مبانی بینایی کامپیوتری

درس ۶ب

فیلتر کردن در حوزه‌ی فرکانس در متلب

Filtering in the Frequency Domain in MATLAB®

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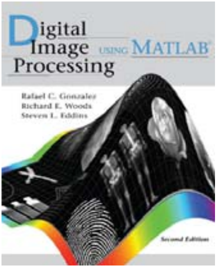
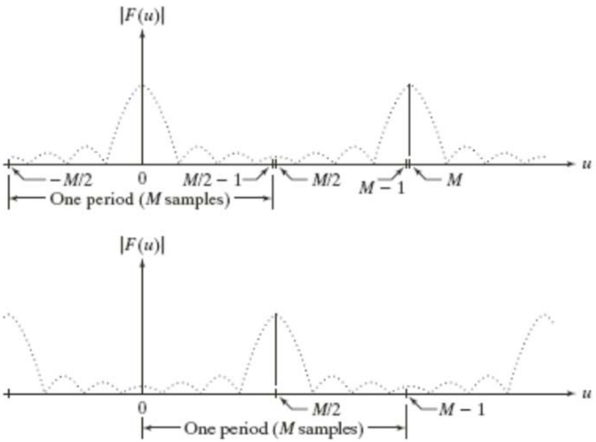
http://courses.fouladi.ir/fcvision



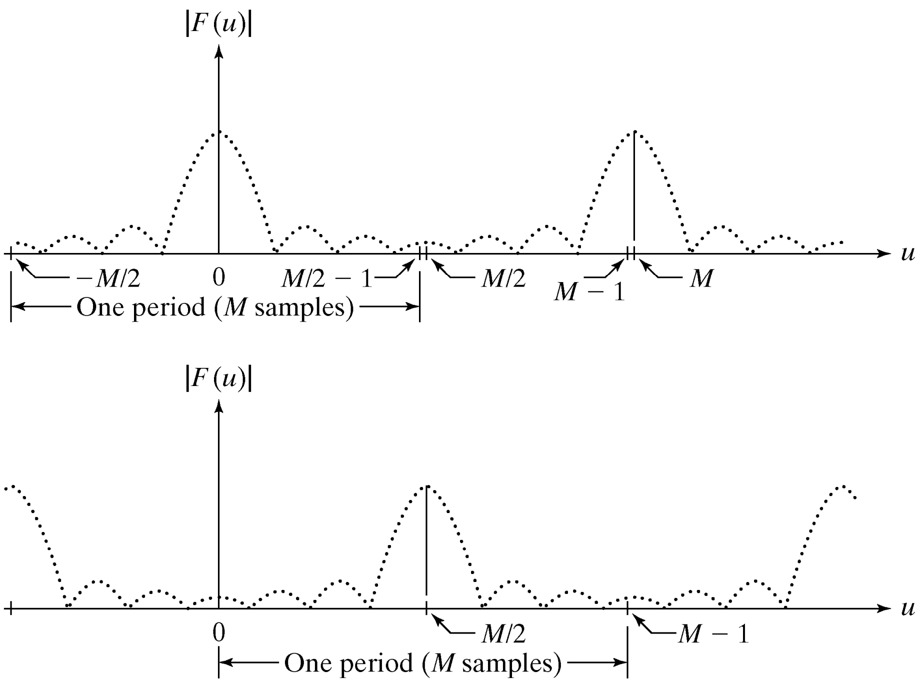
Chapter 4
Filtering in the Frequency Domain



a
b

FIGURE 3.1 (a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$. (b) Centered spectrum in the same interval, obtained by multiplying $f(x)$ by $(-1)^x$ prior to computing the Fourier transform.

خاصیت تناوب در تبدیل فوریه‌ی گسسته‌ی یک‌بعدی

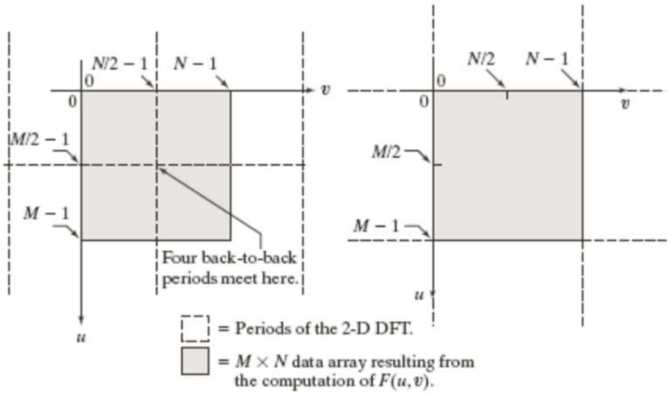


a
b

(a) Fourier spectrum showing back-to-back half periods in the interval $[0, M - 1]$. (b) Centered spectrum in the same interval, obtained by multiplying $f(x)$ by $(-1)^x$ prior to computing the Fourier transform.

MATLAB

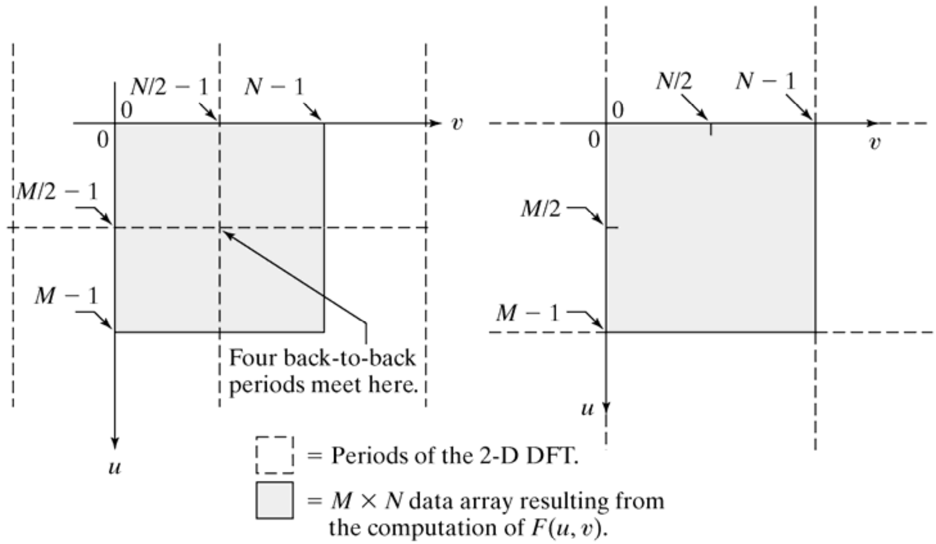
Chapter 4
Filtering in the Frequency Domain



a b

FIGURE 3.2 (a) $M \times N$ Fourier spectrum (shaded), showing four back-to-back quarter periods. (b) Spectrum after multiplying $f(x, y)$ by $(-1)^{x+y}$ prior to computing the Fourier transform. The shaded period is the data that would be obtained by using the DFT.

خاصیت تناوب در تبدیل فوریه‌ی گسسته‌ی دوبعدی



a b

(a) $M \times N$ Fourier spectrum (shaded), showing four back-to-back quarter periods contained in the spectrum data. (b) Spectrum obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ prior to computing the Fourier transform. Only one period is shown shaded because this is the data that would be obtained by an implementation of the equation for $F(u, v)$.

تبدیل فوریه‌ی گسسته‌ی دوبعدی در متلب

با الگوریتم تبدیل سریع فوریه

```
F = fft2(f)
```

```
F = fft2(f, P, Q)
```

با پد کردن تصویر f با صفر به اندازه‌ی لازم تا تبدیل حاصل به اندازه‌ی P*Q درآید.

تبدیل فوریه‌ی گسسته‌ی دوبعدی در متلب

انتقال مبدأ تبدیل به مرکز مستطیل فرکانسی

```
Fc = fftshift(F)
```

چهار ربع F را با هم تعویض می‌کند.

$$\texttt{fftshift}\left(\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}\right) == \begin{array}{cc} 4 & 3 \\ 2 & 1 \end{array}$$

فرمول محاسبه‌ی مرکز برای یک تصویر M*N:

```
[floor(M/2)+1, floor(N/2)+1]
```

تبدیل فوریه‌ی گسسته‌ی دوبعدی در متلب

محاسبه‌ی اندازه طیف و زاویه فاز

```
S = abs(F)
```

اندازه طیف

```
phi = angle(F)
```

زاویه فاز

```
phi = atan2(imag(F), real(F))
```

تبدیل فوریه‌ی گسسته‌ی دوبعدی در متلب

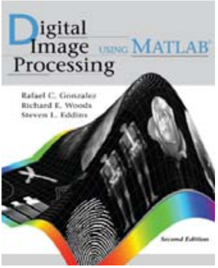
تبدیل معکوس

```
f = ifft2(F)
```

به دلیل خطاهای محاسباتی ناشی از گرد کردن:

```
f = real(ifft2(F))
```


MATLAB



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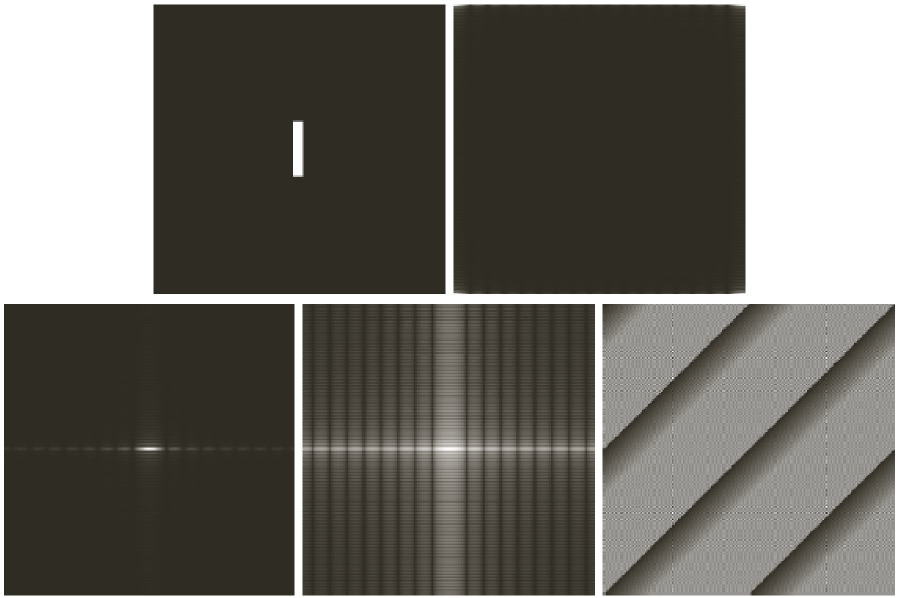
Filtering in the Frequency Domain



