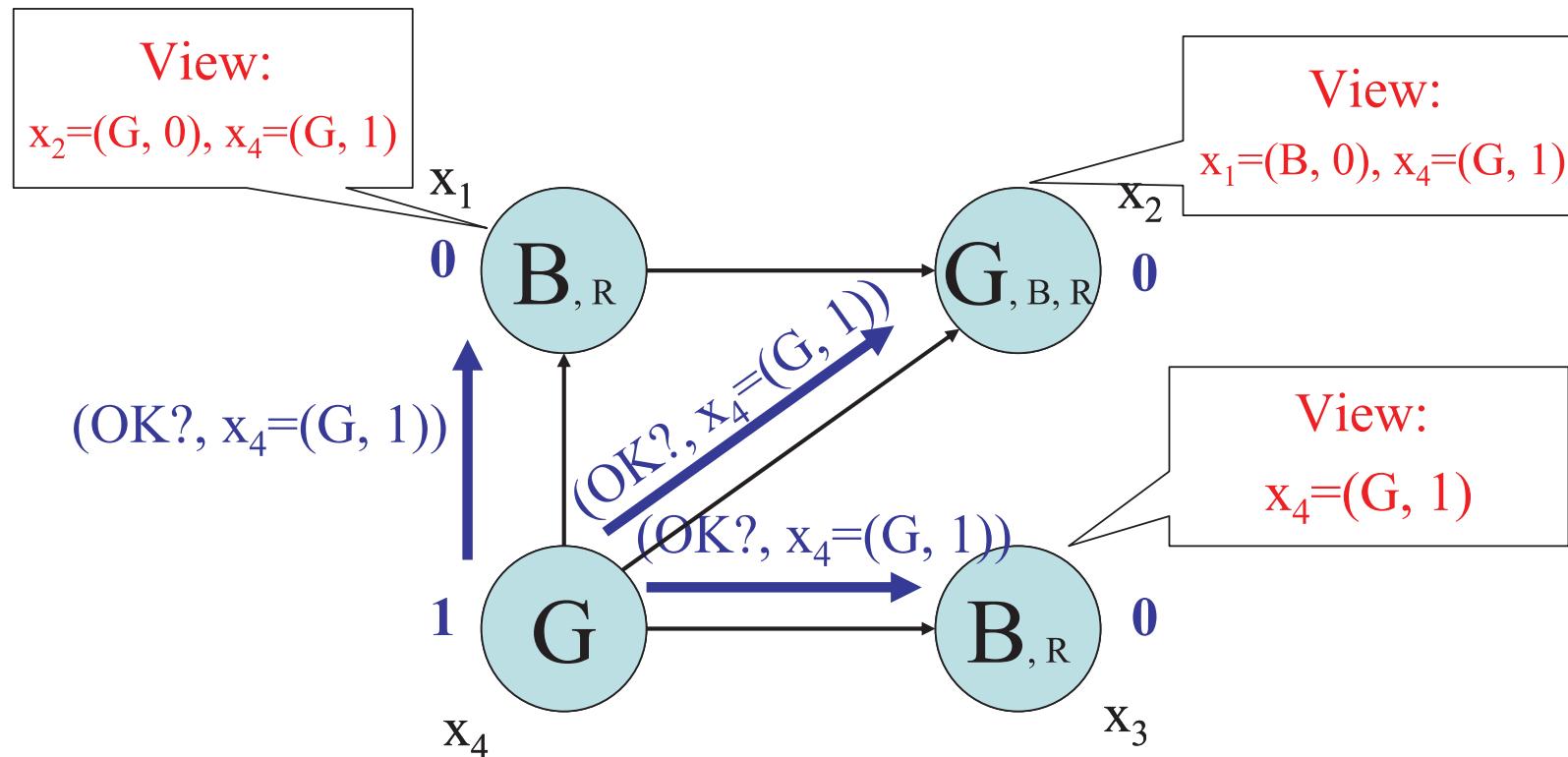
Record new constraint

Agent  $x_3$  records the conflict as a new constraint  
it will be responsible for

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۱۵)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



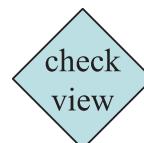
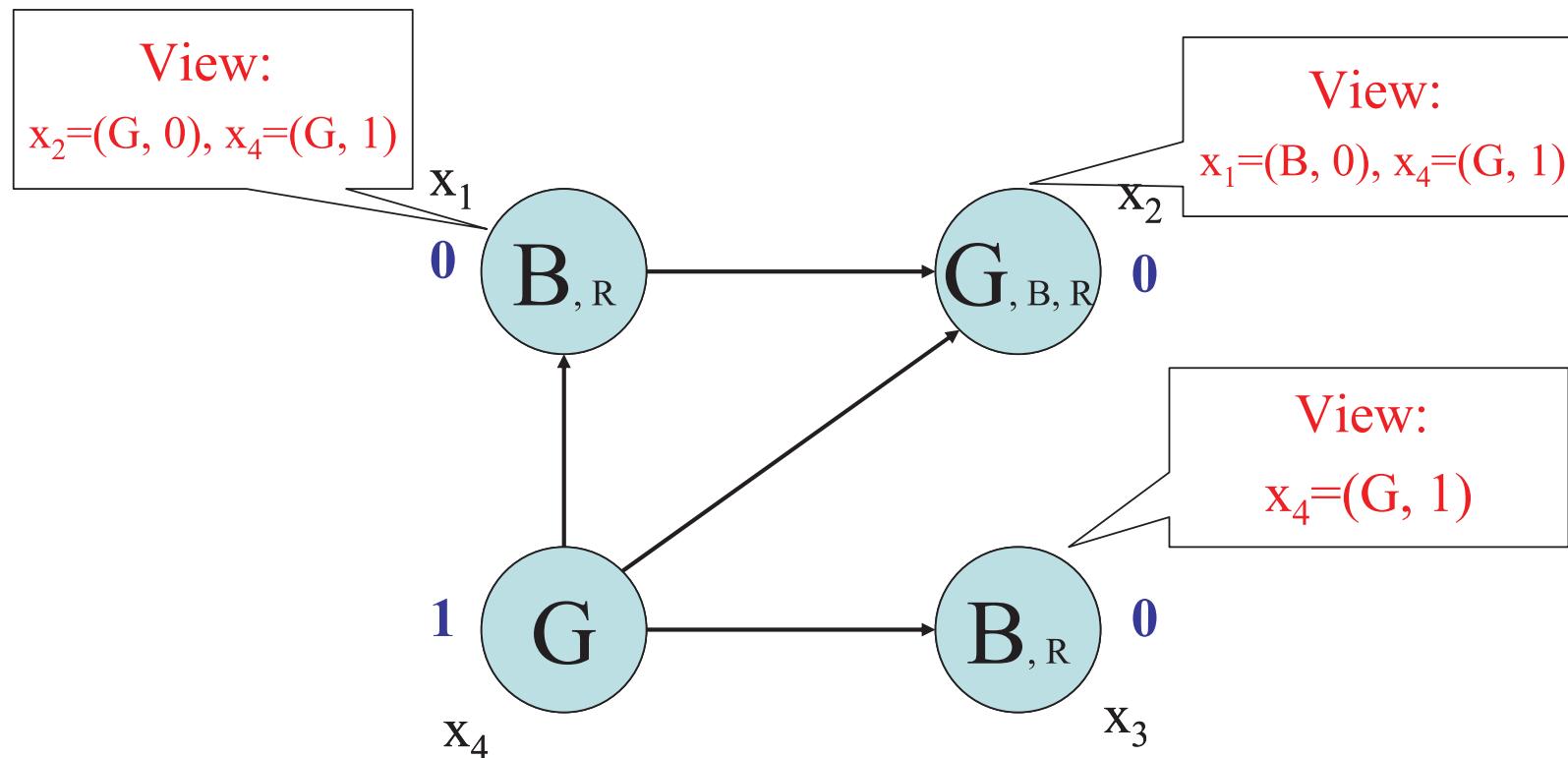
Update view

CONCURRENTLY, all agents receive the OK? messages and update their view

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۱۶ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

