پردازش سیگنال دیجیتال

درس ۳

سیستم‌های خطی تغییرناپذیر با زمان

Linear Time-Invariant Systems

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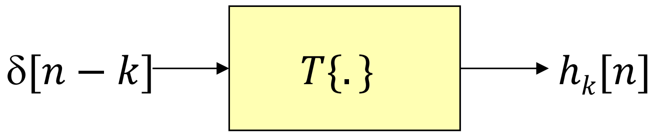
http://courses.fouladi.ir/dsp

Linear-Time Invariant System

- Special importance for their mathematical tractability
- Most signal processing applications involve LTI systems
- LTI system can be completely characterized by their impulse response



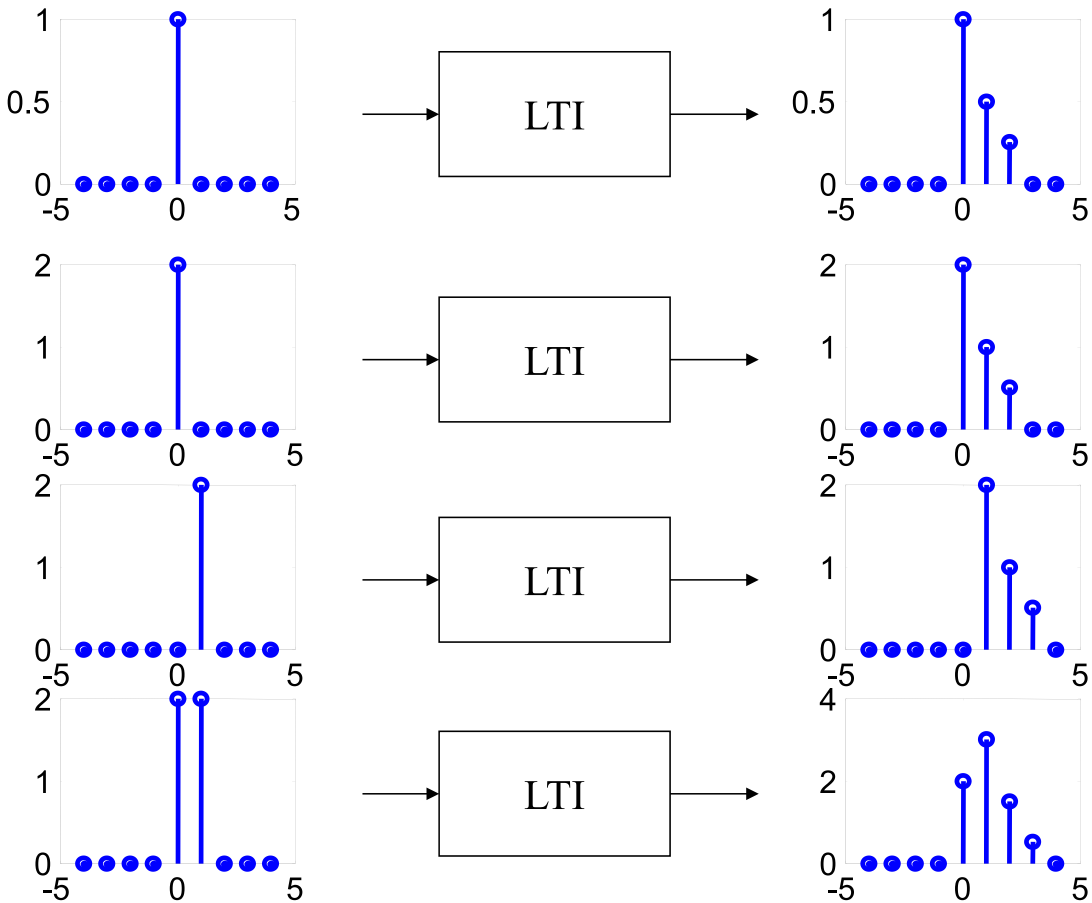
- Represent any input $$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$y[n] = T\left\{ \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \right\} = \sum_{k=-\infty}^{\infty} x[k] T\{\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k] h_k[n]$$

- From time invariance we arrive at convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = x[k] * h[k]$$

LTI System Example



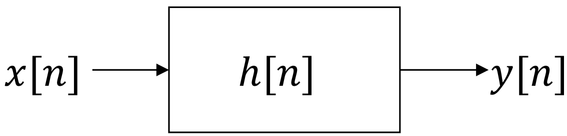
Convolution Demo

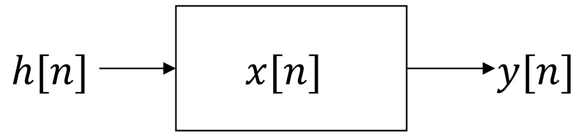
Joy of Convolution Demo from John Hopkins University