a b
c d e

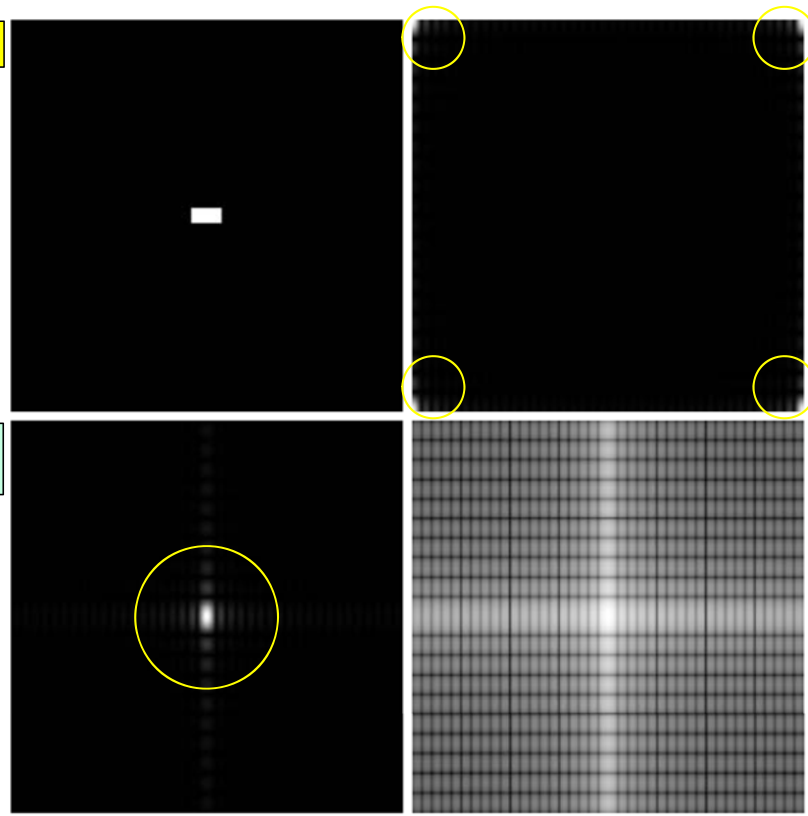
FIGURE 3.3 (a) Image. (b) Fourier spectrum. (c) Centered spectrum. (d) Spectrum visually enhanced by a log transformation. (e) Phase angle image.

خاصیت تناوب در تبدیل فوریه‌ی گسسته‌ی دوبعدی

مثال

f

```
F = fft2(f);
S = abs(F);
imshow(S, [])
```



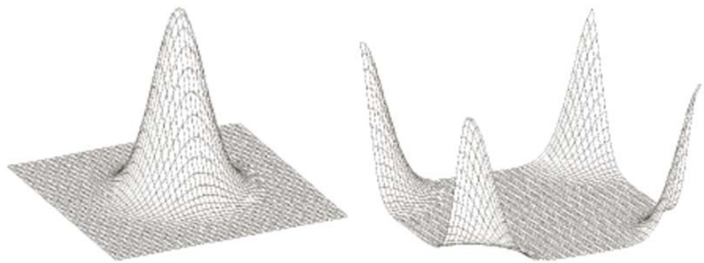
a	b
c	d

(a) A simple image. (b) Fourier spectrum. (c) Centered spectrum. (d) Spectrum visually enhanced by a log transformation.

```
Fc = fftshift(F);
imshow(abs(Fc), [])
```

```
S2 = log(1+abs(Fc));
imshow(S2, [])
```

Chapter 4
Filtering in the Frequency Domain

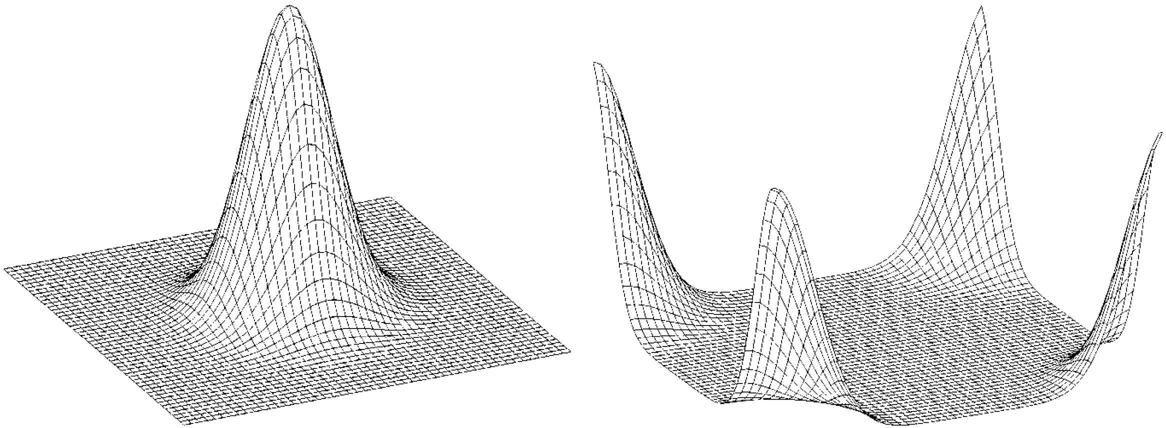


a b

FIGURE 3.4 Transfer functions of (a) a centered lowpass filter, and (b) the format used for DFT filtering. Note that these are frequency domain filters.

فیلتر کردن در حوزه‌ی فرکانس

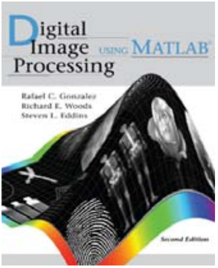
تابع انتقال فیلتر



a b

Transfer functions of (a) a centered lowpass filter, and (b) the format used for DFT filtering. Note that these are frequency domain filters.

MATLAB



Chapter 4
Filtering in the Frequency Domain

a b c

FIGURE 3.5 (a) An image of size 256×256 pixels. (b) Image lowpass-filtered in the frequency domain without padding. (c) Image lowpass-filtered in the frequency domain with padding. Compare the upper portion of the vertical edges in (b) and (c).

فیلتر کردن در حوزه‌ی فرکانس

فیلتر پایین‌گذر

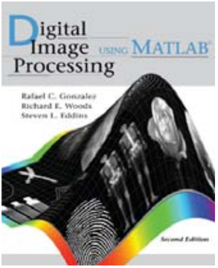
a b c

(a) A simple image of size 256 × 256. (b) Image lowpass-filtered in the frequency domain without padding. (c) Image lowpass-filtered in the frequency domain with padding. Compare the light portion of the vertical edges in (b) and (c).

f

```
[M,N] = size(f);
[f, revertclass] = tofloat(f);
F = fft2(f);
sig = 10;
H = lpfilter('gaussian',M,N,sig);
G = H .* F;
g = ifft(G);
g = revertclass(g);
imshow(g)
```

MATLAB



Chapter 4
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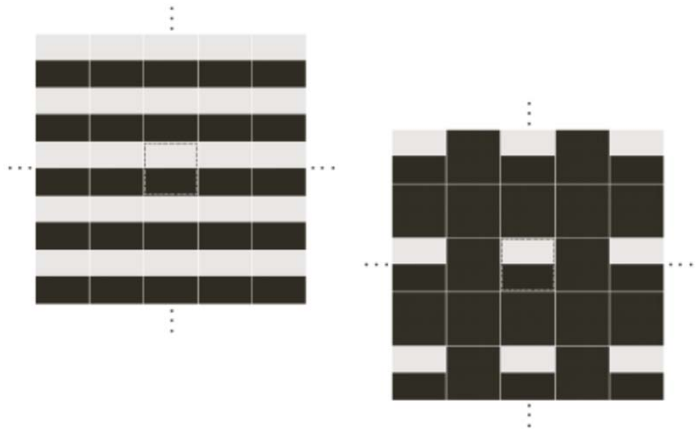
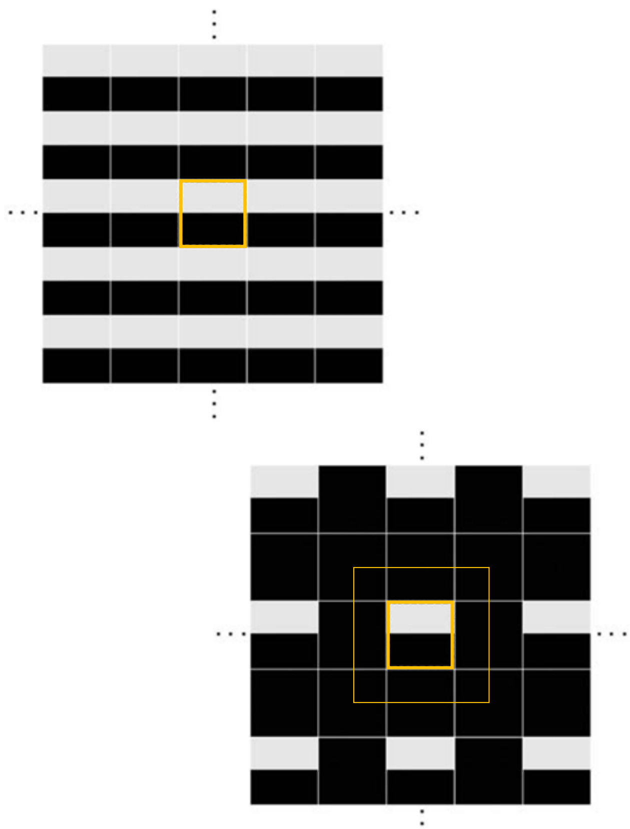


a b

FIGURE 3.6 (a) Implied, infinite periodic sequence of the image in Fig. 3.5(a). The dashed region represents the data processed by `fft2`. (b) The same periodic sequence after padding with 0s. The thin, solid white lines in both images are shown for convenience in viewing; they are not part of the data.