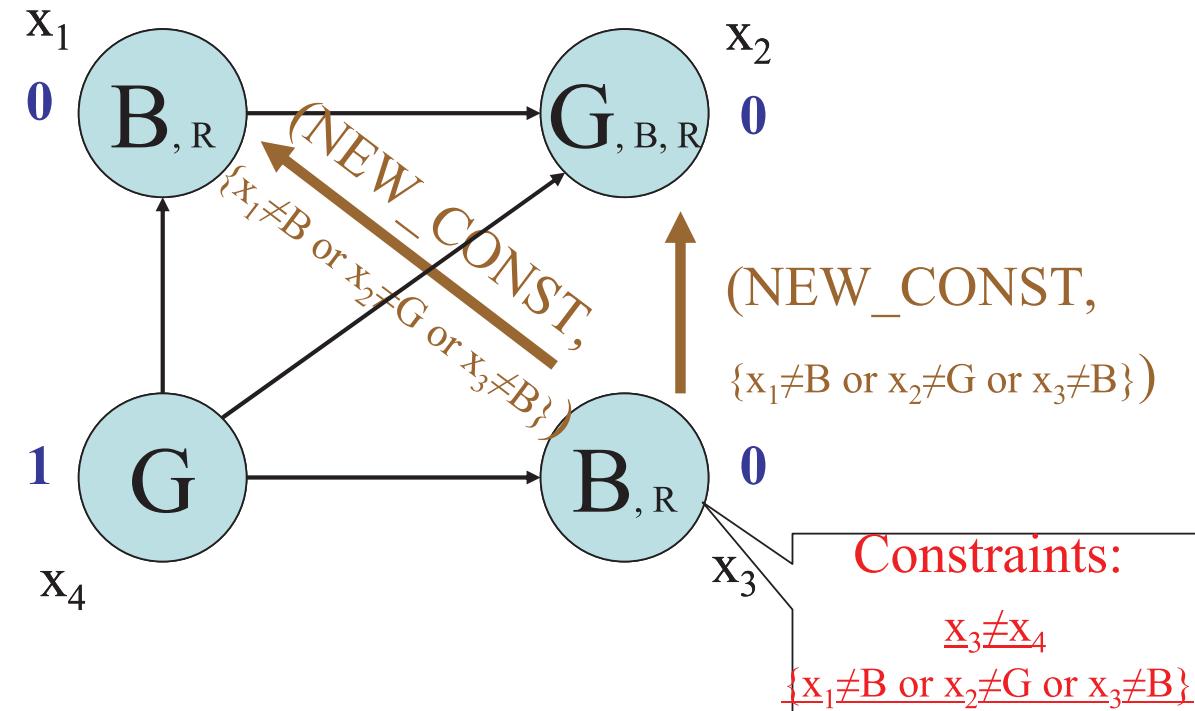


(Only the priority changed, so no new violation is discovered)

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۱۷)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



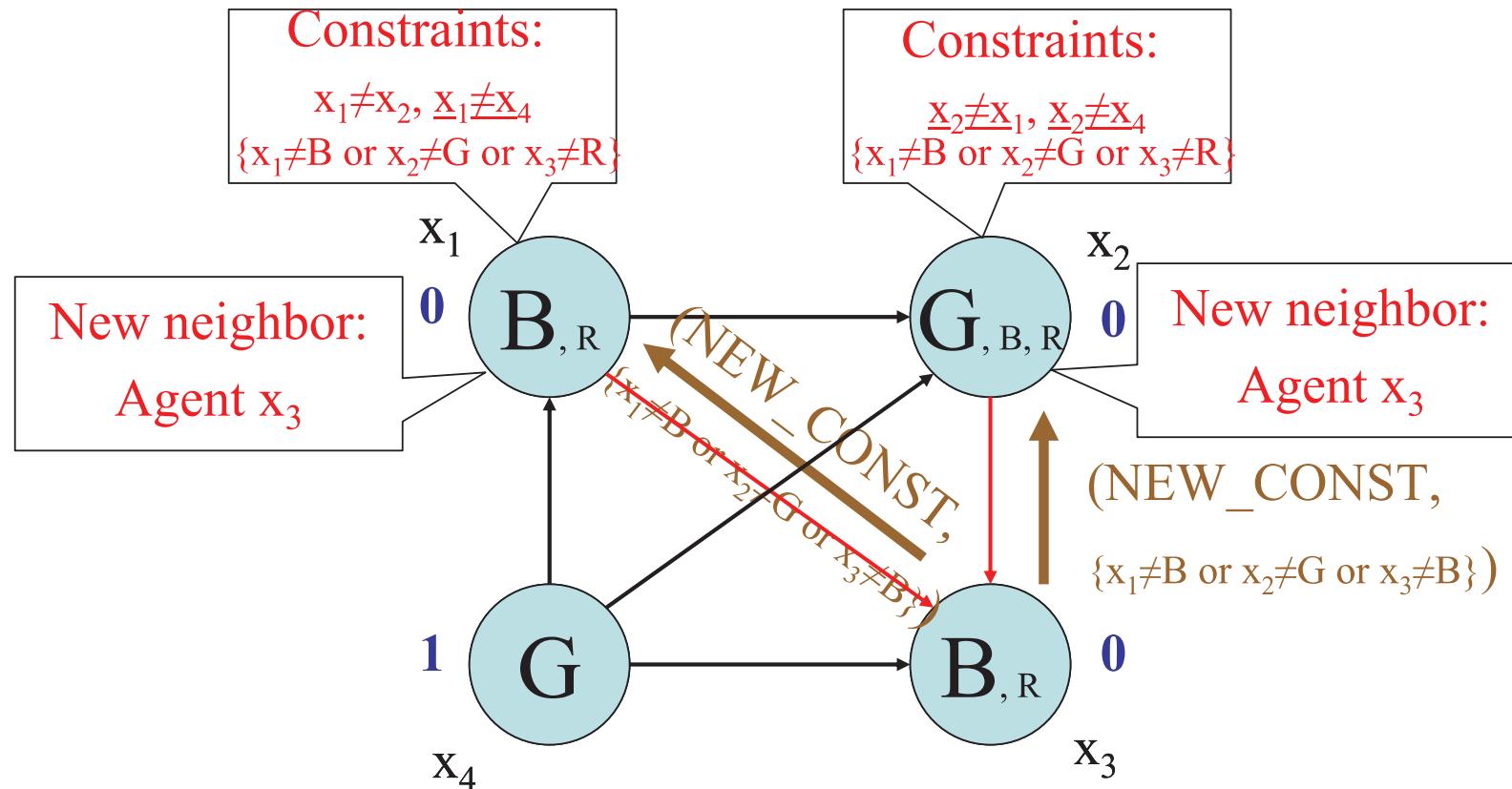
Send  
NEW\_CONST  
messages

CONCURRENTLY, Agent  $x_3$  sends NEW\_CONST messages to all agents involved in the new constraint

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۱۸)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

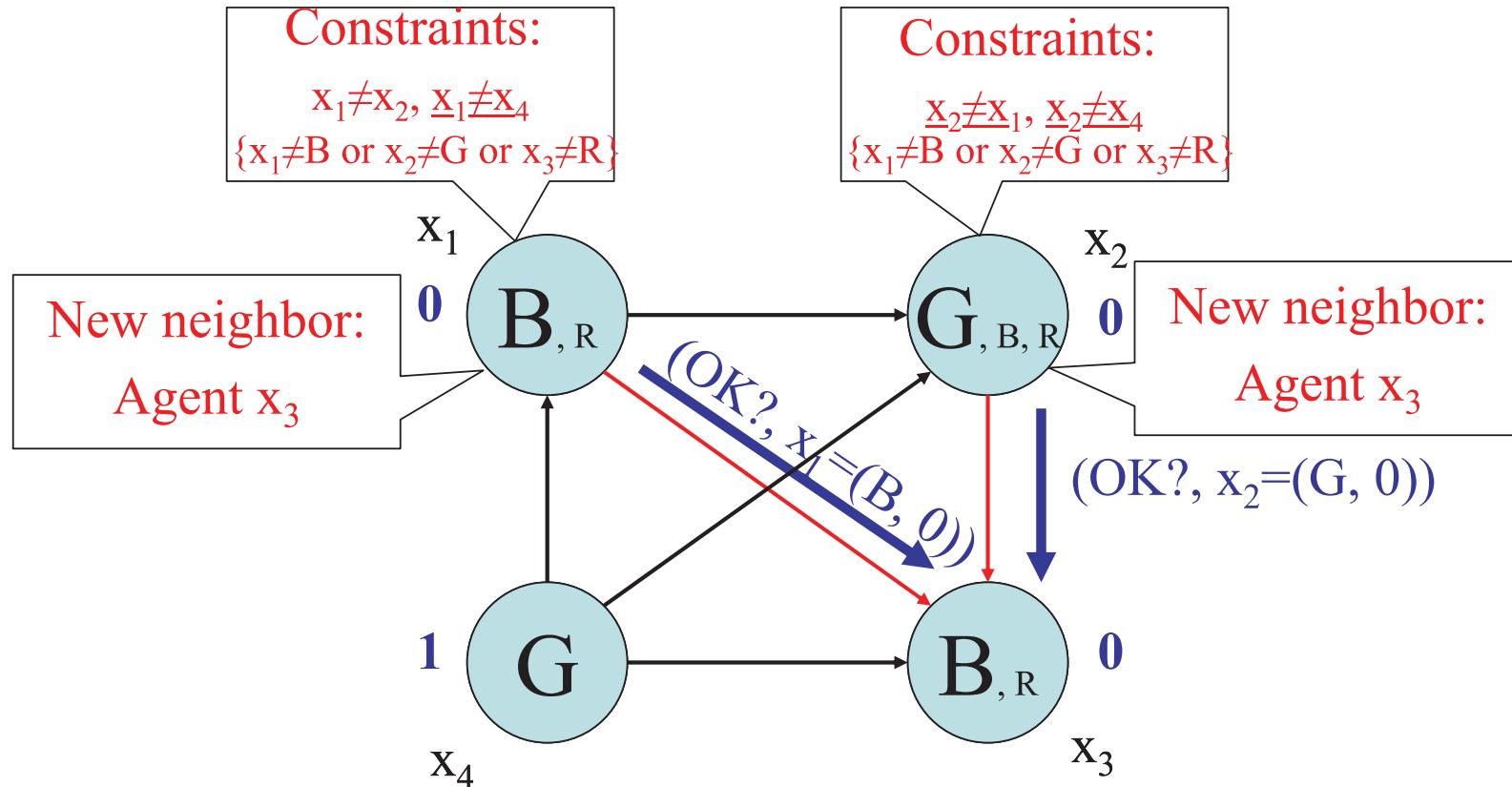


Agents  $x_1$  and  $x_2$  receive NEW\_CONST messages and record new constraint and new neighbor

