پردازش سیگنال دیجیتال

درس ۲۶

مروری بر فیلترهای وفقی

Overview of Adaptive Filters

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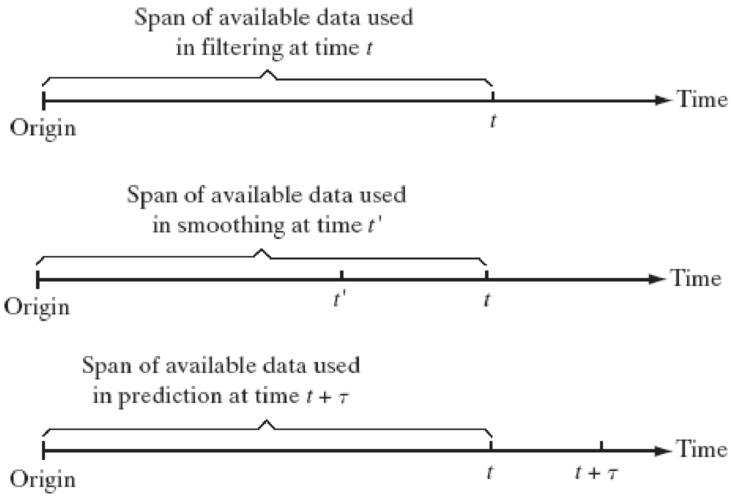
Overview of Adaptive Filters

The Filtering Problem

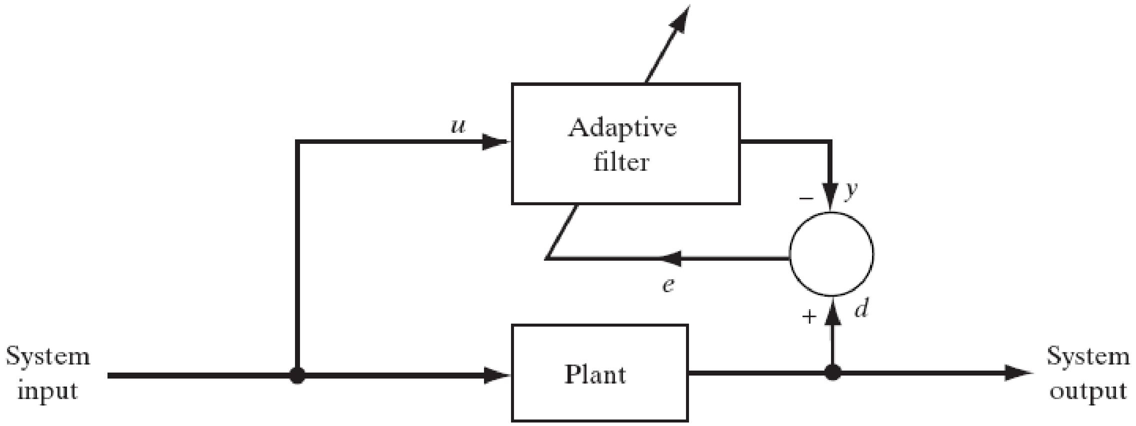
- Filters may be used for three information-processing tasks
  - Filtering
  - Smoothing
  - Prediction



- Given an optimality criteria we often can design **optimal filters**
  - Requires a priori information about the environment
  - Example:
    Under certain conditions the so called **Wiener filter** is optimal in the mean-squared sense
- Adaptive filters are self-designing using a recursive algorithm
  - Useful if complete knowledge of environment is not available a priori

Applications of Adaptive Filters: Identification

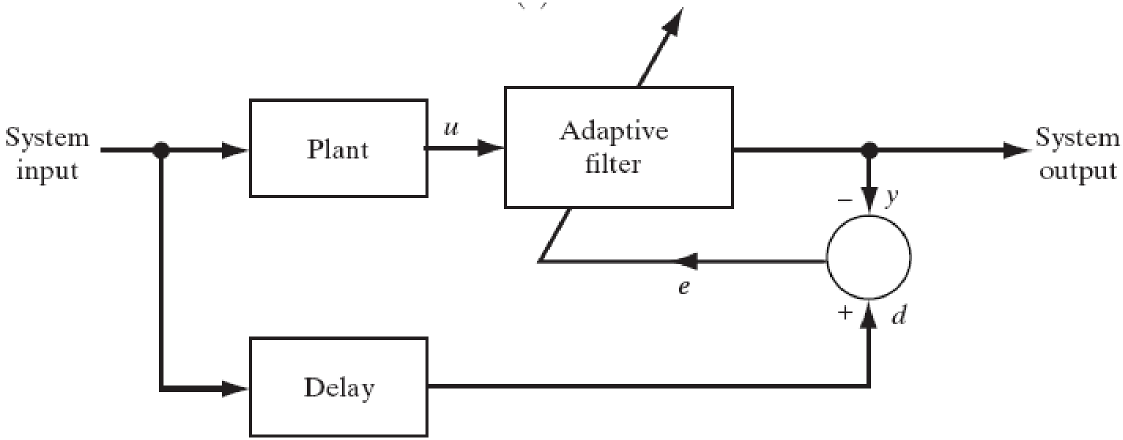
- Used to provide a linear model of an unknown plant