فیلتر کردن در حوزه‌ی فرکانس

پدینگ



a
b

(a) Implied, infinite periodic sequence of the image in Fig. 4.5(a). The dashed region represents the data processed by `fft2`. (b) The same periodic sequence after padding with 0s. The thin white lines in both images are shown for convenience in viewing; they are not part of the data.

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Filtering in the Frequency Domain

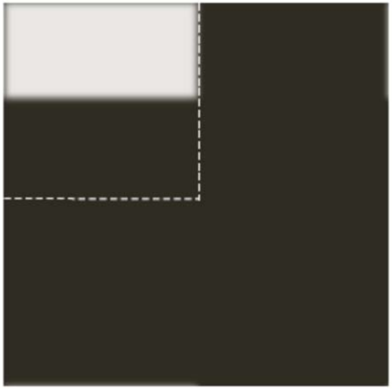
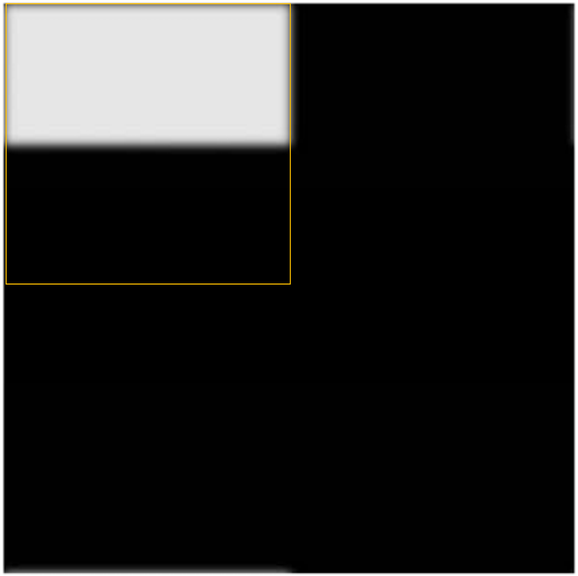


FIGURE 3.7 Full padded image resulting from `ifft2` after filtering. This image is of size 512×512 pixels. The dashed line shows the dimensions of the original, 256×256 image.

فیلتر کردن در حوزه‌ی فرکانس

تصویر نهایی پدشده حاصل از تبدیل فوریه‌ی معکوس



Full padded image resulting from `ifft2` after filtering. This image is of size 512 × 512 pixels.

MATLAB

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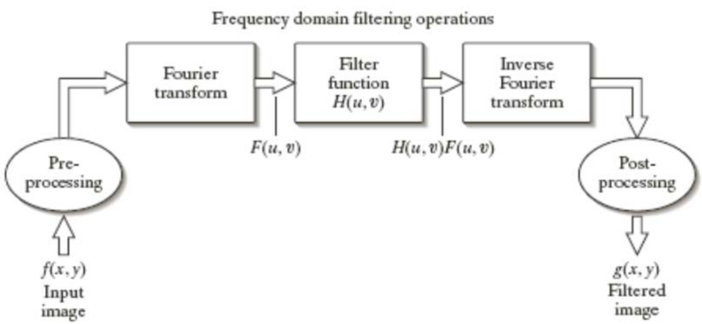
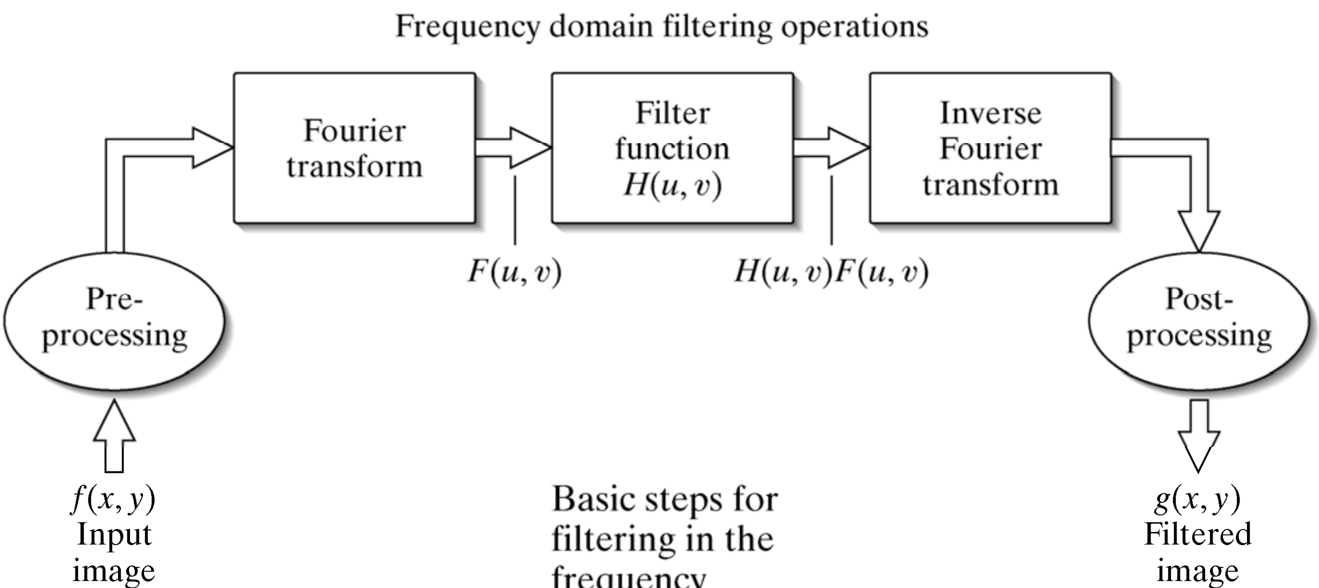


FIGURE 3.8 Basic steps for filtering in the frequency domain.

فیلتر کردن در حوزه‌ی فرکانس

گام‌های پایه



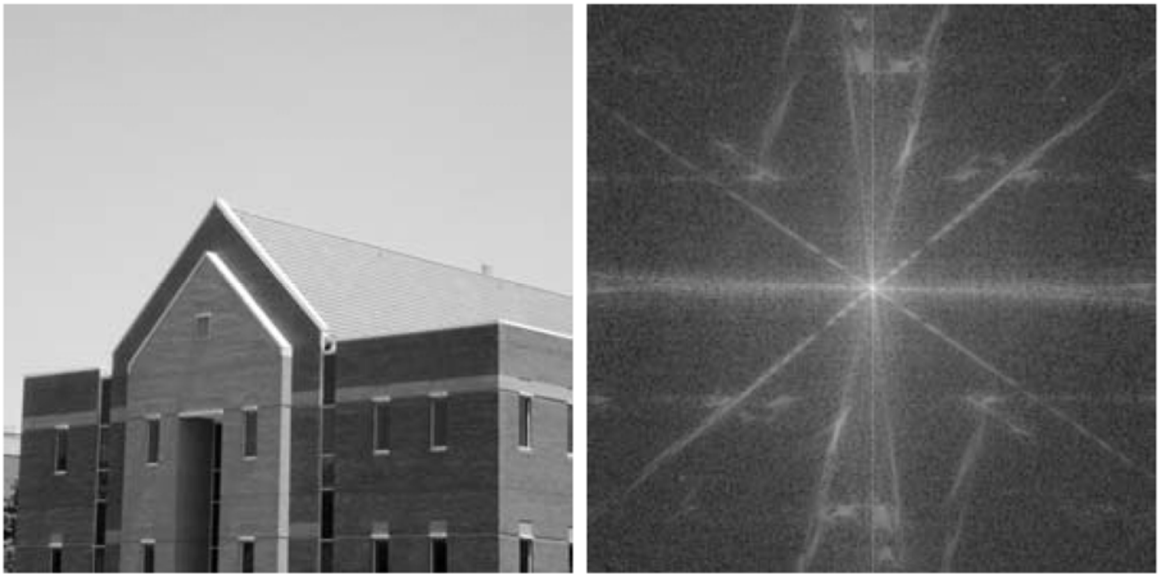
Basic steps for filtering in the frequency domain.

a b

FIGURE 3.9
(a) A gray-scale image. (b) Its Fourier spectrum.