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۱۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

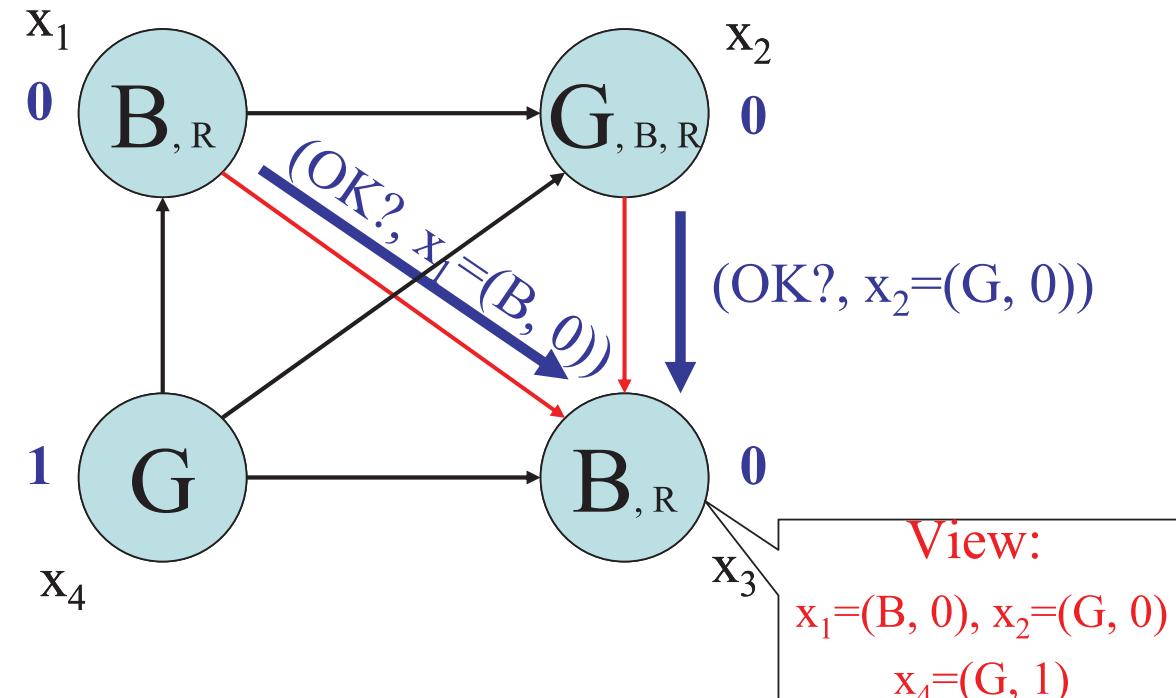


Agents  $x_1$  and  $x_2$  respond to the NEW\_CONST through OK? messages

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۰ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



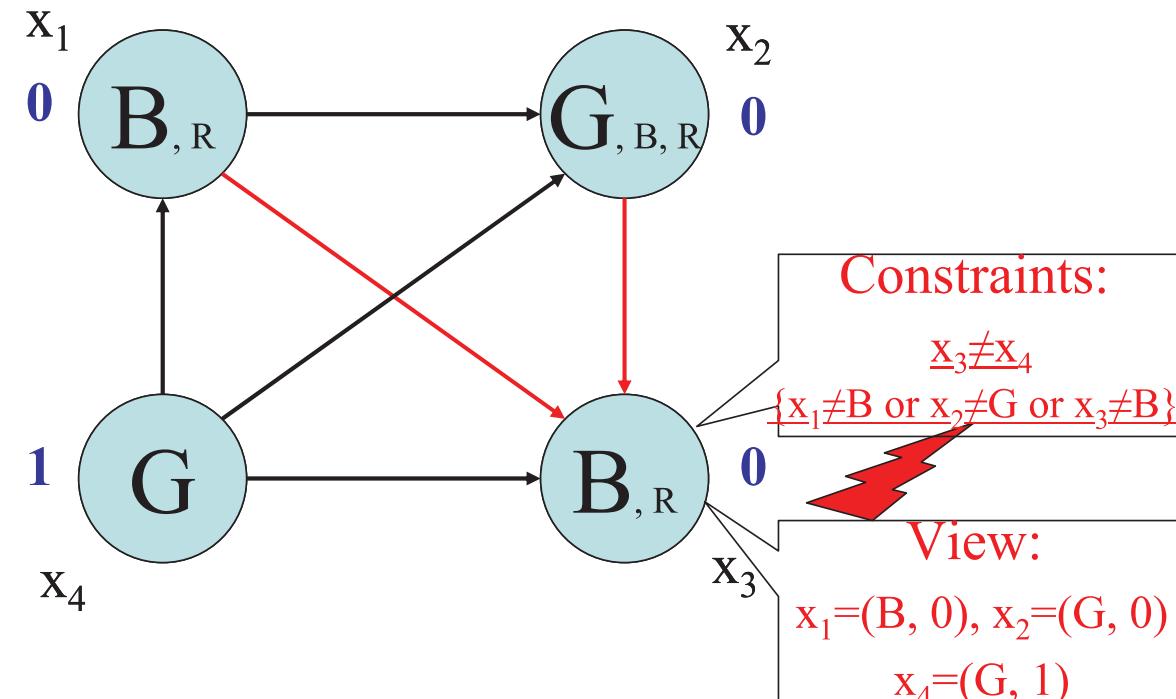
Update view

Agent  $x_3$  receives messages and updates its view

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۱)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



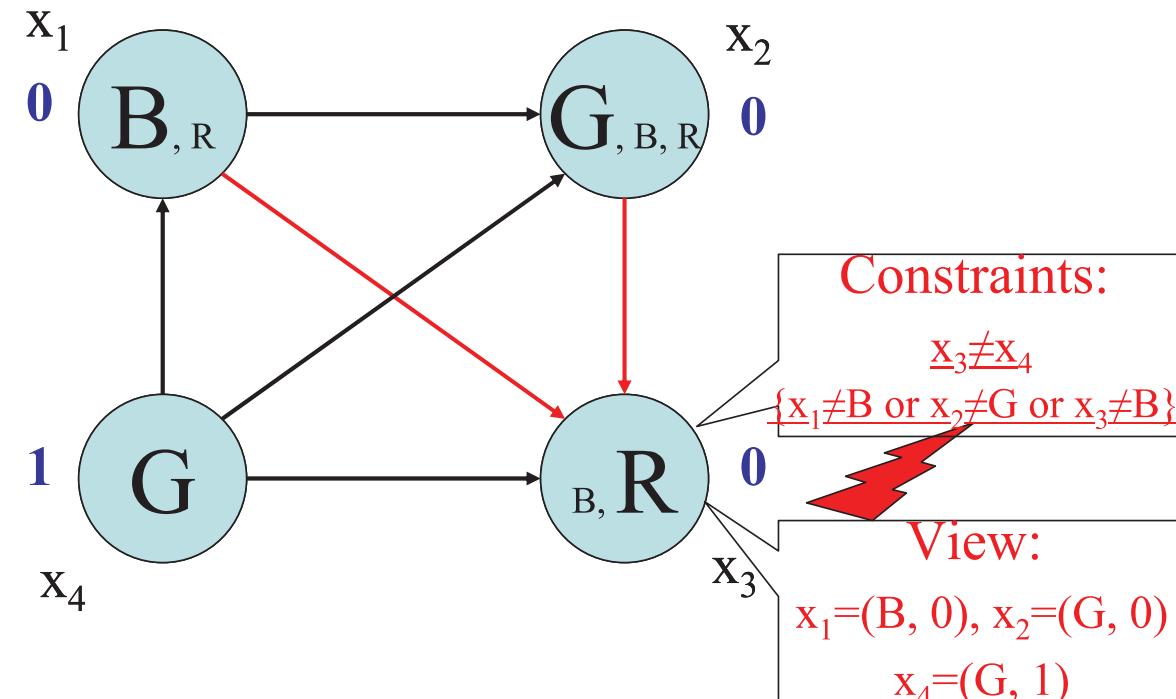
Agent  $x_3$  checks its view, and discovers that one constraint it is responsible for (the new one) is violated



## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۲)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



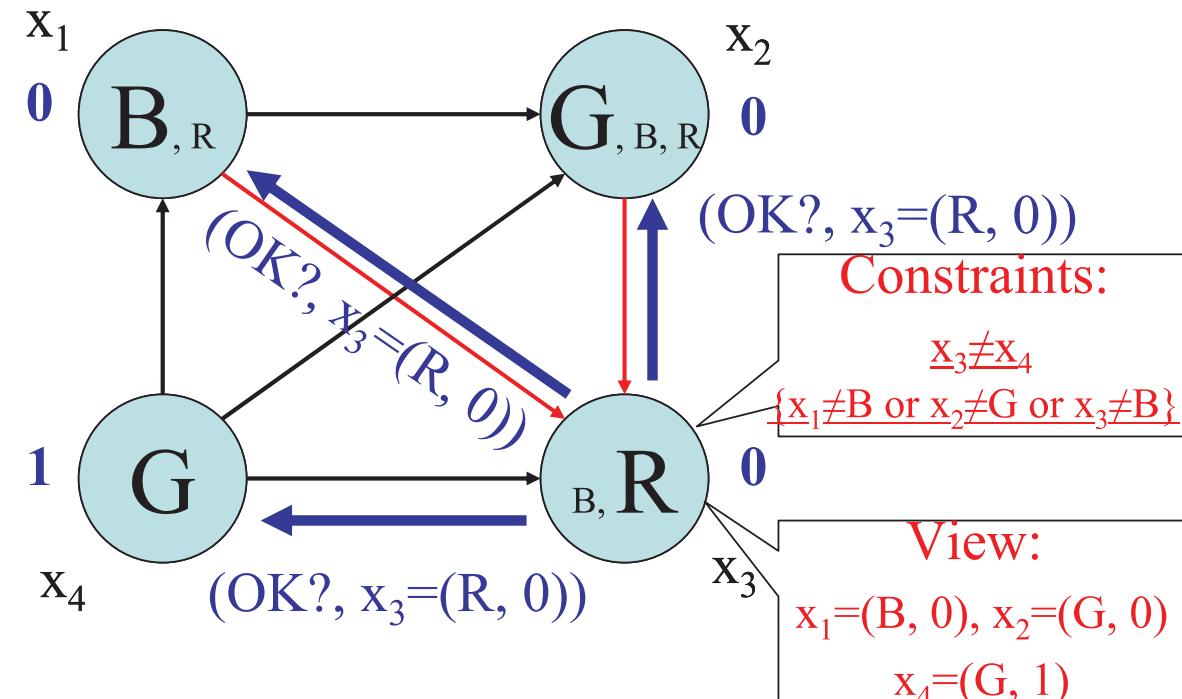
Try  
to choose  
value

Agent  $x_3$  tries to change its value to  $R$ , deleting all violations of constraints it is responsible for, and minimizing the number of violations of others

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۳ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



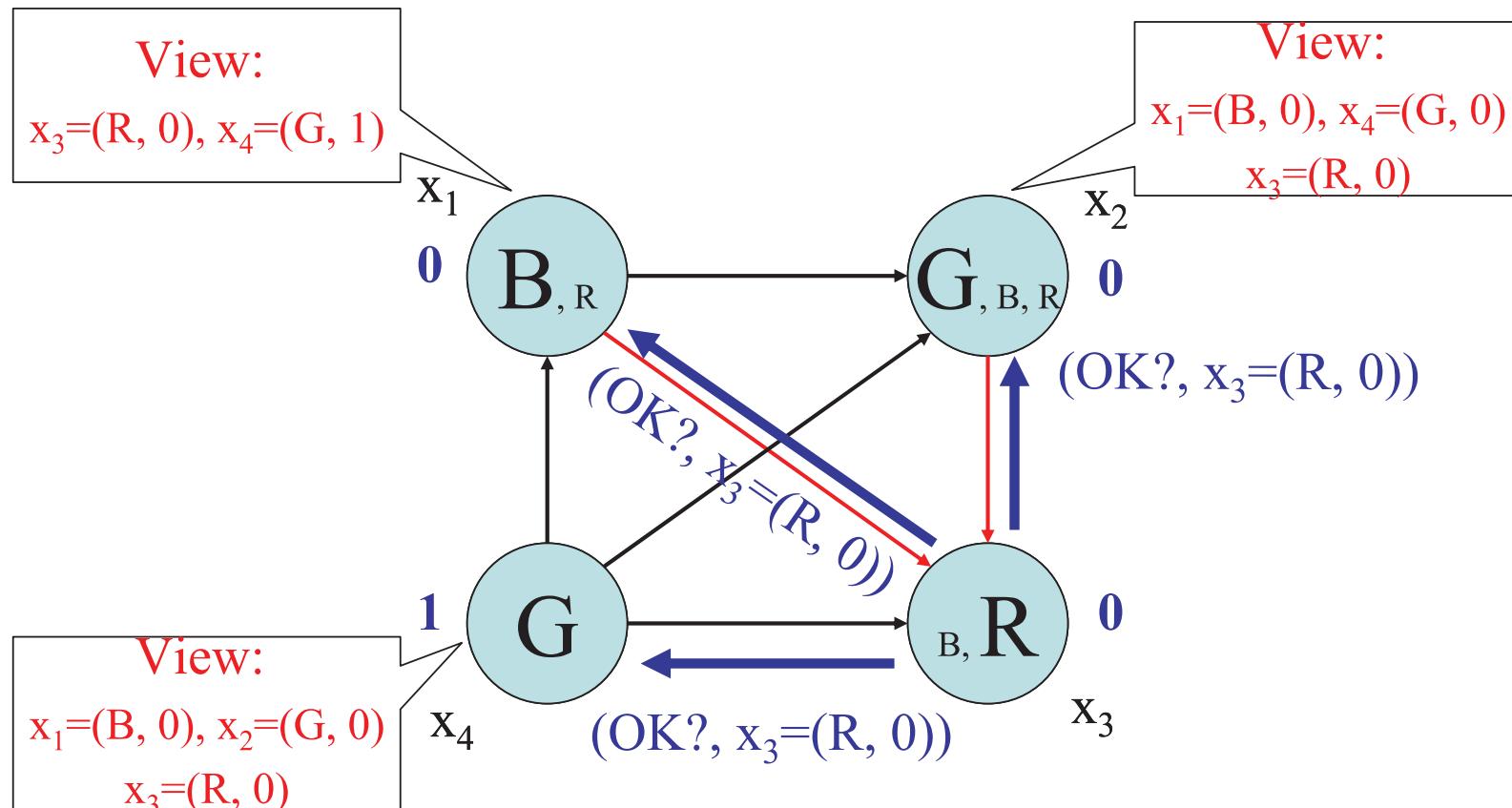
Send OK? messages

There is no more violations of constraints it is responsible for, so Agent  $x_3$  communicates its new value to ALL neighbors

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۴)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