- Parameters
  - u = input of adaptive filter = input to plant
  - y = output of adaptive filter
  - d = desired response = output of plant
  - $e = d - y$ = estimation error
- Applications:
  - System identification

Applications of Adaptive Filters: Inverse Modeling

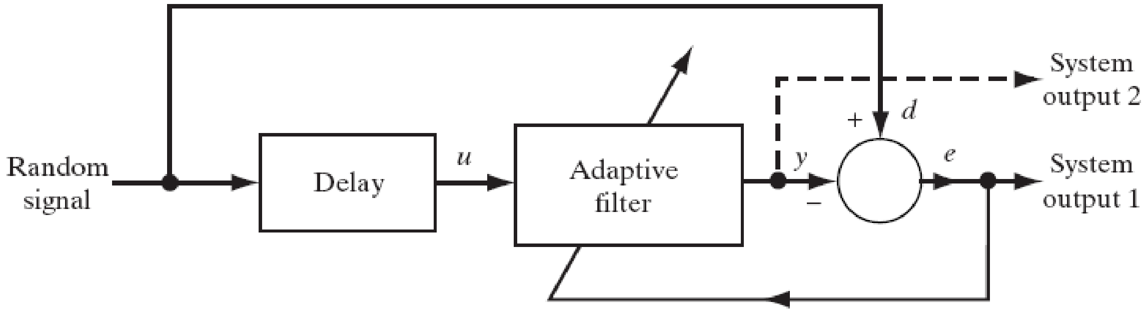
- Used to provide an inverse model of an unknown plant



- Parameters
  - u = input of adaptive filter = output to plant
  - y = output of adaptive filter
  - d = desired response = delayed system input
  - $e = d - y$ = estimation error
- Applications:
  - Equalization

Applications of Adaptive Filters: Prediction

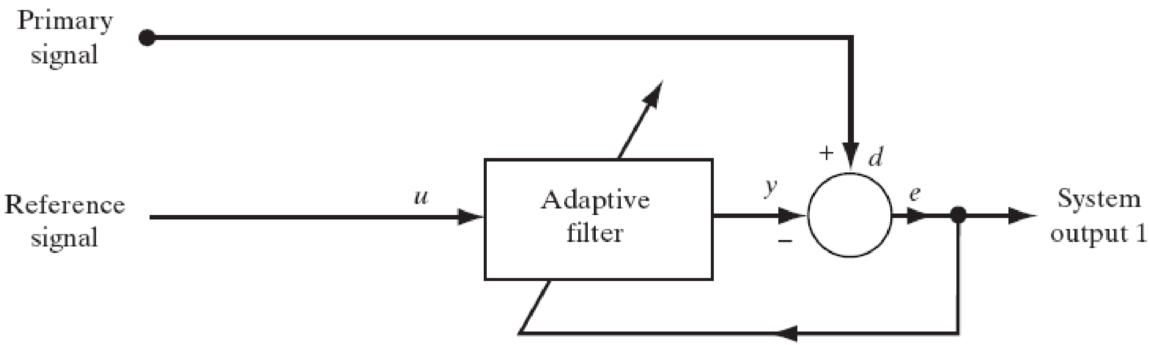
- Used to provide a prediction of the present value of a random signal



- Parameters
  - u = input of adaptive filter = delayed version of random signal
  - y = output of adaptive filter
  - d = desired response = random signal
  - $e = d - y$ = estimation error = system output
- Applications:
  - Linear predictive coding

Applications of Adaptive Filters: Interference Cancellation

- Used to cancel unknown interference from a primary signal



- Parameters
  - u = input of adaptive filter = reference signal
  - y = output of adaptive filter
  - d = desired response = primary signal
  - $e = d - y$ = estimation error = system output
- Applications:
  - Echo cancellation