تبدیل فوریه‌ی گسسته‌ی دوبعدی

مثال



a b

(a) A gray-scale image. (b) Its Fourier spectrum.

```
f = tofloat(f);
F = fft2(f);
S = fftshift(log(1 + abs(F)));
imshow(S, [])
```

MATLAB

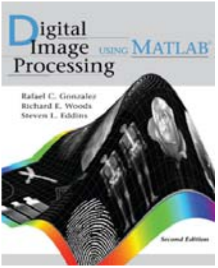
به‌دست آوردن فیلترهای حوزه‌ی فرکانس از روی فیلترهای مکانی

```
H = freqz2(h, R, C)
```

محاسبه‌ی پاسخ فرکانسی فیلترهای FIR:

h فیلتر مکانی دوبعدی؛ H فیلتر دوبعدی حوزه‌ی فرکانس متناظر؛ R تعداد سطرها؛ C تعداد ستون‌ها

MATLAB



Chapter 4
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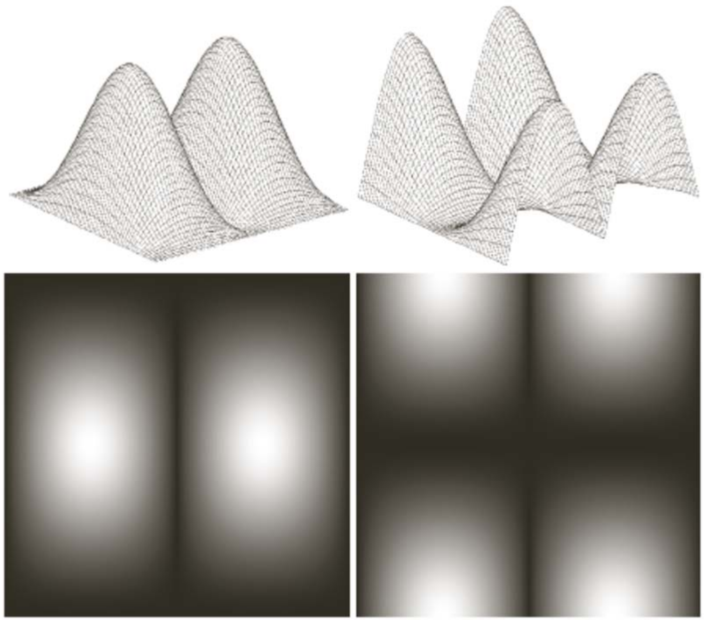


a b
c d

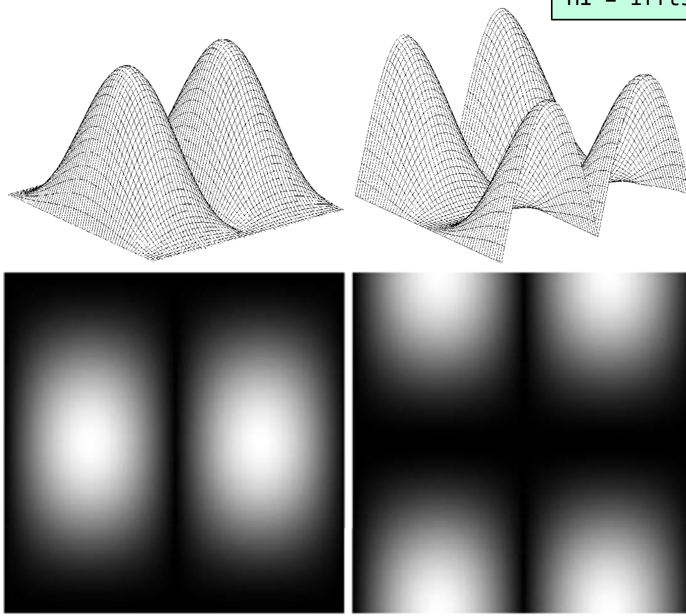
FIGURE 3.10 (a) Absolute value of the frequency domain filter corresponding to a vertical Sobel spatial filter. (b) The same filter after processing with function `ifftshift`. Figures (c) and (d) show the filters as images.

به‌دست آوردن فیلترهای حوزه‌ی فرکانس از روی فیلترهای مکانی

مثال

```
h = fspecial('sobel')'
h =
    1   0  -1
    2   0  -2
    1   0  -1
freqz2(h)
```

```
PQ = paddedsize(size(f));
H = freqz2(h, PQ(1), PQ(2));
H1 = ifftshift(H);
```



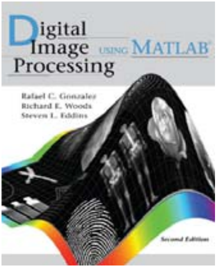
```
imshow(abs(H), [])
```

```
imshow(abs(H1), [])
```

a	b
c	d

(a) Absolute value of the frequency domain filter corresponding to a vertical Sobel mask. (b) The same filter after processing with function `fftshift`. Figures (c) and (d) are the filters in (a) and (b) shown as images.

MATLAB



Chapter 4
Filtering in the Frequency Domain

a b
c d

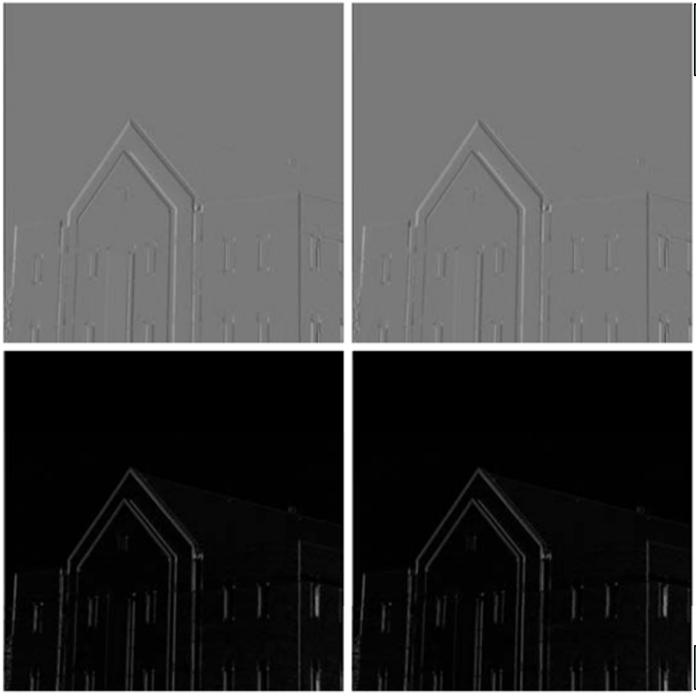
FIGURE 3.11 (a) Result of filtering Fig. 3.9(a) in the spatial domain with a vertical Sobel mask. (b) Result obtained in the frequency domain using the filter shown in Fig. 3.10(b). Figures (c) and (d) are the absolute values of (a) and (b), respectively.

به‌دست آوردن فیلترهای حوزه‌ی فرکانس از روی فیلترهای مکانی

مثال

```
gs = imfilter(f, h);
imshow(gs, [])
```

```
gf = dftfilt(f, H1);
imshow(gf, [])
```



```
imshow(abs(gs), [])
```

```
imshow(abs(gf), [])
```

a b
c d

(a) Result of filtering Fig. 4.9(a) in the spatial domain with a vertical Sobel mask. (b) Result obtained in the frequency domain using the filter shown in Fig. 4.10(b). Figures (c) and (d) are the absolute values of (a) and (b), respectively.

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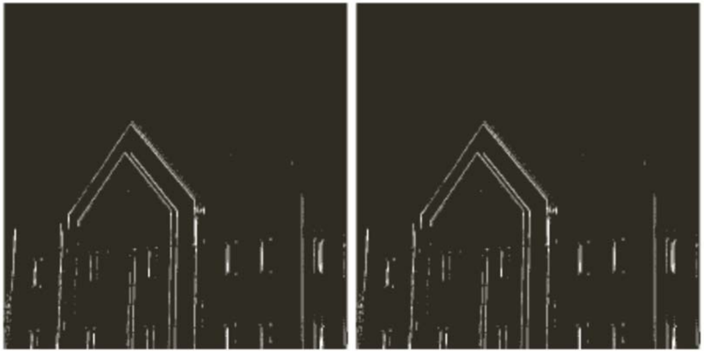
Filtering in the Frequency Domain

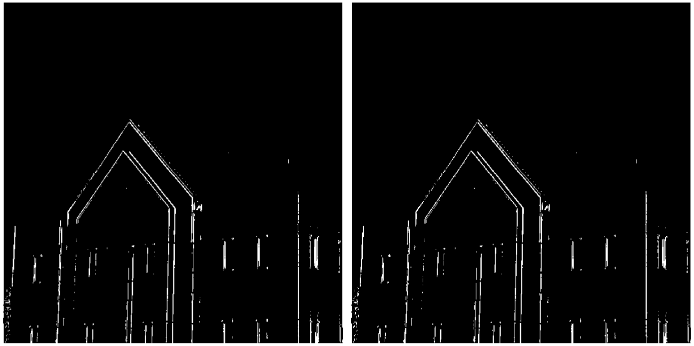


a b

FIGURE 3.12 Thresholded versions of Figs. 3.11(c) and (d), respectively, to show the principal edges more clearly.