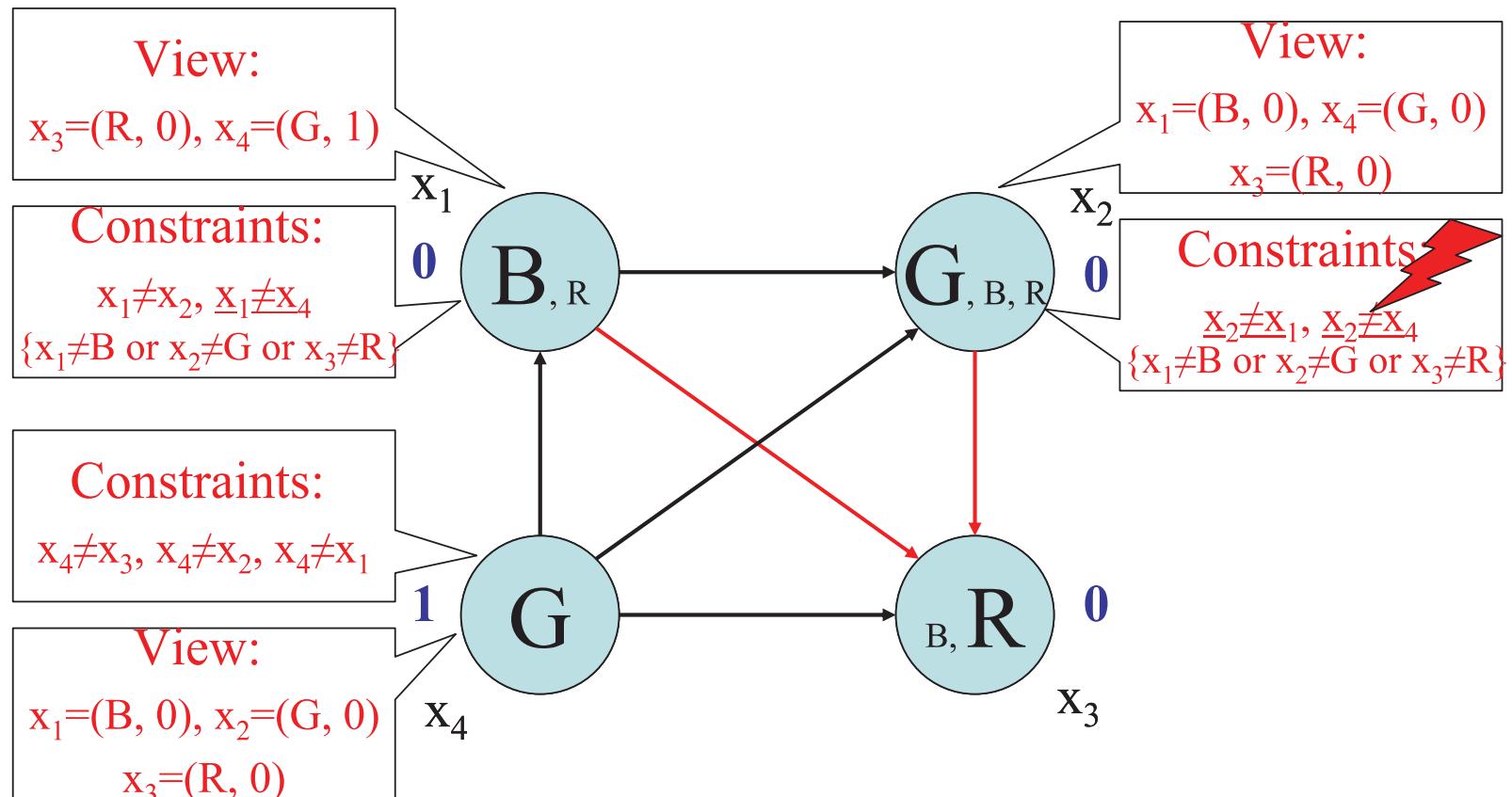
Update view

All agents receive the OK? messages  
and update their view

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۵)

## THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

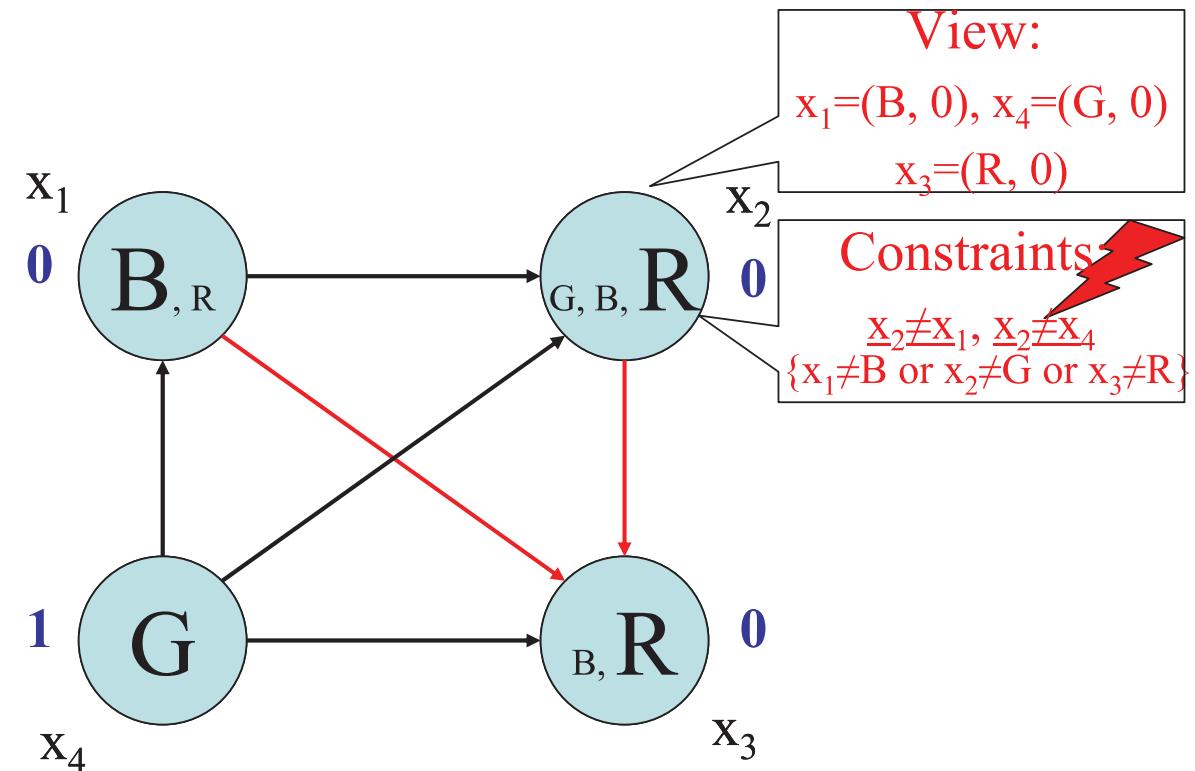


Agent  $x_1$ ,  $x_2$  and  $x_4$  check their view against the constraints they are responsible for, and Agent  $x_2$  discovers a violation

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۶ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



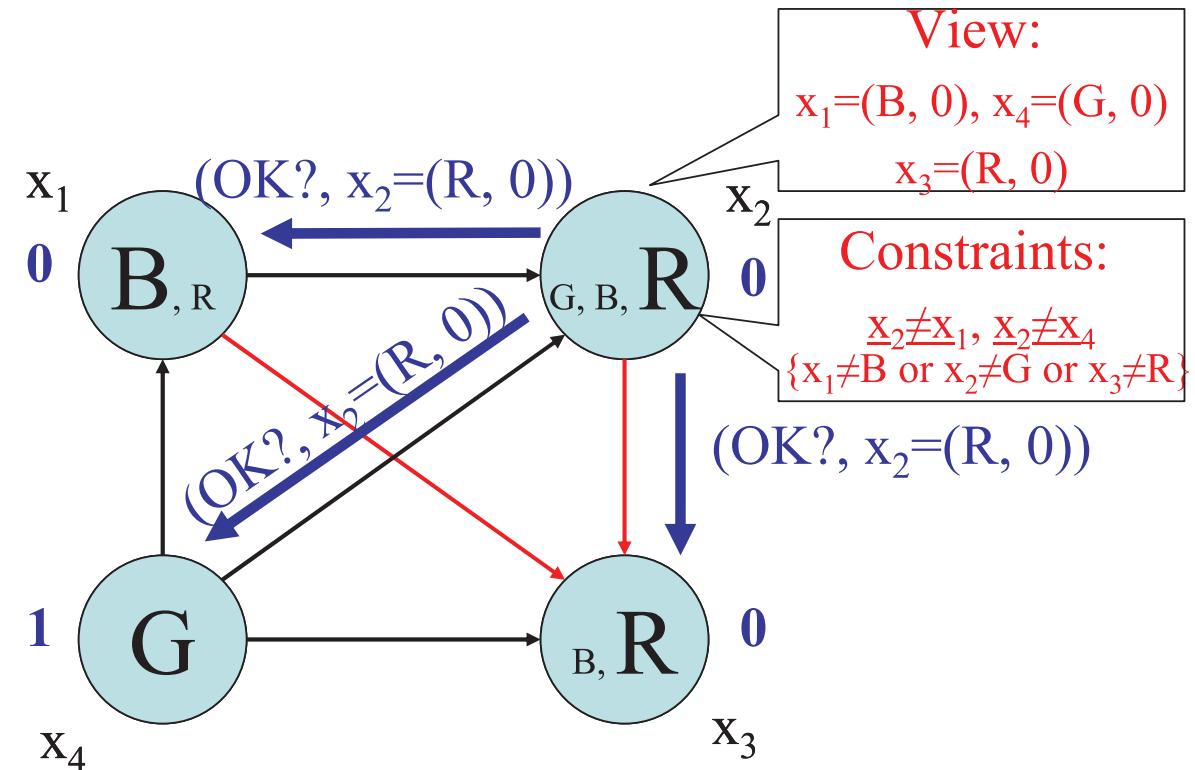
Try  
to choose  
value

Agents  $x_2$  tries to change its value to R, deleting all violations of constraints it is responsible for, and minimizing the number of violations of others

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۷ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