Stochastic Gradient Approach

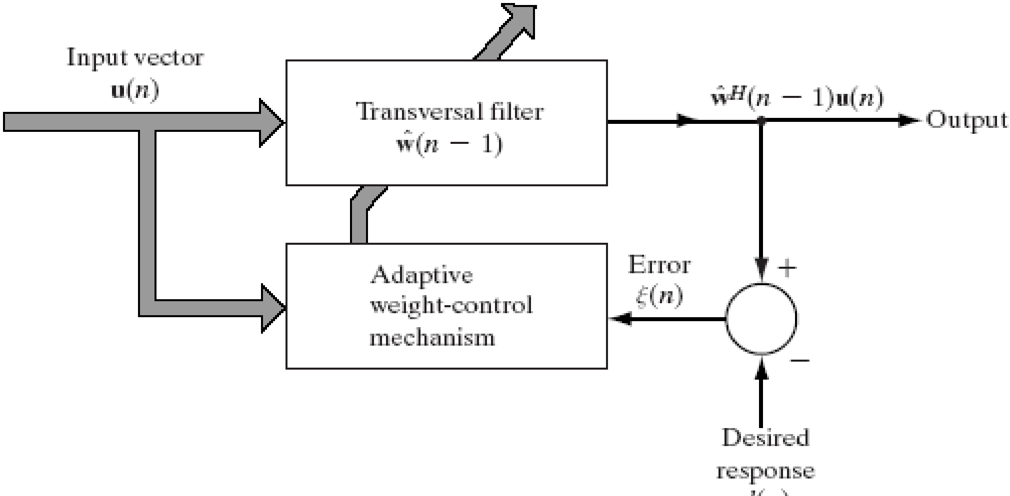
- Most commonly used type of Adaptive Filters
- Define cost function as mean-squared error
  - Difference between filter output and desired response
- Based on the method of **steepest descent**
  - Move towards the minimum on the error surface to get to minimum
  - Requires the gradient of the error surface to be known
- Most popular adaptation algorithm is LMS
  - Derived from steepest descent
  - Doesn’t require gradient to be know: it is estimated at every iteration

- Least-Mean-Square (LMS) Algorithm

$$\begin{pmatrix} \text{update value} \\ \text{of tap - weigth} \\ \text{vector} \end{pmatrix} = \begin{pmatrix} \text{old value} \\ \text{of tap - weight} \\ \text{vector} \end{pmatrix} + \begin{pmatrix} \text{learning -} \\ \text{rate} \\ \text{parameter} \end{pmatrix} \begin{pmatrix} \text{tap} - \\ \text{input} \\ \text{vector} \end{pmatrix} \begin{pmatrix} \text{error} \\ \text{signal} \end{pmatrix}$$

Least-Mean-Square (LMS) Algorithm

- The LMS Algorithm consists of two basic processes
  - Filtering process
    - Calculate the output of FIR filter by convolving input and taps
    - Calculate estimation error by comparing the output to desired signal
  - Adaptation process
    - Adjust tap weights based on the estimation error



(a)

LMS Algorithm Steps

- Filter output

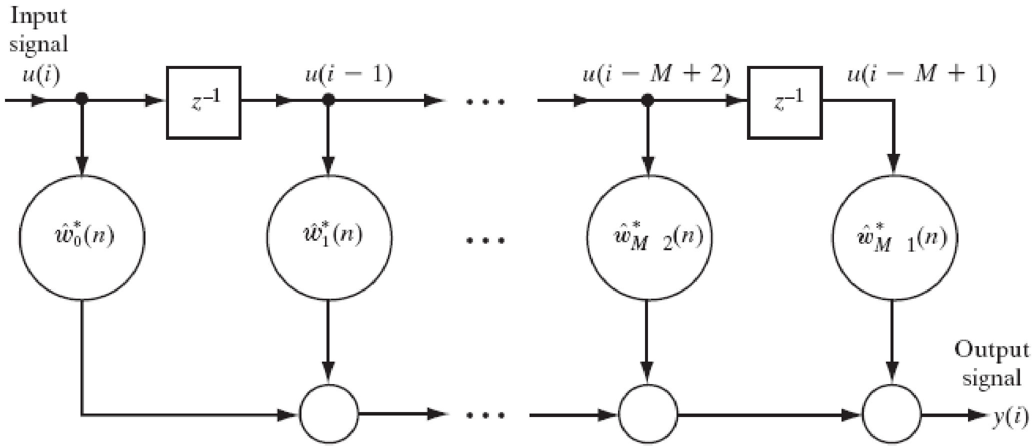
$$y[n]=\sum_{k=0}^{M-1}u[n-k]w_k^*[n]$$

- Estimation error

$$e[n]=d[n]-y[n]$$

- Tap-weight adaptation

$$w_k[n+1]=w_k[n]+\mu u[n-k]e^*[n]$$



Stability of LMS

- The LMS algorithm is convergent in the mean square if and only if the step-size parameter satisfy

$$0 < \mu < \frac{2}{\lambda_{max}}$$

- Here λ_{max} is the largest eigenvalue of the correlation matrix of the input data
- More practical test for stability is

$$0 < \mu < \frac{2}{\text{input signal power}}$$

- Larger values for step size
  - Increases adaptation rate (faster adaptation)
  - Increases residual mean-squared error