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فصل چهارم

پردازش گسسته-زمان سیگنال‌های پیوسته-زمان

Discrete-Time Processing of Continuous-Time Signals

◊ مسئله‌های تحلیلی ـ تشریحی

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1. **[Oppenheim/Schafer/Buck Problem #4.29]** Suppose that $x[n] \overset{\mathcal{F}}{\leftrightarrow} X\left(e^{j\omega}\right)$, and $x_u[n]$ is the signal that results when $x[n]$ is upsampled by 2. Then $x_u[n] \overset{\mathcal{F}}{\leftrightarrow} X\left(e^{j2\omega}\right)$. Let $Y_u\left(e^{j\omega}\right) = H_1\left(e^{j\omega}\right) X\left(e^{j\omega}\right)$ and $y_u[n] \overset{\mathcal{F}}{\leftrightarrow} Y_u\left(e^{j\omega}\right)$. Then $Y_1\left(e^{j\omega}\right)$ is the signal that results from downsampling $Y_u\left(e^{j\omega}\right)$ $(y_u[n])$ by 2. Therefore,

$$\begin{aligned} Y_1\left(e^{j\omega}\right) &= \frac{1}{2}\left[Y_u\left(e^{j\omega/2}\right) + Y_u\left(e^{j(\omega-2\pi)/2}\right)\right] \\ &= \frac{1}{2}\left[X\left(e^{j2\omega/2}\right) H_1\left(e^{j\omega/2}\right) + X\left(e^{2j(\omega-2\pi)/2}\right) + H_1\left(e^{j(\omega-2\pi)/2}\right)\right] \\ &= \frac{1}{2}\left[X\left(e^{j\omega}\right) H_1\left(e^{j\omega/2}\right) + X\left(e^{j(\omega-2\pi)}\right) H_1\left(e^{j(\omega/2-\pi)}\right)\right] \\ &= \frac{1}{2}\left[H_1\left(e^{j\omega/2}\right) + H_1\left(e^{j(\omega/2-\pi)}\right)\right] X\left(e^{j\omega}\right) \end{aligned}$$

Therefore, $H_2\left(e^{j\omega}\right) = \frac{1}{2}\left[H_1\left(e^{j\omega/2}\right) + H_1\left(e^{j(\omega/2-\pi)}\right)\right]$.

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2. **[Oppenheim/Schafer/Buck Problem #4.34]** The first thing to note is that the system shown in Figure P4.34-1 simply convolves $x[n]$ with $h[n]$ to get $y[n]$, where $h[n] \overset{\mathcal{F}}{\leftrightarrow} H\left(e^{j\omega}\right)$. Note that the sampling rate does not figure anywhere in this, and so may be chosen arbitrarily. So we pick T to be 1. Since $\omega = \Omega T$, we have $H(j\Omega) = e^{-j\Omega T/2} = e^{-j\Omega/2}$. For the second part, let $\omega_0 = 5\pi/2$. Then

$$\cos(\omega_0 n - \pi/4) = \frac{e^{j(\omega_0 n-\pi/4)} + e^{-j(\omega_0 n-\pi/4)}}{2} = \frac{e^{-j\pi/4}}{2}e^{j\omega_0 n} + \frac{e^{j\pi/4}}{2}e^{-j\omega_0 n}$$

Now, $e^{-j\omega_0 n}$ and $e^{j\omega_0 n}$ are eigenfunctions of the system, and are scaled by $H\left(e^{j\omega_0}\right) = e^{j\pi/4}$ and $H\left(e^{j(-\omega_0)}\right) = e^{-j\pi/4}$ respectively. Therefore,

$$\begin{aligned} y[n] &= \frac{e^{-j\pi/4}}{2}e^{j\omega_0 n} H\left(e^{j\omega_0}\right) + \frac{e^{j\pi/4}}{2}e^{-j\omega_0 n} H\left(e^{j(-\omega_0)}\right) \\ &= \frac{e^{-j\pi/4}}{2}e^{j\omega_0 n}e^{-j\pi/4} + \frac{e^{j\pi/4}}{2}e^{-j\omega_0 n}e^{j\pi/4} \\ &= \cos(\omega_0 n - \pi/2) \end{aligned}$$

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3. **[Oppenheim/Schafer/Buck Problem #4.36]** The continuous frequency Ω and the discrete frequency ω are related by $\Omega T = \omega$. We need $X\left(e^{j\omega}\right) = 0$ for $\omega > \pi/2$. We have $X_c(j\Omega) = 0$ for $\Omega > 2\pi(100)$. Therefore, we need $(2\pi \cdot 100)T \leq \pi/2$ or $T \geq \frac{1}{400}$. For the second part, we have to chose $T' = 2T$.

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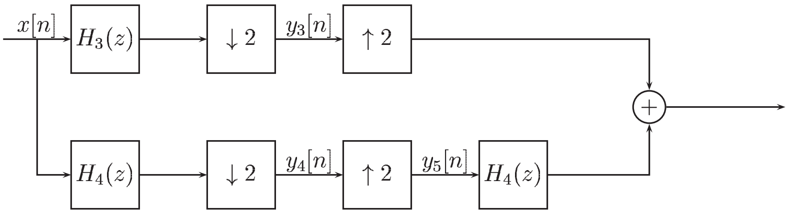
4. **[Oppenheim/Schafer/Buck Problem #4.37]** Going from $s[n]$ to $s_1[n]$ requires changing the sampling rate by a factor of $3/5$. Therefore, we use the system shown in Figure 4.28 of the text, with $L = 3$, and $M = 5$.



5. **[Oppenheim/Schafer/Buck Problem #4.42]** It is not possible. The Nyquist criterion requires strict inequality (so we need $T < \frac{1}{500}$). As a counter example, consider the signal $x_c(t) = \sin(2\pi \cdot 250 \cdot t)$, with corresponding sampled sequence $x[n] = x_c(n/500) = \sin(2\pi \cdot (250) \cdot n/500) = \sin(n\pi) = 0$. You are not going to be recovering anything from this signal. The remaining answers are now obvious.

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6. **[Oppenheim/Schafer/Buck Problem #4.46]** $y_i[n] = x[3n+i]$. Therefore,

$$x[n] = \begin{cases} y_0[n/3] & \text{if } 3|n \\ y_1[(n-1)/3] & \text{if } 3|(n-1) \\ y_2[(n-2)/3] & \text{else} \end{cases}$$

The answer for part (b) is yes as well because the output of the filters have bandwidth at most $\pi/3$, and so no aliasing will take place. To reconstruct, upsample $y_1[n]$, $y_2[n]$ and $y_3[n]$ by 3, then pass the upsampled signal through $H_i(z)$, and add. For part (c), the answer is yes. We note that $Y_3\left(e^{j\omega}\right) = \frac{1}{2}\left[X(e^{j\omega/2}) + X(e^{j(\omega/2-\pi)})\right]$, and

$$Y_4\left(e^{j\omega}\right) = \begin{cases} \frac{1}{2}\left[X(e^{j\omega/2}) - X(e^{j(\omega/2-\pi)})\right] & \text{if } 0 \leq \omega < \pi \\ \frac{1}{2}\left[-X(e^{j\omega/2}) + X(e^{j(\omega/2-\pi)})\right] & \text{else} \end{cases}$$

A simple calculation now shows that the system shown in the figure reconstructs $x[n]$.