به‌دست آوردن فیلترهای حوزه‌ی فرکانس از روی فیلترهای مکانی

مثال



a b

Thresholded versions of Figs. 4.11(c) and (d), respectively, to show the principal edges more clearly.

```
imshow(abs(gs) > 0.2*abs(max(gs(:))))
```

```
imshow(abs(gf) > 0.2*abs(max(gf(:))))
```

Chapter 4
Filtering in the Frequency Domain

TABLE 3.1 Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[D(u,v)/D_0\right]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای پایین‌گذر

Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)>D_0\end{cases}$	$H(u,v)=\dfrac{1}{1+[D(u,v)/D_0]^{2n}}$	$H(u,v)=e^{-D^2(u,v)/2D_0^2}$

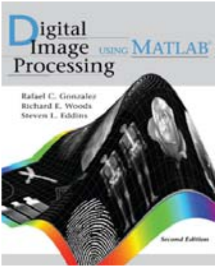
تولید مستقیم فیلترها در حوزه‌ی فرکانس

مثال: فیلتر پایین‌گذر گاوسی

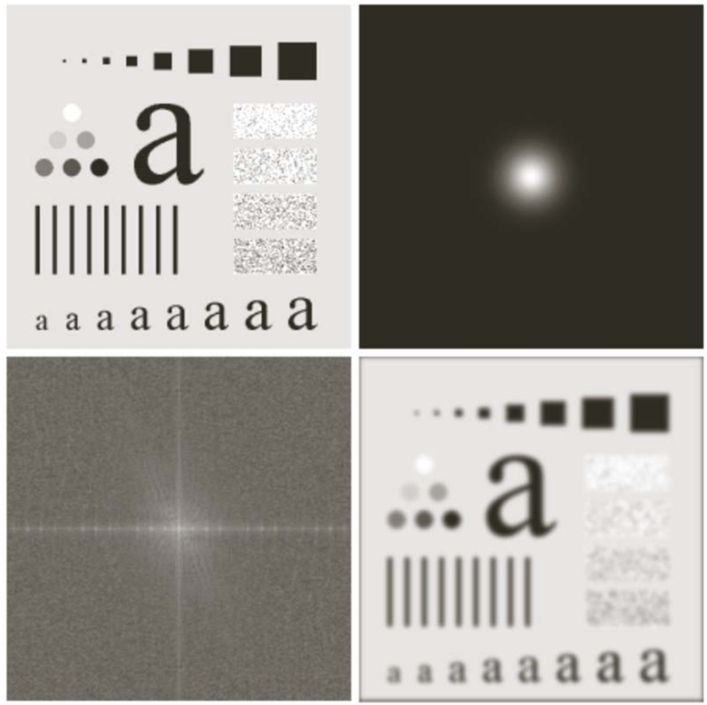
Gaussian
$H(u, v) = e^{-D^2(u,v)/2D_0^2}$

```
[f, revertclass] = tofolat(f);
PQ = paddedsize(size(f));
[U, V] = dftuv(PQ(1), PQ(2));
D = hypot(U, V); % compute D = sqrt(U.^2 + V.^2)
D0 = 0.05 * PQ(2);
F = fft2(f, PQ(1), PQ(2)); % needed for the spectrum.
H = exp(-(D.^2)/(2*(D0^2)));
g = dftfilt(f, H);
g = revertclass(g);
```


MATLAB



Chapter 4
Filtering in the Frequency Domain



a b
c d

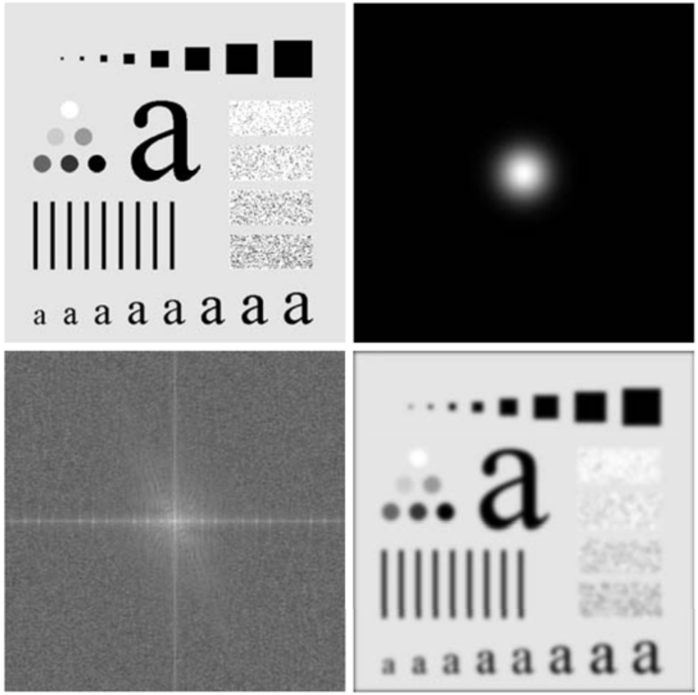
FIGURE 3.13 Lowpass filtering. (a) Original image. (b) Gaussian lowpass filter shown as an image. (c) Spectrum of (a). (d) Filtered image.

تولید مستقیم فیلترها در حوزه‌ی فرکانس

مثال: فیلتر پایین‌گذر گاوسی

`f`

`imshow(fftshift(H))`



`imshow(log(1 + abs(fftshift(F))), [])`

`imshow(dftfilt(f, H))`

a b
c d

Lowpass filtering. (a) Original image. (b) Gaussian lowpass filter shown as an image. (c) Spectrum of (a). (d) Processed image.

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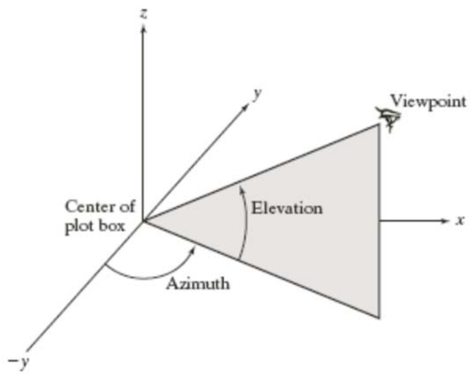
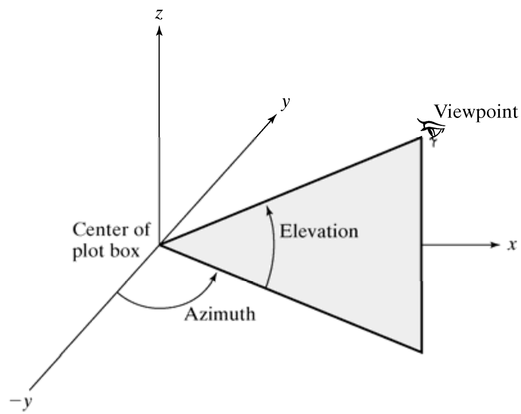


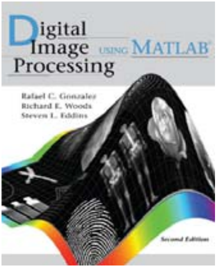
FIGURE 3.14 Viewing geometry for function `view`.

هندسه‌ی نما برای تابع view در متلب



Geometry for function view.

```
view(az, el)
```



Chapter 4
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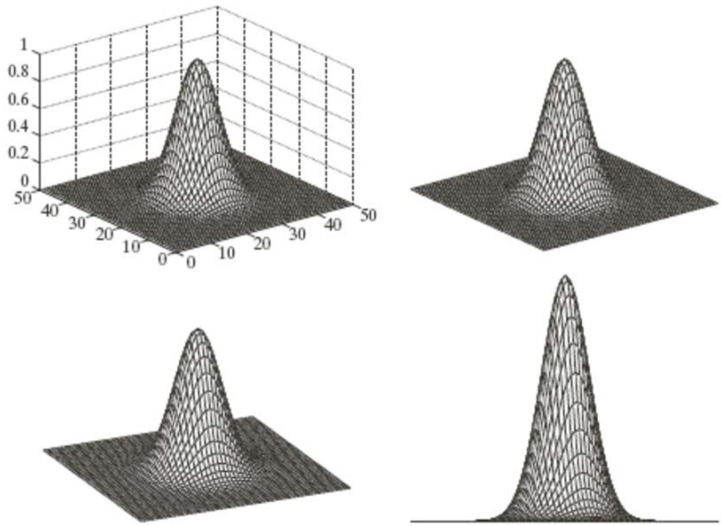


a b
c d

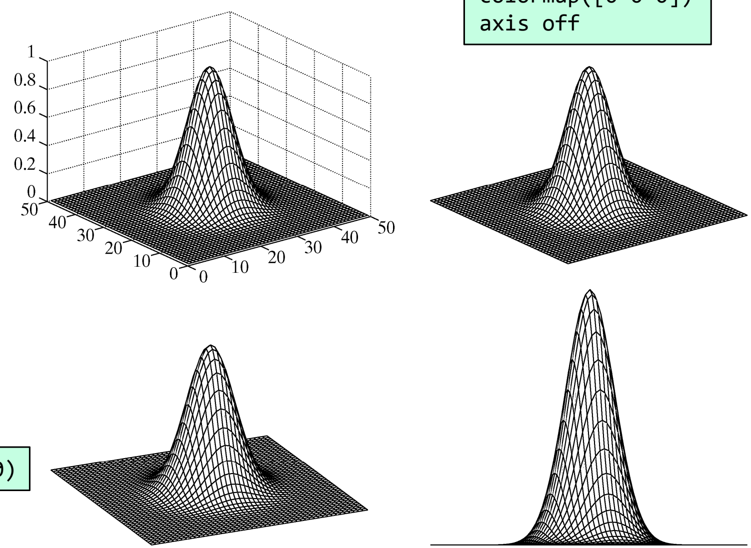
FIGURE 3.15 (a) A plot obtained using function `mesh`. (b) Axes and grid removed. (c) A different perspective view obtained using function `view`. (d) Another view obtained using the same function.