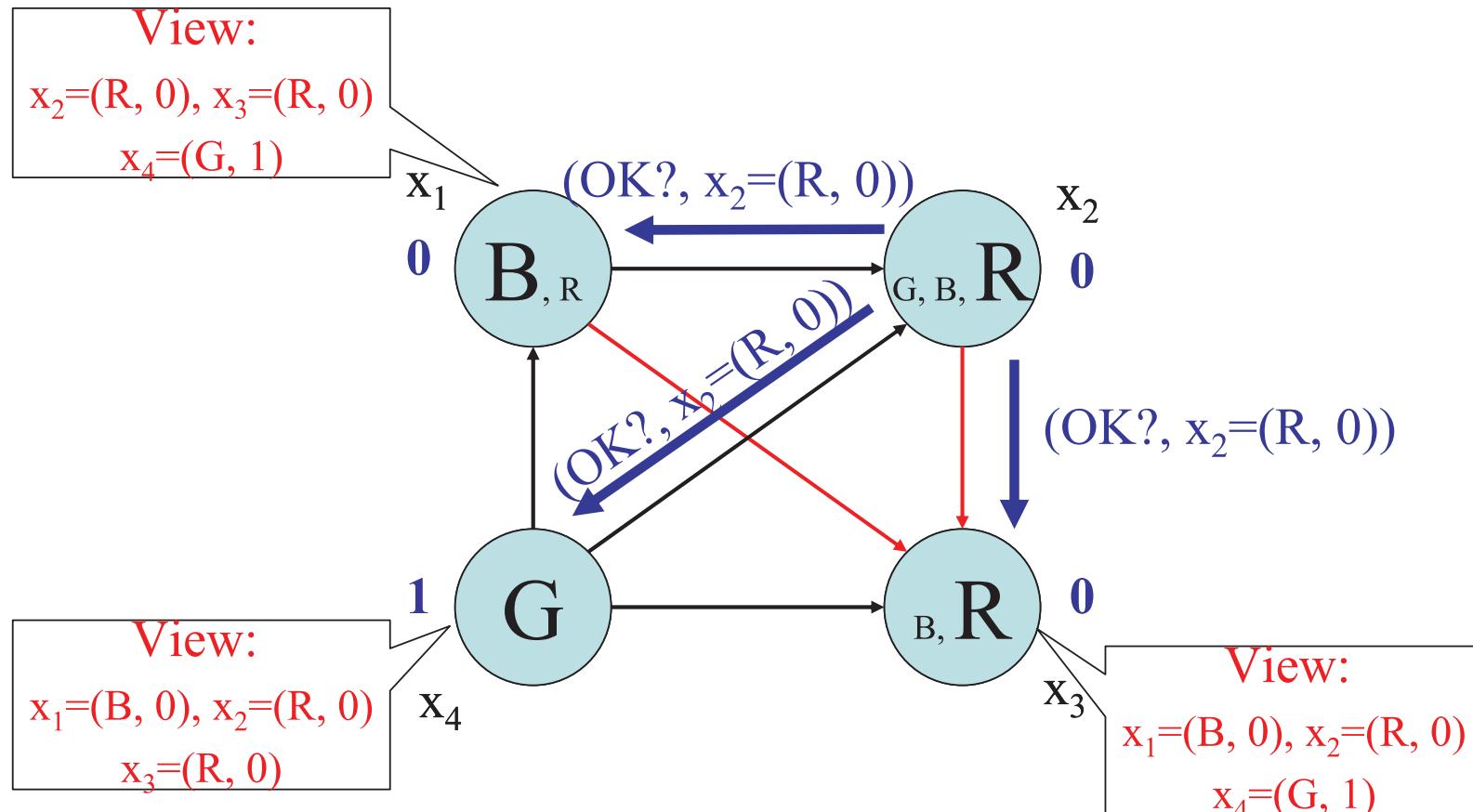
Send OK? messages

There is no more violations of constraints it is responsible for, so Agent  $x_3$  communicates its new value to ALL neighbors

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۸ از ۲۹)

### THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

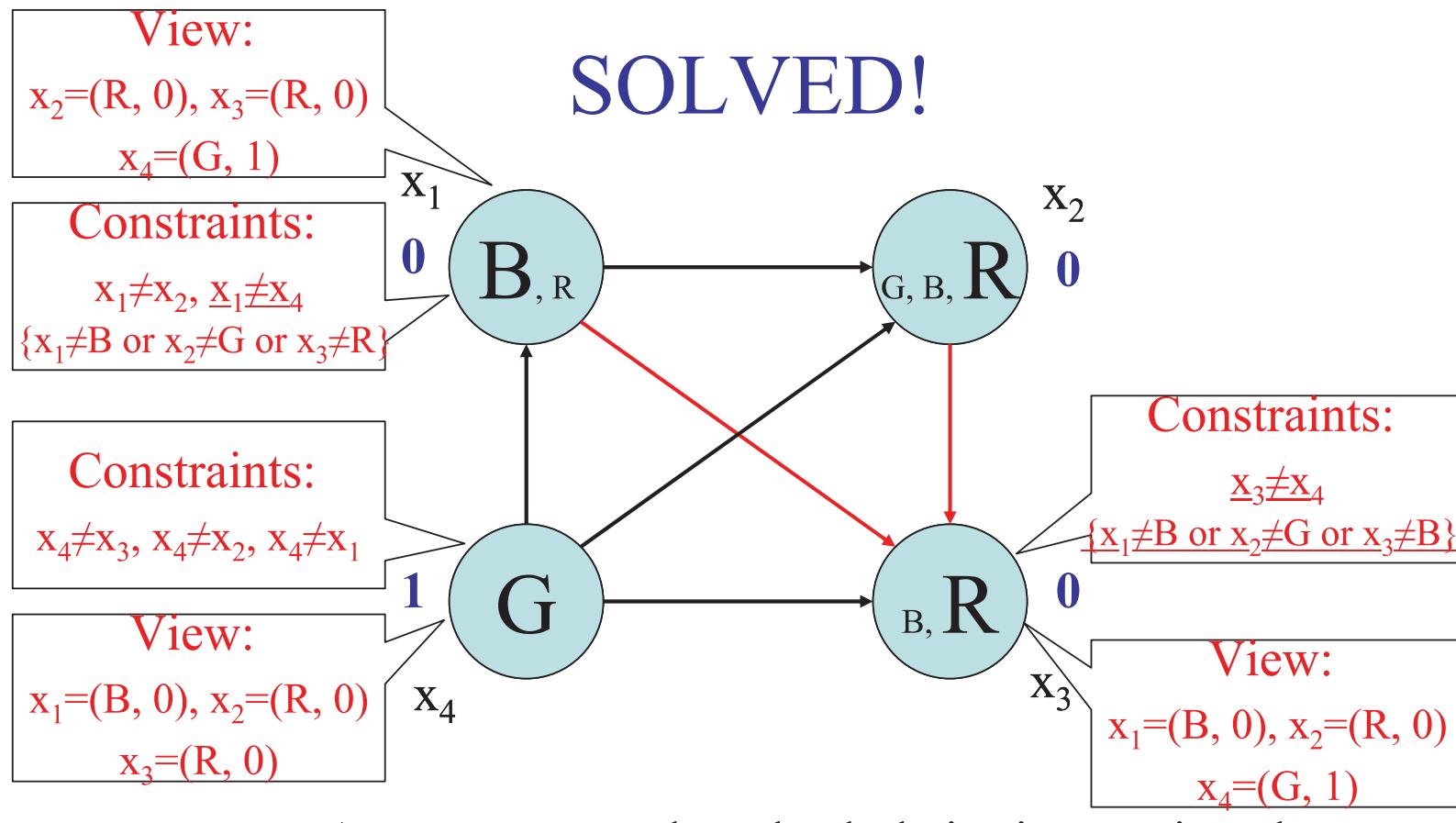


All agents receive the OK? messages  
and update their view

## الگوریتم جستجوی تعهد ضعیف ناهمگام

مثال (۲۹ از ۲۹)

## THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM



Agents  $x_1, x_3$  and  $x_4$  check their view against the constraints they are responsible for, and no new violation is discovered

