هوالحکیم

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فصل نهم

محاسبه‌ی تبدیل فوریه‌ی گسسته

Computation of The Discrete Fourier Transform

◊ مسئله‌های تحلیلی ـ تشریحی

1. **[Oppenheim/Schafer/Buck Problem #9.2]** The gain along the path shown is $W_N^0 \cdot -1 \cdot W_N^2 = -W_N^2$. There is only one path between the each input and each output sample. Part (c) is just tedious computation.

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2. **[Oppenheim/Schafer/Buck Problem #9.3]** The input should be placed in $A[r]$ in bit reversed order $(A[0], A[1], \ldots, A[7]) = (x[0], x[4], x[2], x[6], x[1], x[5], x[3], x[7])$. The output from $D[r]$ is normal order. If $x[n] = (-W_N)^n$, then

$$\begin{aligned} D[k] &= \sum_{n=0}^{7} (-W_8)^n W_8^{nk} \\ &= \sum_{n=0}^{7} (-1)^n W_8^{n(k+1)} \\ &= \sum_{n=0}^{7} W_8^{-4n} W_8^{n(k+1)} \\ &= \sum_{n=0}^{7} W_8^{n(k-3)} \end{aligned}$$

Therefore, $D[k] = \delta[k-3]$. More tedious computation shows that

$$C[k] = \begin{cases} \frac{D[k]+D[k+4]}{2} & \text{if } 0 \le k < 4 \\ \frac{D[k-4]-D[k]}{2} \cdot W_8^{k-4} & \text{if } 4 \le k < 8 \end{cases}$$

Substitute for $D[k] = X[k]$.

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3. **[Oppenheim/Schafer/Buck Problem #9.4]** In any stage, $N/2$ butterflies must be computed. There are 2^{m-1} different coefficients in the m'th stage. The difference equation is given by $y[n] = W_{2^m} y[n-1] + x[n]$ has impulse response $h[n] = W_{2^m}^n u[n]$. Noting that $W_{2^m} = e^{-j2\pi/2^m}$, we see that $h[n]$ has period 2^m. Therefore, the frequency of the oscillator is $2\pi/2^m$.

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4. **[Oppenheim/Schafer/Buck Problem #9.6]** It is not possible to say. This could have come from either decimation in time or decimation in frequency.