هندسه‌ی نما برای تابع view در متلب

مثال

```
H = fftshift(lpfilter('gaussian', 500, 500, 50));
mesh(double(H(1:10:500, 1:10:500)))
axis tight
```

```
colormap([0 0 0])
axis off
```




```
view(-25,30)
```

```
view(-25,0)
```

a b
c d

(a) A plot obtained using function mesh. (b) Axes and grid removed. (c) A different perspective view obtained using function view. (d) Another view obtained using the same function.

Chapter 4
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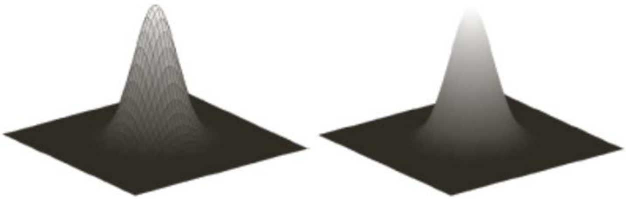
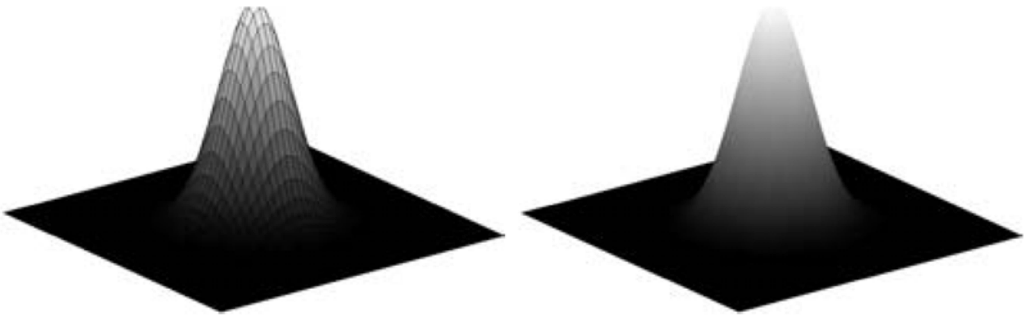


a b

FIGURE 3.16 (a) Plot obtained using function `surf`. (b) Result of using the command `shading interp`.

هندسه‌ی نما برای تابع view در متلب

مثال



a b

(a) Plot obtained using function `surf`. (b) Result of using the command `shading interp`.

```
H = fftshift(lpfilter('gaussian', 500, 500, 50));
surf(double(H(1:10:500, 1:10:500)))
axis tight
colormap(gray)
axis off
```

```
shading interp
```

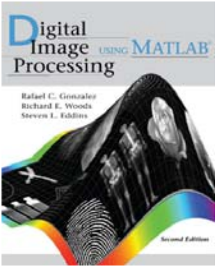


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TABLE 3.2 Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[D_0/D(u,v)\right]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

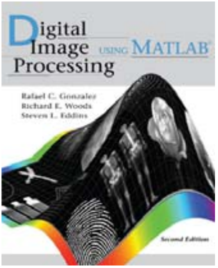
تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای بالاگذر

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

MATLAB



Chapter 4
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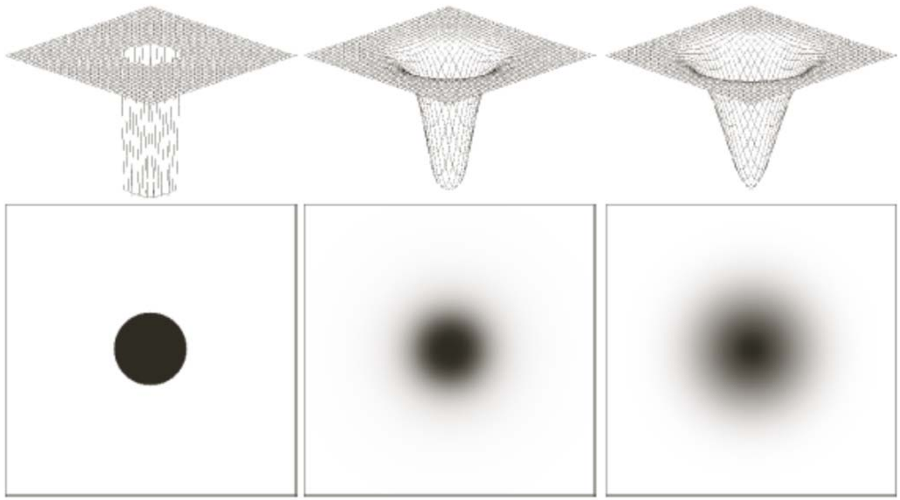
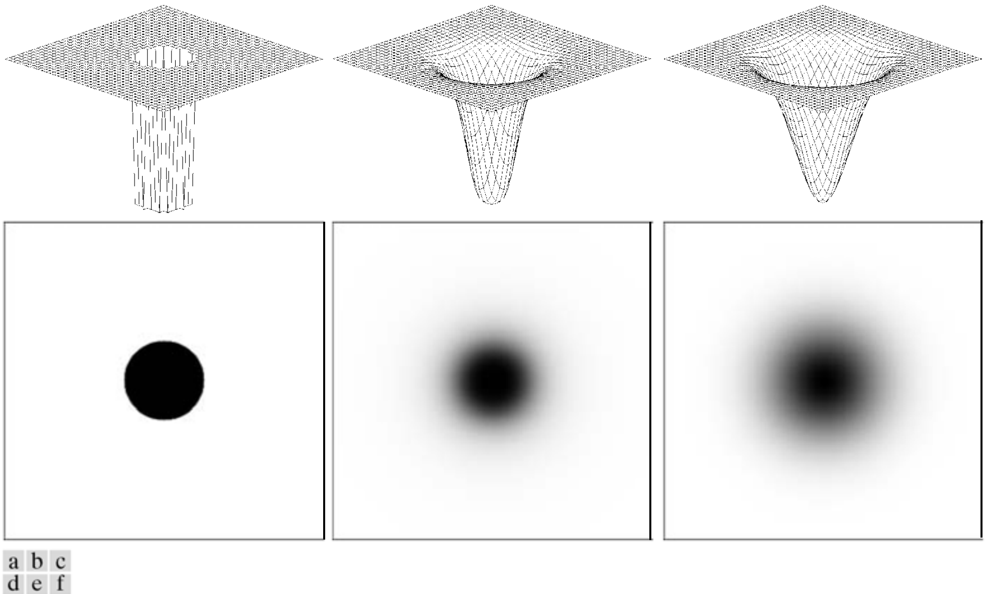


a b c
d e f

FIGURE 3.17 Top row: Perspective plots of ideal, Butterworth, and Gaussian highpass filters. Bottom row: Corresponding images. White represents 1 and black is 0.

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای بالاگذر



Top row: Perspective plots of ideal, Butterworth, and Gaussian highpass filters. Bottom row: Corresponding images.

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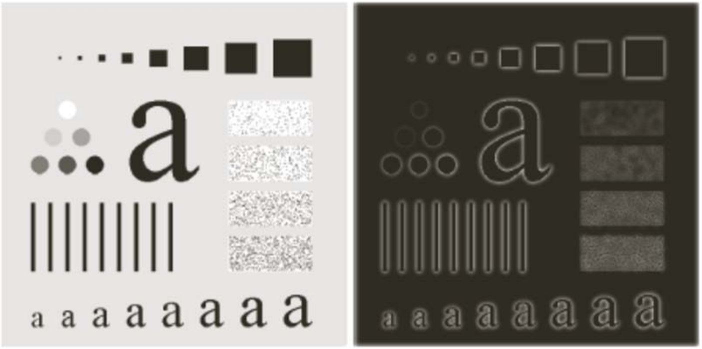


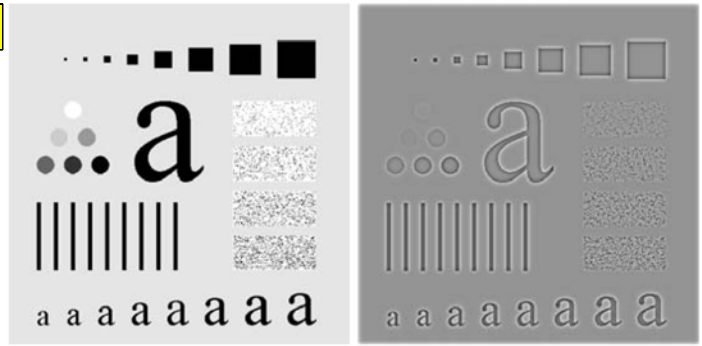
a b

FIGURE 3.18 (a) Original image. (b) Result of Gaussian high-pass filtering.