## مسائل بهینه‌سازی قید توزیع شده

### DISTRIBUTED CONSTRAINT OPTIMIZATION PROBLEMS

Given

Variables  $\{x_1, x_2, \dots, x_n\}$ , each assigned to an agent

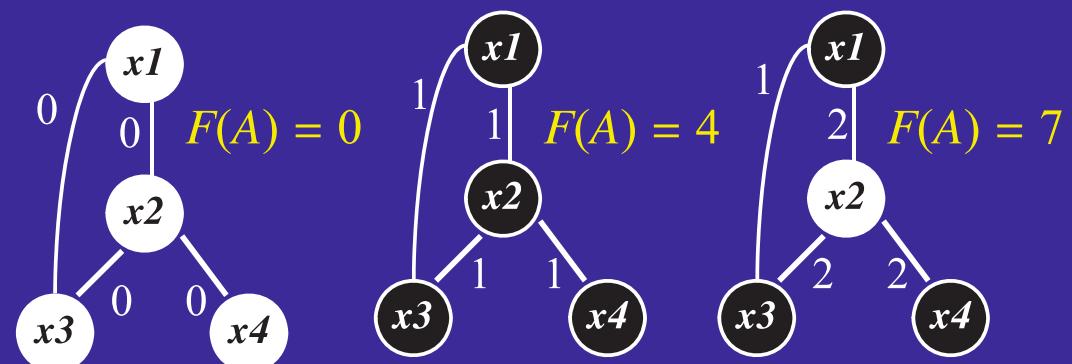
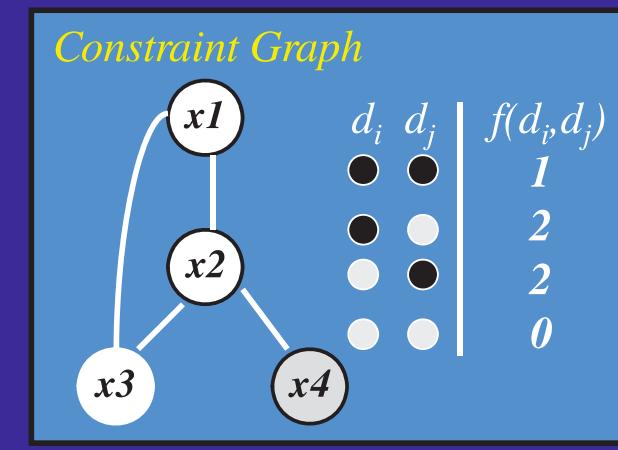
Finite, discrete domains  $D_1, D_2, \dots, D_n$ ,

For each  $x_i, x_j$ , valued constraint  $f_{ij}: D_i \times D_j \rightarrow N$ .

Goal

Find complete assignment  $A$  that minimizes  $F(A)$  where,

$$F(A) = \sum f_{ij}(d_i, d_j), \quad x_i \leftarrow d_i, \quad x_j \leftarrow d_j \text{ in } A$$

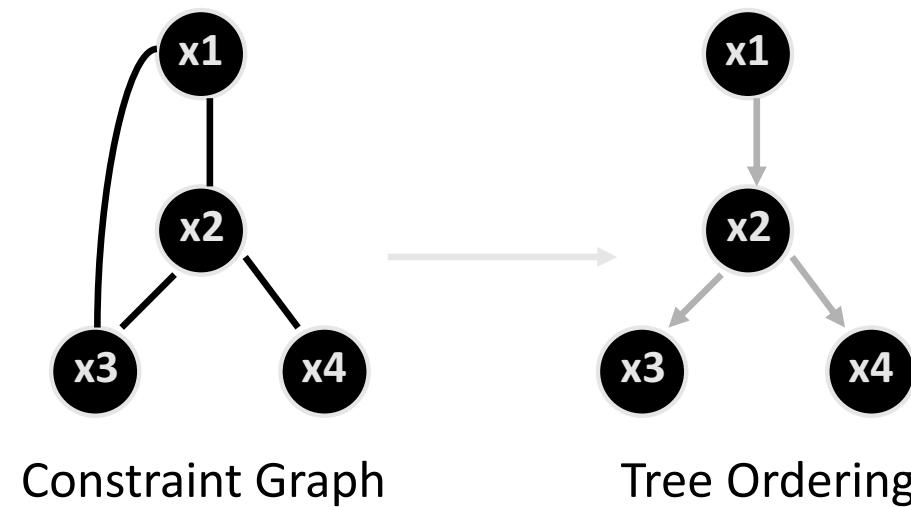


## الگوریتم جستجوی تعهد ضعیف ناهمگام

THE ASYNCHRONOUS WEAK-COMMITMENT SEARCH ALGORITHM

## الگوریتم بهینه‌سازی توزیع شده ناهمگام

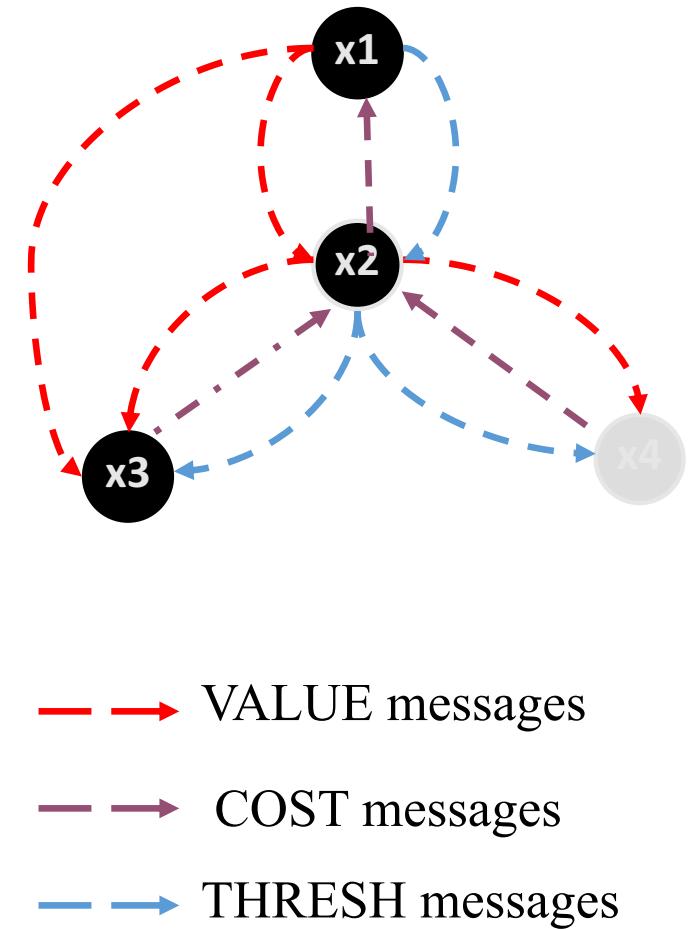
### ADOPT (ASYNCHRONOUS DISTRIBUTED OPTIMIZATION)



## الگوریتم بهینه‌سازی توزیع شده ناهمگام

### ADOPT (ASYNCHRONOUS DISTRIBUTED OPTIMIZATION)

- Agents are ordered in a tree
  - constraints between ancestors/descendents
  - no constraints between siblings
  
- Basic Algorithm:
  - choose value with min cost
  - Loop until **termination-condition** true:
    - When receive message:
      - choose value with min cost
      - send **VALUE** message to descendents
      - send **COST** message to parent
      - send **THRESHOLD** message to child

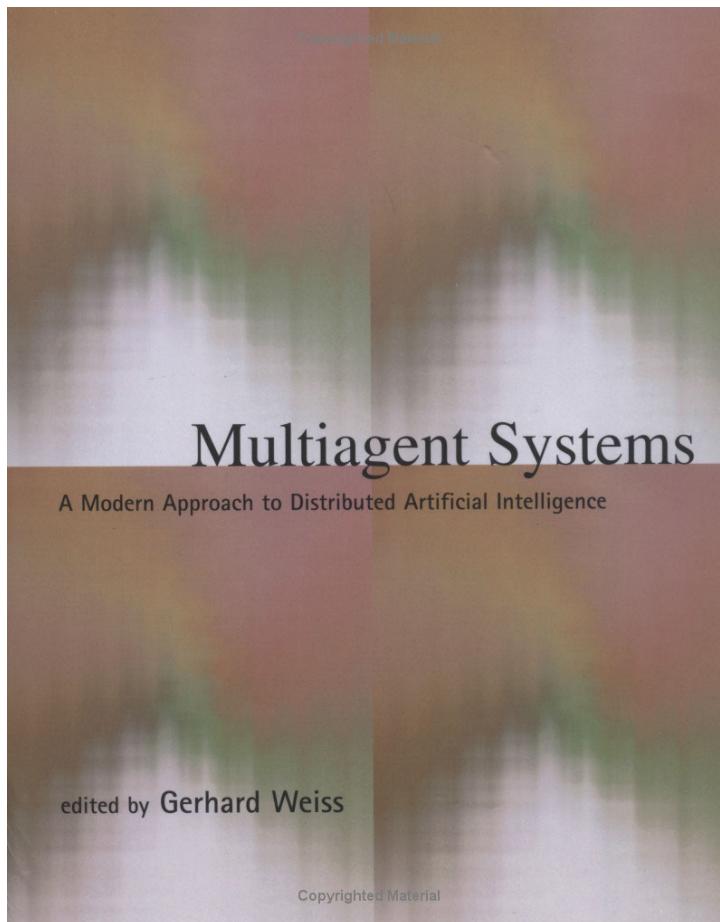


## **سیستم‌های چند عاملی**

ارضای قید توزیع شده

# **منابع**

## منبع اصلی



Gerhard Weiss (ed.),  
**Multiagent Systems: A Modern Approach to  
 Distributed Artificial Intelligence,**  
 MIT Press, 1999.  
**Chapter 4**

## 4 Search Algorithms for Agents

Makoto Yokoo and Toru Ishida

### 4.1 Introduction

In this chapter, we introduce several search algorithms that are useful for problem solving by multiple agents. Search is an umbrella term for various problem solving techniques in AI. In search problems, the sequence of actions required for solving a problem cannot be known *a priori* but must be determined by a trial-and-error exploration of alternatives. Since virtually all AI problems require some sort of search, search has a long and distinguished history in AI.

