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دانشگاه تهران
پردیس البرز
گروه فنی

راه‌حل تکلیف شماره‌ی ۹

فصل هشتم

تبدیل فوریه‌ی گسسته

The Discrete Fourier Transform

◊ مسئله‌های تحلیلی ـ تشریحی

1) (8.22) For a finite-length sequence $x[n]$, with length equal to N, the periodic repetition of $x[-n]$ is represented by

$$x[((-n))_N] = x[((-n + \ell N))_N], \quad \ell : integer$$

where the right side is justified since $x[n]$ (and $x[-n]$) is periodic with period N. The above statement holds true for any choice of ℓ. Therefore, for $\ell = 1$:

$$x[((-n))_n] = x[((-n + N))_N]$$

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2. **[Oppenheim/Schafer/Buck Problem #8.23]** If $N > P$, zero pad the sequence to get exactly N terms. If $x_1[n]$ is the sequence which is zero-padded, then $X_1(z) = X(z)$, and so sampling $X_1(z)$ on he unit circle is equivalent to sampling $X(z)$ on he unit circle.

   Suppose that $N < P$. We have $X(z) = \sum_{n=0}^{P-1} x[n]z^{-n}$. We need the values of $X(z)|_{z=e^{j2\pi k/N}}$. Supposing that this is obtained by taking the N-po n DFT of some sequence $x_1[n]$. Then we get This is

$$X(z)|_{z=e^{j2\pi k/N}} = \sum_{n=0}^{P-1} x[n]e^{-j2\pi nk/N} = \sum_{n=0}^{N-1} x_1[n]e^{-j2\pi nk/N}$$

   In particular, we need $x_1[n] = \sum_{k\in\mathbb{Z}} x[n+kN]$. While this is expressed as an nfinite sum, it is really a finite sum because only finitely many terms are non-zero.

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3. **[Oppenheim/Schafer/Buck Problem #8.31]** We need $X_1[k] = X(z)|_{z=\frac{1}{2}e^{j(2\pi k/10+\pi/10)}}$. Now, $X(z) = \sum_{n=0}^{N-1} x[n]z^{-n}$, and so,

$$\sum_{n=0}^{N-1} x_1[n]e^{-j2\pi kn/N} = X_1[k] = \sum_{n=0}^{N-1} x[n]\left(\frac{1}{2}e^{j\pi/10}\right)^{-n} e^{-j2\pi kn/10}$$

   Hence we need $x_1[n] = x[n]\left(\frac{1}{2}e^{j\pi/10}\right)^{-n}$.

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4. **[Oppenheim/Schafer/Buck Problem #8.34]** The statemen   s fa se. Note that $x_e[n]$ is a sequence of length $2N-1$, so the statemen does not even make sense (since the DFT of $x_e[n]$ will have $2N-1$ points, while the DFT of $x[n]$ wi   on y have N points. The correc condition s DFT $\left\{\frac{x[n]+x[N-n]}{2}\right\} = \Re\{X[k]\}$.

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5. **[Oppenheim/Schafer/Buck Problem #8.37]** If $g_1[n] = x[N-1-n]$, then

$$G_1[k] = \sum_{n=0}^{N-1} x[N-1-n]{W_N}^{kn} = {W_N}^{k(N-1)} \sum_{m=0}^{N-1} x[m]{W_N}^{-km} H_7[k]$$

since ${W_N}^{k(N-1)} = e^{j2\pi k(N-1)/N} = e^{-j2\pi k/N}$. Similarly, we have

$$G_2[k] = \sum_{m=0}^{N-1} x[m]{W_N}^{(k+N/2)m} = H_8[k]$$

$$G_3[k] = \sum_{n=0}^{N-1} x[n]({W_{2N}}^{kn} + {W_{2N}}^{k(n+N)}) = (1 + {W_{2N}}^{kN}) \sum_{n=0}^{N-1} x[n]{W_{2N}}^{kn}$$

Noting that ${W_{2N}}^{kn} = {W_N}^{kn/2}$, we get $G_3[k] = H_3[k]$. For $G_4[k]$, note that it has to be per odic with period $N/2$, and so it can only be $H_6[k]$ (since it has to be one of the listed signals). Also, $G_5[k] = H_2[k]$, $G_6[k] = H_1[k]$ and $G_7[k] = H_5[k]$.