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلتر بالاگذر: مثال

f



a b

(a) Original image.
(b) Result of Gaussian highpass filtering.

```
PQ = paddedsize(size(f));
D0 = 0.05 * PQ(1);
H = hpfilter('gaussian', PQ(1), PQ(2), D0);
g = dftfilt(f, H);
Figure, imshow(g)
```

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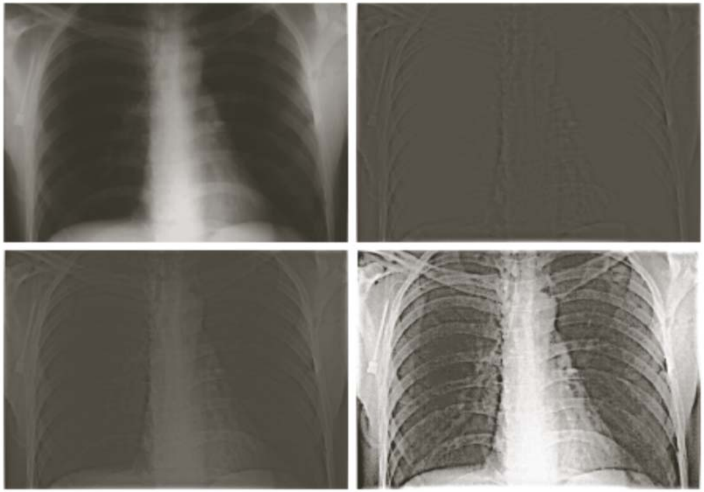
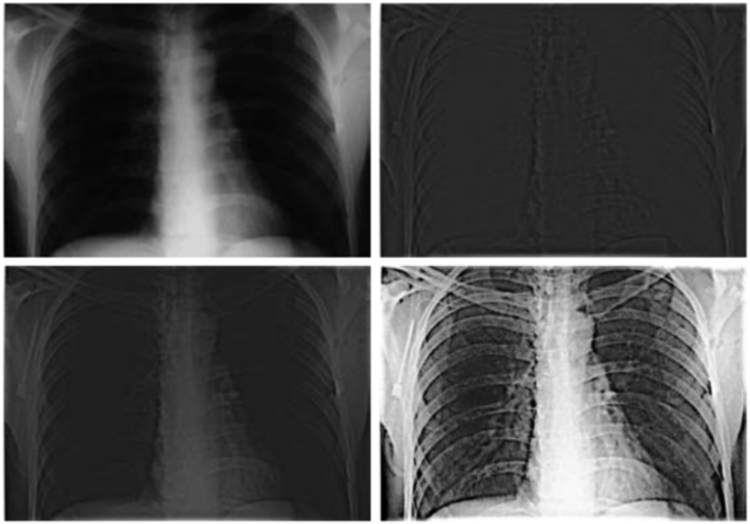


a b
c d

FIGURE 3.19 High-frequency emphasis filtering. (a) Original image. (b) Highpass filtering result. (c) High-frequency emphasis result. (d) Image (c) after histogram equalization. (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلتر تأکید بر فرکانس بالا



a b
c d

High-frequency emphasis filtering. (a) Original image. (b) Highpass filtering result. (c) High-frequency emphasis result. (d) Image (c) after histogram equalization.

```
PQ = paddedsize(size(f));
D0 = 0.05 * PQ(1);
HBW = hpfilter('btw', PQ(1), PQ(2), D0, 2);
H = 0.5 + 2 * HBW;
gbw = dftfilt(f, HBW, 'fltpoint');
gbw = gscale(gbw);
ghf = dftfilt(f, H, 'fltpoint');
ghf = gscale(ghf);
ghe = histeq(ghf, 256)
```


MATLAB

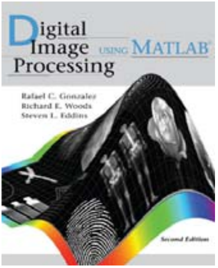


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TABLE 3.3 Bandreject filters. W is the "width" of the band, $D(u,v)$ is the distance from the center of the filter, D_0 is the radius of the center of the band, and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v)=\begin{cases}0 & \text{for } D_0-\frac{W}{2}\le D(u,v)\le D_0+\frac{W}{2}\\ 1 & \text{otherwise}\end{cases}$	$H(u,v)=\dfrac{1}{1+\left[\dfrac{WD(u,v)}{D^2(u,v)-D_0^2}\right]^{2n}}$	$H(u,v)=1-e^{-\left[\frac{D^2(u,v)-D_0^2}{WD(u,v)}\right]^2}$

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای رد باند

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

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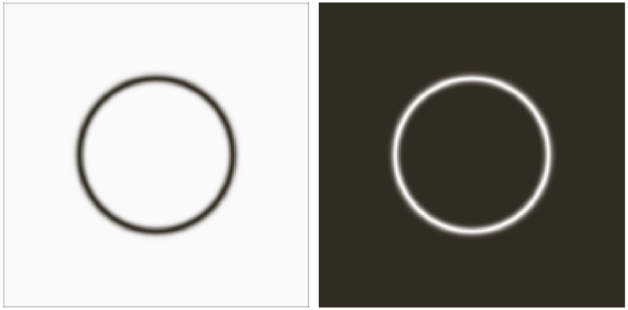
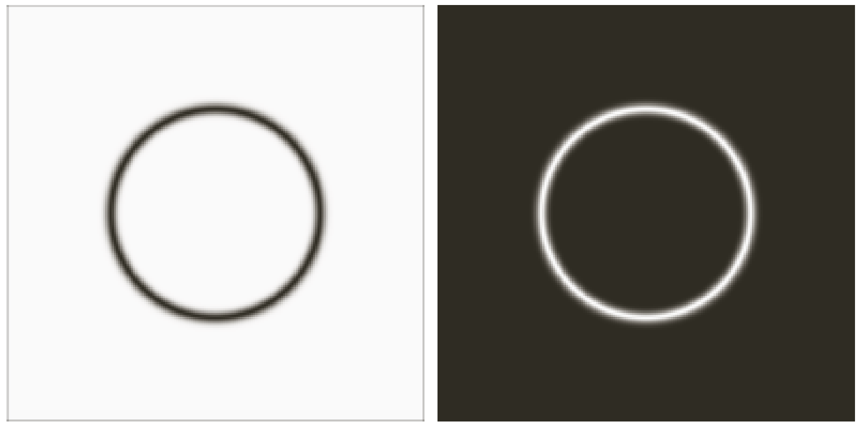
a b

FIGURE 3.20 (a) A Gaussian bandreject filter. (b) Corresponding bandpass filter. The filters were generated using $M = N = 800$, $D_0 = 200$, and $W = 20$ in function `bandfilter`.



تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای رد باند: مثال (فیلتر رد باند گاوسی)



a b

(a) A Gaussian bandreject filter. (b) Corresponding bandpass filter. The filters were generated using $M = N = 800$, $D_0 = 200$, and $W = 20$ in function `bandfilter`.

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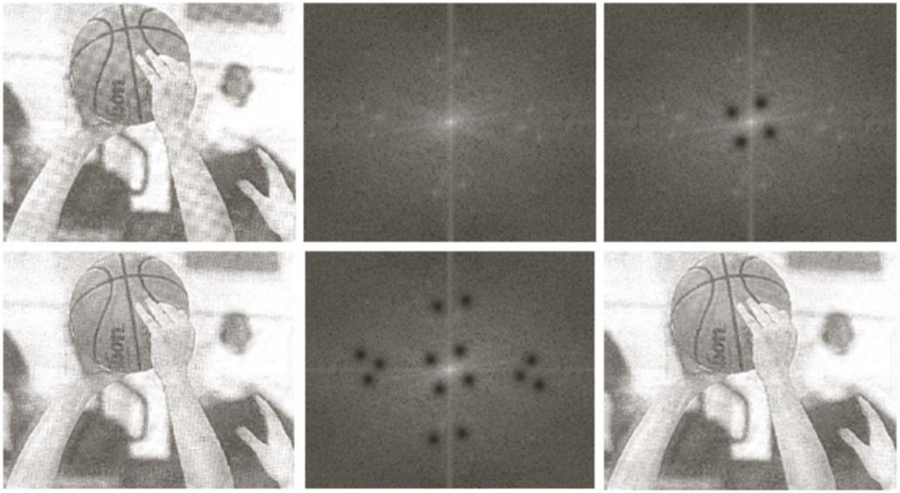
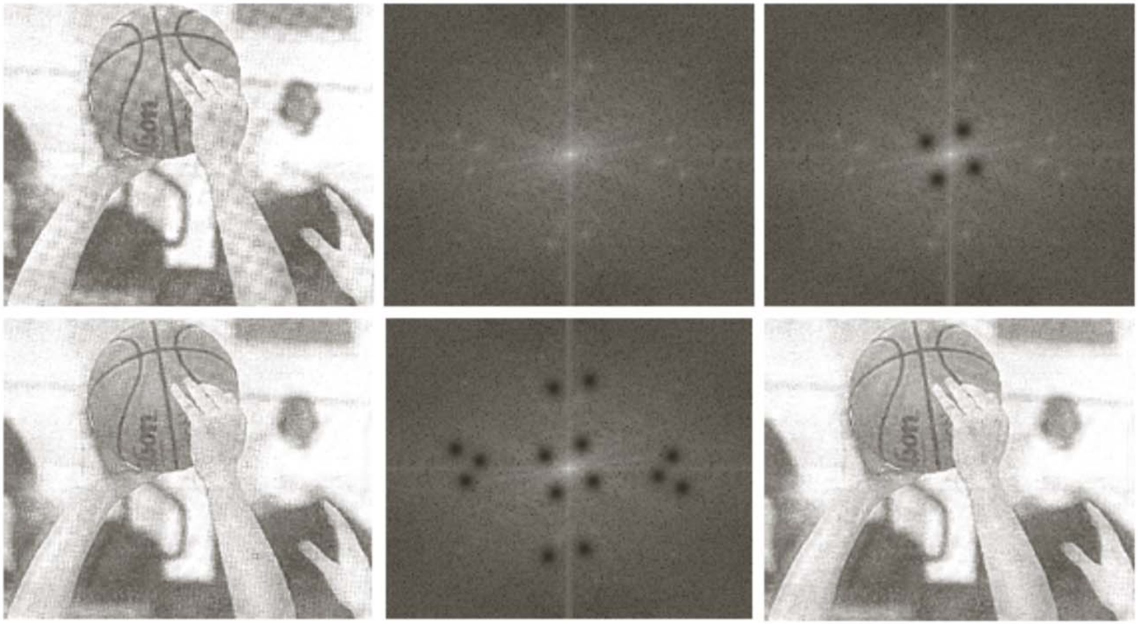


a b c
d e f

FIGURE 3.21 (a) Scanned, 72 dpi newspaper image of size 232 × 288 pixels corrupted by a moiré pattern. (b) Spectrum. (c) Gaussian notch filters applied to the low-frequency bursts caused by the moiré pattern. (d) Filtered result. (e) Using more filters to eliminate higher frequency "structured" noise. (f) Filtered result.