The problems that have been addressed by search algorithms can be divided into three classes: path-finding problems, constraint satisfaction problems, and two-player games.

A typical example of the first class, i.e., path-finding problems, is a puzzle called the *n-puzzle*. Figure 4.1 shows the 8-puzzle, which consists of eight numbered tiles arranged on a  $3 \times 3$  board (in a generalized case, there are  $n = k^2 - 1$  tiles on a  $k \times k$  board). The allowed moves are to slide any tile that is horizontally or vertically adjacent to the empty square into the position of the empty square. The objective is to transform the given initial configuration to the goal configuration by making allowed moves. Such a problem is called a path-finding problem, since the objective is to find a path (a sequence of moves) from the initial configuration to the goal configuration.

A constraint satisfaction problem (CSP) involves finding a goal configuration rather than finding a path to the goal configuration. A typical example of a CSP is a puzzle called 8-queens. The objective is to place eight queens on a chess board ( $8 \times 8$  squares) so that these queens will not threaten each other. This problem is called a constraint satisfaction problem since the objective is to find a configuration that satisfies the given conditions (constraints).

Another important class of search problems is two-player games, such as chess. Since two-player games deal with situations in which two *competitive* agents exist, it is obvious that these studies have a very close relation with DAI/multiagent systems where agents are competitive.

On the other hand, most algorithms for the other two classes (constraint satisfaction and path-finding) were originally developed for single-agent problem solving.

## منبع کمکی



### Multiagent Systems

Algorithmic, Game-Theoretic, and Logical Foundations

YOAV SHOHAM

KEVIN LEYTON-BROWN

CAMBRIDGE

Copyrighted Material

Yoav Shoham and Kevin Leyton-brown,  
**Multiagent Systems: Algorithmic, Game-Theoretic,  
 and Logical Foundations**,  
 Cambridge University Press, 2009.  
**Chapter 1**

## 1

### *Distributed Constraint Satisfaction*

sensor network

In this chapter and the next we discuss cooperative situations in which agents collaborate to achieve a common goal. This goal can be viewed as shared between the agents or, alternatively, as the goal of a central designer who is designing the various agents. Of course, if such a designer exists, a natural question is why it matters that there are multiple agents; they can be viewed merely as end sensors and effectors for executing the plan devised by the designer. However, there exist situations in which a problem needs to be solved in a distributed fashion, either because a central controller is not feasible or because one wants to make good use of the distributed resources. A good example is provided by *sensor networks*. Such networks consist of multiple processing units, each with local sensor capabilities, limited processing power, limited power supply, and limited communication bandwidth. Despite these limitations, these networks aim to provide some global service. Figure 1.1 shows an example of a fielded sensor network used for monitoring environmental quantities like humidity, temperature and pressure in an office environment. Each sensor can monitor only its local area and, similarly, can communicate only with other sensors in its local vicinity. The question is what algorithm the individual sensors should run so that the center can still piece together a reliable global picture.

Distributed algorithms have been widely studied in computer science. We concentrate on distributed problem-solving algorithms of the sort studied in artificial intelligence. We divide the discussion into two parts. In this chapter we cover distributed constraint satisfaction, where agents attempt in a distributed fashion to find a feasible solution to a problem with global constraints. In the next chapter we look at agents who try not only to satisfy constraints, but also to optimize some objective function subject to these constraints.

Later in this book we will encounter additional examples of distributed problem solving. Each of them requires specific background, however, which is why they are not discussed here. Two of them stand out in particular.

- In Chapter 7 we encounter a family of techniques that involve learning, some of them targeted at purely cooperative situations. In these situations the agents learn through repeated interactions how to coordinate a choice of action. This material requires some discussion of noncooperative game theory (discussed in

# منبع کمکی

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**Chapter 2**

CONSTRAINT SATISFACTION  
PROBLEM

## Chapter 2

### Distributed Constraints

Most multiagent systems are characterized by a set of autonomous agents each with local information and ability to perform an action when the set of actions of all must be coordinated so as to achieve a desired global behavior. In all these cases you own all the agents in question and can program them to do whatever you want. Thus, there is no need to properly incentivize them as there would be in an open system, which we study in later Chapters. However, there is still the problem of coordination. Since each agent only has local information it might be hard for it to decide what to do.

In this chapter we look at some algorithms for performing distributed search in cooperative multiagent systems where each agent has some local information and where the goal is to get all the agents to set themselves to a state such that the set of states in the system is optimal. For example, imagine a group of small sensors in a field. Each sensor can communicate only with those that are near him and has to decide which of its modalities to use (sense temperature, point radar North, point radar South, etc.) so that the group of sensors gets a complete view of the important events in the field. Or, imagine a group of kids who have been told to stand in a circle, each one must move find a spot to stand on but the position of that spot depends on everyone else's location. In these examples the agents have local information, can take any one of their available actions at any time, but the utility of their actions depends on the actions of others. We find that many multiagent problems can be reduced to a distributed constraints problem. Thus, the algorithms we present in this chapter have many different applications.

#### 2.1 Distributed Constraint Satisfaction

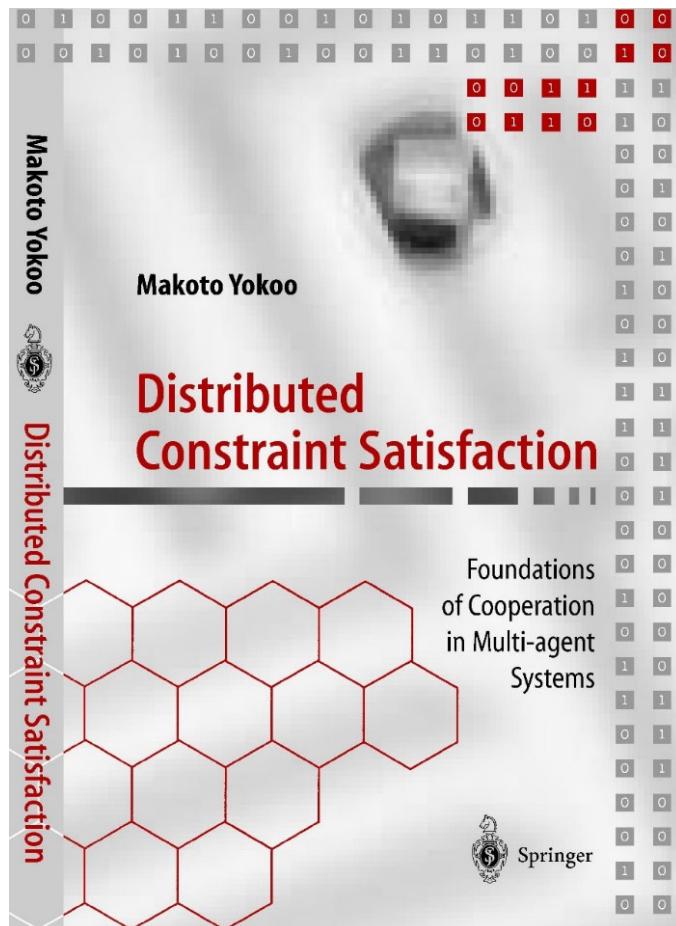
We start by formally describing the problem. In a constraint satisfaction problem (CSP) we are given a set of variables, each with its own domain along with a set of constraints. The goal is to set every variable to a value from its domain such that no constraints are violated. Formally,

**Definition 2.1** (Constraint Satisfaction Problem). *Given a set of variables  $x_1, x_2, \dots, x_n$  with domains  $D_1, D_2, \dots, D_n$  and a set of boolean constraints  $P$  of the form  $p_k(x_{k1}, x_{k2}, \dots, x_{kj}) \rightarrow [0, 1]$ , find assignments for all the variables such that no constraints are violated.*

The most widely studied instance of a constraint satisfaction problem is the graph coloring problem, shown in figure 2.1. In this problem we are given a graph and a set of colors. The problem is to find out if there is a way to color each node with one of the given colors such that no two nodes that are connected by an edge have the same color. We can easily map the graph coloring problem to the formal definition of a CSP by considering each node to be a variable, the domains to be the set of colors, and each edge becomes a constraint between its two nodes that is true only if their values are different. Notice that, in graph coloring constraints are only over two variables instead of over any set of variables as we find in the general constraint satisfaction problem.

The constraint satisfaction problem is NP-complete. As such, we use search algorithms to find a solution and hope that the actual running time will be less than

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