Adversarial Search
(Game playing)

Chapter 6

Outline

- ♢ Games
- ♢ Perfect play
  - – minimax decisions
  - – α–β pruning
- ♢ Resource limits and approximate evaluation
- ♢ Games of chance
- ♢ Games of imperfect information

Games vs. search problems

"Unpredictable" opponent ⇒ solution is a strategy
specifying a move for every possible opponent reply

Time limits ⇒ unlikely to find goal, must approximate

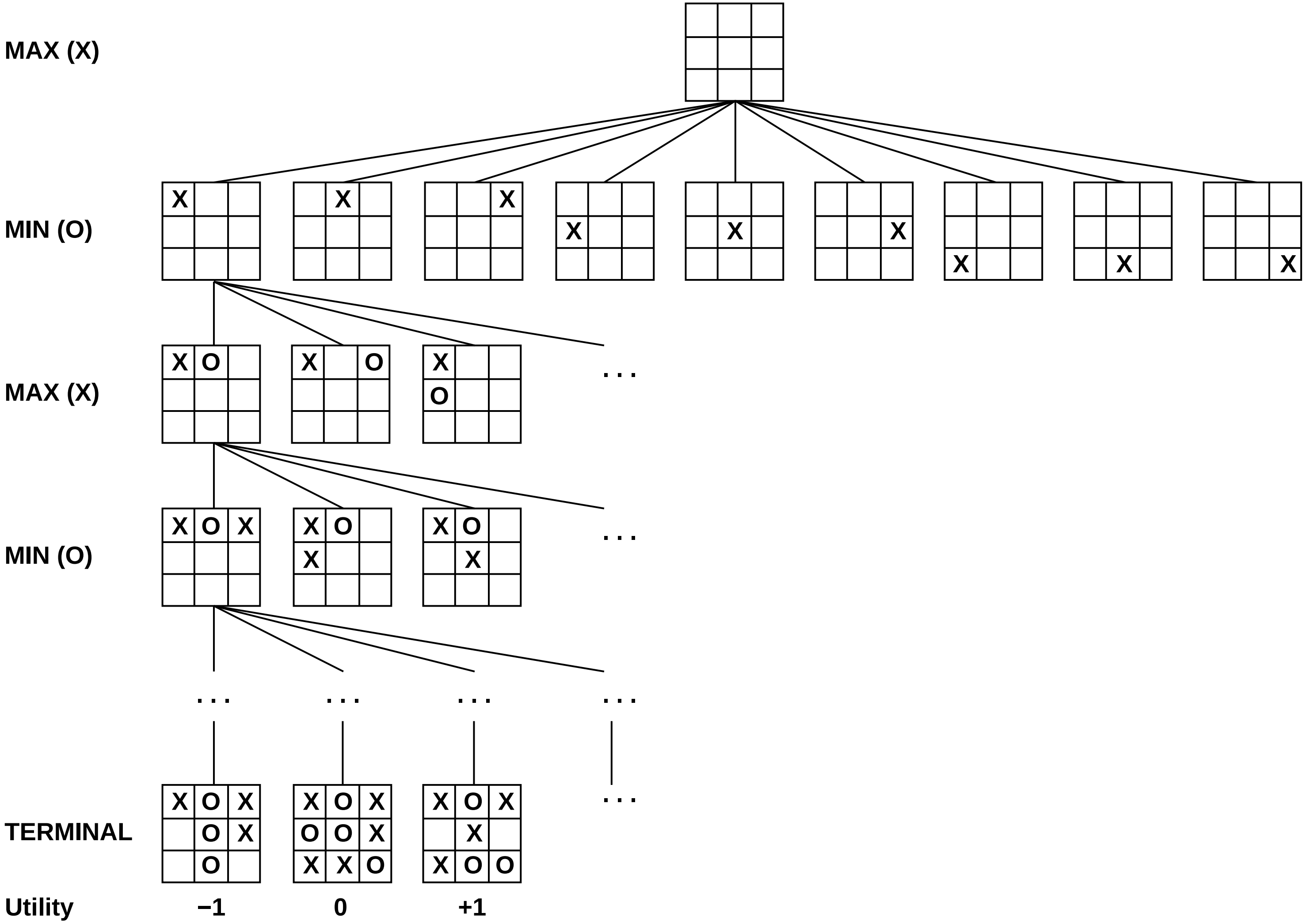
Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Game tree (2-player, deterministic, turns)