Properties of LTI Systems

- Convolution is **commutative**

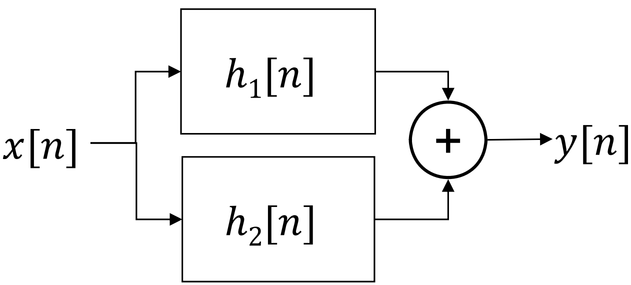
$$x[k]*h[k]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=-\infty}^{\infty}h[k]x[n-k]=h[k]*x[k]$$

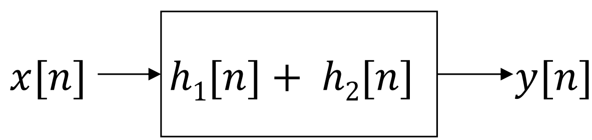




- Convolution is **distributive**

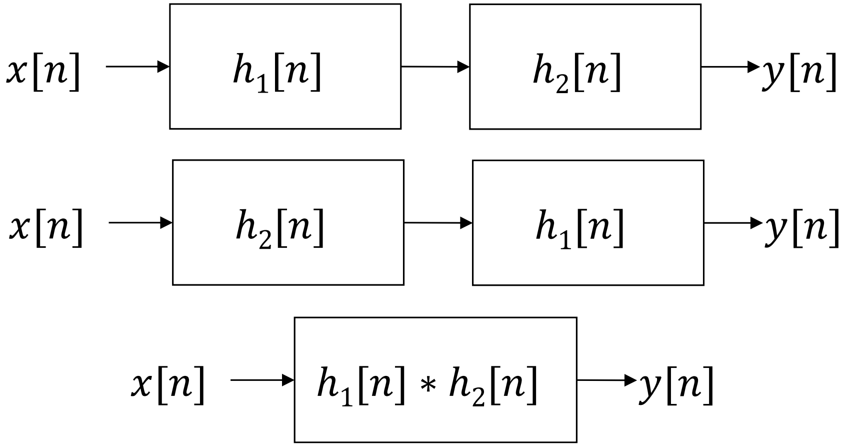
$$x[k]*\left(h_1[k]+h_2[k]\right)=x[k]*h_1[k]+x[k]*h_2[k]$$





Properties of LTI Systems

- **Cascade** connection of LTI systems



Stable and Causal LTI Systems

- An LTI system is (BIBO) **stable** if and only if
  - Impulse response is absolute summable

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

  - Let's write the output of the system as

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

  - If the input is bounded

$$|x[n]| \leq B_x$$

  - Then the output is bounded by

$$|y[n]| \leq B_x \sum_{k=-\infty}^{\infty} |h[k]|$$

  - The output is bounded if the absolute sum is finite
- An LTI system is **causal** if and only if

$$h[k] = 0 \quad \text{for} \quad k < 0$$

Linear Constant-Coefficient Difference Equations

- An important class of LTI systems of the form

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- The output is not uniquely specified for a given input
  - The initial conditions are required
  - Linearity, time invariance, and causality depend on the initial conditions
  - If initial conditions are assumed to be zero
    system is linear, time invariant, and causal
- Example
  - Moving Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

  - Difference Equation Representation

$$\sum_{k=0}^{0} a_k y[n-k] = \sum_{k=0}^{3} b_k x[n-k] \quad \text{where } a_k = b_k = 1$$

Eigenfunctions of LTI Systems

- Complex exponentials are **eigenfunctions** of LTI systems:

$$x[n] = e^{j\omega n}$$

- Let's see what happens if we feed $x[n]$ into an LTI system:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)}$$

$$y[n] = \left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}\right)e^{j\omega n} = H\left(e^{j\omega}\right)e^{j\omega n}$$

eigenfunction ($e^{j\omega n}$)

eigenvalue ($H\left(e^{j\omega}\right)$)

- The eigenvalue is called the frequency response of the system

$$H\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

- $H(e^{j\omega})$ is a complex function of frequency
  – Specifies amplitude and phase change of the input

Eigenfunction Demo

LTI System Demo