پردازش سیگنال دیجیتال

درس ۲

سیگنال‌ها و سیستم‌های گسسته–زمان

Discrete-Time Signals and Systems

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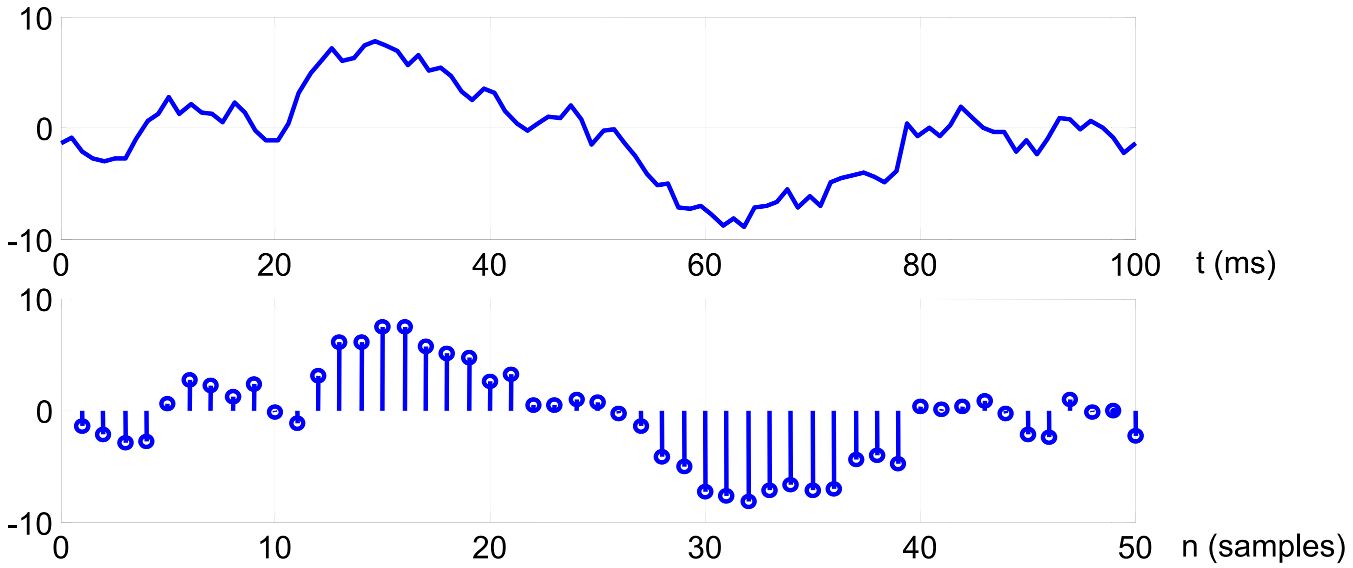
Discrete-Time Signals: Sequences

- Discrete-time signals are represented by sequence of numbers
  - The n^{th} number in the sequence is represented with $x[n]$
- Often times sequences are obtained by sampling of continuous-time signals
  - In this case $x[n]$ is value of the analog signal at $x_c(nT)$
  - Where T is the sampling period



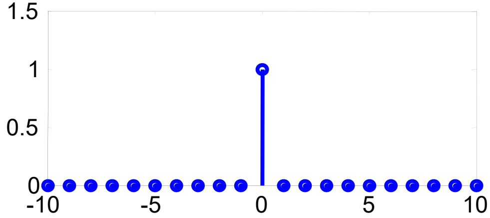
Basic Sequences and Operations

- Delaying (Shifting) a sequence

$$y[n] = x[n - n_o]$$

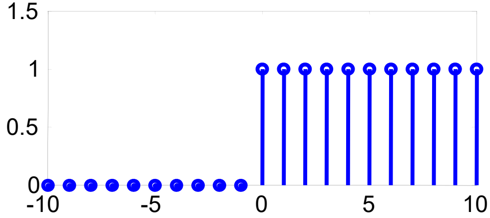
- Unit sample (impulse) sequence

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$



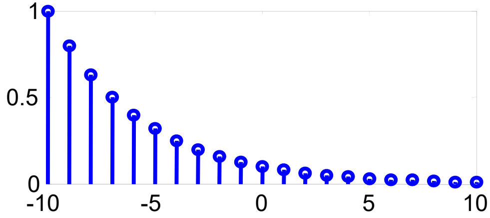
- Unit step sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



- Exponential sequences

$$x[n] = A\alpha^n$$



Sinusoidal Sequences

- Important class of sequences

$$x[n] = \cos(\omega_o n + \phi)$$

- An exponential sequence with complex $\alpha = |\alpha| e^{j\omega_o}$ and $A = |A| e^{j\phi}$

$$x[n] = A\alpha^n = |A| e^{j\phi} |\alpha|^n e^{j\omega_o n} = |A||\alpha|^n e^{j(\omega_o n + \phi)}$$

$$x[n] = |A||\alpha|^n \cos(\omega_o n + \phi) + j|A||\alpha|^n \sin(\omega_o n + \phi)$$

