هوالحکیم

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دانشگاه تهران
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گروه فنی

راه‌حل تکلیف شماره‌ی ۸

فصل هفتم

تکنیک‌های طراحی فیلتر

Filter Design Techniques

◊ مسئله‌های تحلیلی ـ تشریحی

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1. **[Oppenheim/Schafer/Buck Problem #7.21]** We have $\Omega_p = \frac{2}{T_d}\tan(\omega_p/2)$, and so $T_d = \frac{2}{\Omega_p}$ an$(\pi/4) = \frac{2}{\Omega_p}$. For part (b), we see that $\omega_p = 2\arctan\left(\frac{\Omega_p T_d}{2}\right)$, and $\omega_s = 2\arctan\left(\frac{\Omega_s T_d}{2}\right)$. Therefore, $\Delta\omega = 2\left[\arctan\left(\frac{\Omega_s T_d}{2}\right) - \arctan\left(\frac{\Omega_p T_d}{2}\right)\right]$.

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2. **[Oppenheim/Schafer/Buck Problem #7.22]** $H(z) = H_c(s)|_{s=\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{T_d}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$, with ROC $|z| > 1$ (by causality). Therefore, $h[n] = \frac{T_d}{2}(u[n] + u[n-1])$. The difference equation is $y[n] - y[n-1] = \frac{T_d}{2}(x[n] + x[n-1])$. The system is not implementable since it has a pole on the unit circle, and hence not stab e. Not that the system does not have a four er ransform (strictly speaking). However, if we gnore this inconvenient fact, we get $He^{j\omega} = \frac{T_d}{2j}\cot(\omega/2)$ (no ice the value when $\omega = 0$). Th s s a good approximation at low frequencies (since both the continuous and discrete versions get very arge) but is not a good approximation at high frequencies. The differentiator is similar. To get one to be the inverse of the other, we need to use the same value of T_d.

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3. **[Oppenheim/Schafer/Buck Problem #7.26]** See the prob em session notes.

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4. **[Oppenheim/Schafer/Buck Problem #7.32]** By Parseva , we have

$$\epsilon^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|Ee^{j\omega}\right|^2 d\omega = \sum_{k\in\mathbb{Z}}|e[n]|^2 = \sum_{k=0}^{M}|h_d[n] - h[n]| + \sum_{k\notin[0,M]}|h_d[n]|^2$$

To minimize this expression, we set $h[n] = h_d[n]$ for $n \in [0, M]$ yielding the rectangular window.

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5. **[Oppenheim/Schafer/Buck Problem #7.33]** $H_d e^{j\omega} = [1 - 2u(\omega)]e^{j(\pi/2 - \tau\omega)}$ for $-\pi < \omega < \pi$. The phase discontinuity at 0 requires the fi er to have a 0 at $z = 0$. This means that we need to use a type III or a type IV filter here. $h_d[n] = \frac{2}{\pi}\frac{\sin^2[\pi(n-\tau)/2]}{n-\tau}$ f $n \neq \tau$, and $h_d[n] = 0$ if $n = \tau$. We should choose $\tau = M/2$ to ensure symmetry/anti-symmetry. For part (d) the delay is 10.5, while for part (e) the delay is 10 samples.

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3) (7.26)

a) Since

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$

and we desire

$$H(e^{j\omega})|_{\omega=0} = H_c(j\Omega)|_{\Omega=0},$$

we see that

$$H(e^{j\omega})|_{\omega=0} = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)|_{\omega=0} = H_c(j\Omega)|_{\Omega=0}$$

requires

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} H_c\left(j\frac{2\pi k}{T_d}\right) = 0.$$

b) Since the bilinear transform maps $\Omega = 0$ to $\omega = 0$, the condition will hold for any choice of $H_c(j\Omega)$