تولید مستقیم فیلترها در حوزه‌ی فرکانس

فیلترهای رد باند: مثال (کاهش الگوهای مویره)



a b c
d e f

(a) Scanned, 72 dpi newspaper image of size 232 × 288 pixels corrupted by a moiré pattern. (b) Spectrum. (c) Gaussian notch filters applied to the low-frequency bursts caused by the moiré pattern. (d) Filtered result. (e) Using more filters to eliminate higher frequency "structured" noise. (f) Filtered result.

MATLAB

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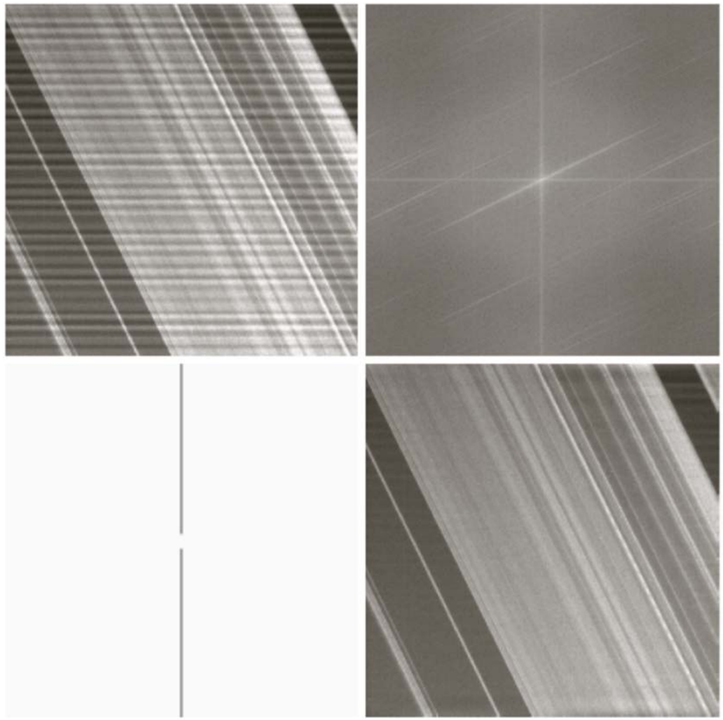
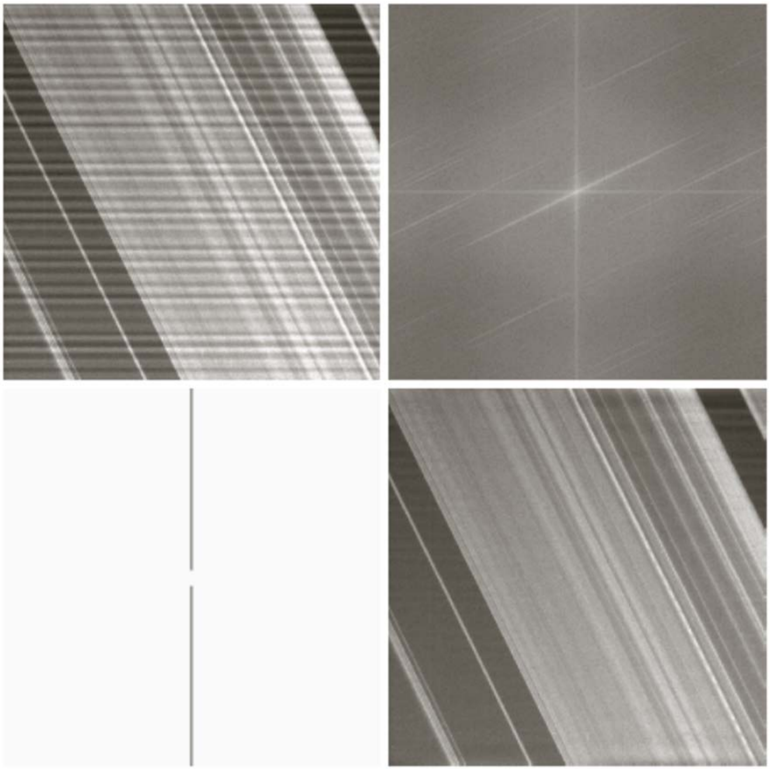


a b
c d

FIGURE 3.22
(a) 674×674 image of the Saturn rings, corrupted by periodic interference. (b) Spectrum: The bursts of energy on the vertical axis are caused by the interference. (c) Result of multiplying the DFT by a notch reject filter. (d) Result of computing the IDFT of (c). Note the improvement over (a). (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

تولید مستقیم فیلترها در حوزه‌ی فرکانس

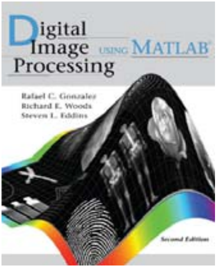
فیلترهای رد باند: مثال (کاهش الگوهای تداخل افقی متناوب)



a b
c d

(a) 674 × 674 image of the Saturn rings, corrupted by periodic interference. (b) Spectrum: The bursts of energy on the vertical axis are caused by the interference. (c) Result of multiplying the DFT by a notch reject filter. (d) Result of computing the IDFT of (c). Note the improvement over (a).

MATLAB



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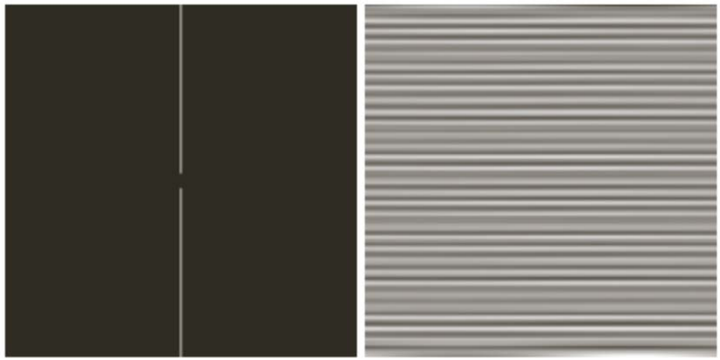


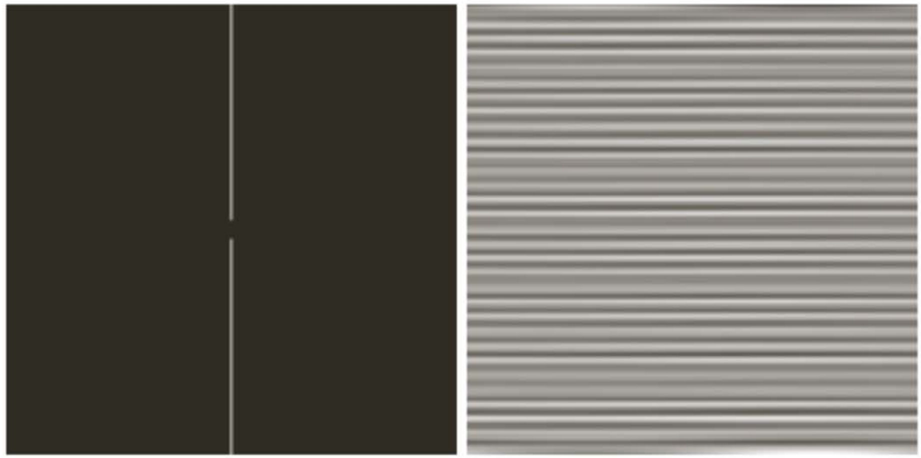
a b

FIGURE 3.23 (a) Notchpass filter. (b) Spatial interference pattern obtained by notchpass filtering.

تولید مستقیم فیلترها در حوزه‌ی فرکانس

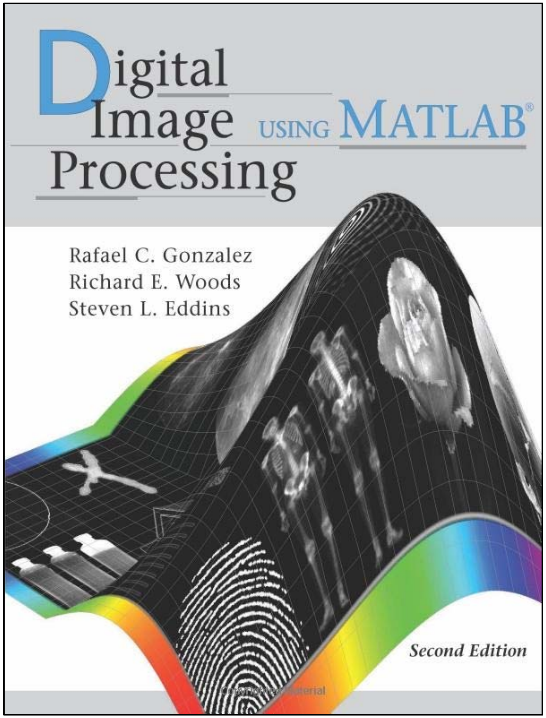
فیلترهای گذر باند: مثال (استخراج الگوهای تداخل افقی متناوب)

a b



(a) Notchpass filter. (b) Spatial interference pattern obtained by notchpass filtering.

منبع اصلی



Rafael C. Gonzalez, Richard E. Woods, Steven L. Eddins,
Digital Image Processing Using MATLAB®,
Second Edition, Pearson Prentice Hall, 2008.
Chapter 4