- $x[n]$ is a sum of weighted sinusoids
- Different from continuous-time, discrete-time sinusoids
  - Have ambiguity of $2\pi k$ in frequency

$$\cos((\omega_o + 2\pi k)n + \phi) = \cos(\omega_o n + \phi)$$

  - Are not necessary periodic with $2\pi/\omega_o$

$$\cos(\omega_o n + \phi) = \cos(\omega_o n + \omega_o N + \phi) \quad \text{only if} \quad N = \frac{2\pi k}{\omega_o} \text{ is an integer}$$

Rotating Phasors Demo

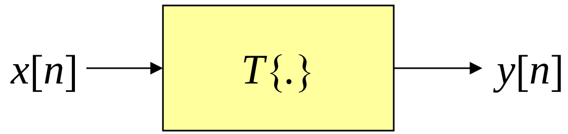
http://www.ewh.ieee.org/soc/es/Aug1996/011/cd/Demos/Phasors/index.htm

http://www.gpds.ene.unb.br/mylene/PSMM/DSPFIRST/chapters/2sines/demos/phasors/index.htm

Discrete-Time Systems

- **Discrete-Time Sequence** is a mathematical operation that maps a given input sequence $x[n]$ into an output sequence $y[n]$

$$y[n] = T\{x[n]\}$$



- Example Discrete-Time Systems
  - **Moving (Running) Average**

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

  - **Maximum**

$$y[n] = \max\{x[n], x[n-1], x[n-2]\}$$

  - **Ideal Delay System**

$$y[n] = x[n - n_o]$$

Memoryless System

- **Memoryless System**
  - A system is memoryless if the output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n

- Example Memoryless Systems
  - Square

$$y[n] = (x[n])^2$$

  - Sign

$$y[n] = \text{sgn}\{x[n]\}$$

- Counter Example
  - Ideal Delay System

$$y[n] = x[n - n_o]$$

Linear Systems

- **Linear System:** A system is linear if and only if

$$T\{x_1[n]+x_2[n]\}=T\{x_1[n]\}+T\{x_2[n]\} \quad \text{(additivity)}$$

and

$$T\{ax[n]\}=aT\{x[n]\} \quad \text{(scaling)}$$

- Examples
  - Ideal Delay System

$$y[n]=x[n-n_o]$$

$$\begin{aligned}
T\{x_1[n]+x_2[n]\} &= x_1[n-n_o]+x_2[n-n_o] \\
T\{x_2[n]\}+T\{x_1[n]\} &= x_1[n-n_o]+x_2[n-n_o] \\
T\{ax[n]\} &= ax_1[n-n_o] \\
aT\{x[n]\} &= ax_1[n-n_o]
\end{aligned}$$

Time-Invariant Systems

- **Time-Invariant (shift-invariant) Systems**
  - A time shift at the input causes corresponding time-shift at output

$$y[n] = T\{x[n]\} \Rightarrow y[n-n_o] = T\{x[n-n_o]\}$$

- Example
  - Square

$$y[n] = (x[n])^2$$

Delay the input the output is $y_1[n] = (x[n-n_o])^2$

Delay the output gives $y[n-n_o] = (x[n-n_o])^2$

- Counter Example
  - Compressor System

$$y[n] = x[Mn]$$

Delay the input the output is $y_1[n] = x[Mn-n_o]$

Delay the output gives $y[n-n_o] = x[M(n-n_o)]$

Causal System

- **Causality**
  - A system is causal it's output is a function of only the current and previous samples

- Examples
  - Backward Difference

$$y[n] = x[n] - x[n-1] \qquad y[n] = \Delta x[n]$$

- Counter Example
  - Forward Difference

$$y[n] = x[n+1] - x[n] \qquad y[n] = \nabla x[n]$$

Stable System

- **Stability** (in the sense of bounded-input bounded-output **BIBO**)
  - A system is stable if and only if every bounded input produces a bounded output

$$|x[n]| \le B_x < \infty \Rightarrow |y[n]| \le B_y < \infty$$

- Example
  - Square

$$y[n] = (x[n])^2$$

if input is bounded by $|x[n]| \le B_x < \infty$

output is bounded by $|y[n]| \le B_x^2 < \infty$

- Counter Example
  - Logarithm

$$y[n] = \log_{10}(|x[n]|)$$

even if input is bounded by $|x[n]| \le B_x < \infty$

output not bounded for $\mathrm{x}[\mathrm{n}] = 0 \Rightarrow \mathrm{y}[0] = \log_{10}(|x[n]|) = -\infty$