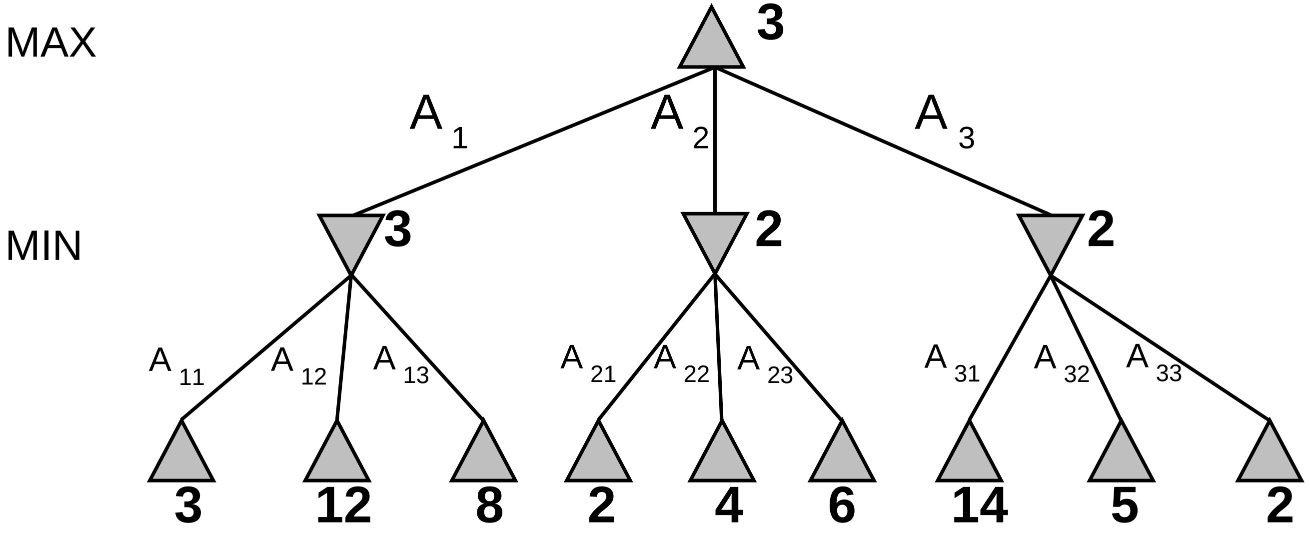


Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state, game) returns an action
    action, state ← the a, s in SUCCESSORS(state)
            such that MINIMAX-VALUE(s, game) is maximized
    return action
```

```
function MINIMAX-VALUE(state, game) returns a utility value
    if TERMINAL-TEST(state) then
        return UTILITY(state)
    else if MAX is to move in state then
        return the highest MINIMAX-VALUE of SUCCESSORS(state)
    else
        return the lowest MINIMAX-VALUE of SUCCESSORS(state)
```

Properties of minimax

Complete??

Properties of minimax

Complete?? Only if tree is finite (chess has specific rules for this).
NB a finite strategy can exist even in an infinite tree!

Optimal??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity??

Properties of minimax

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Time complexity?? $O(b^m)$

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

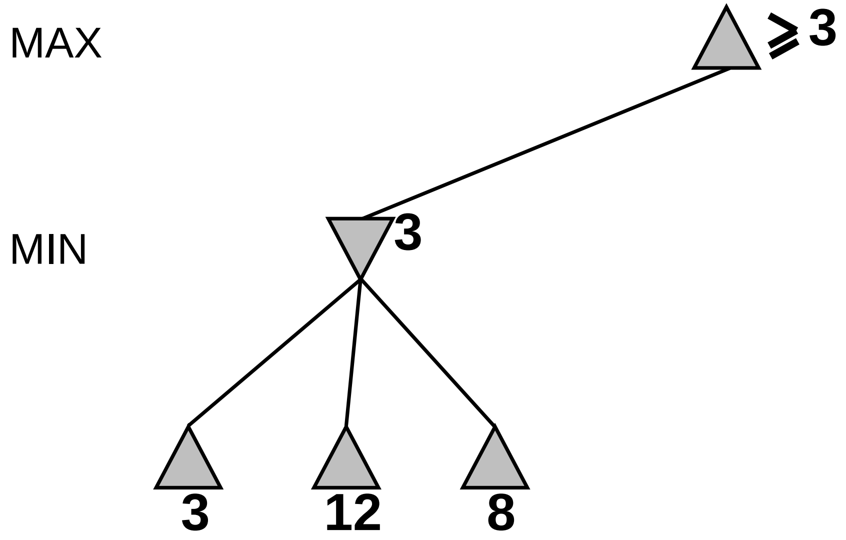
Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

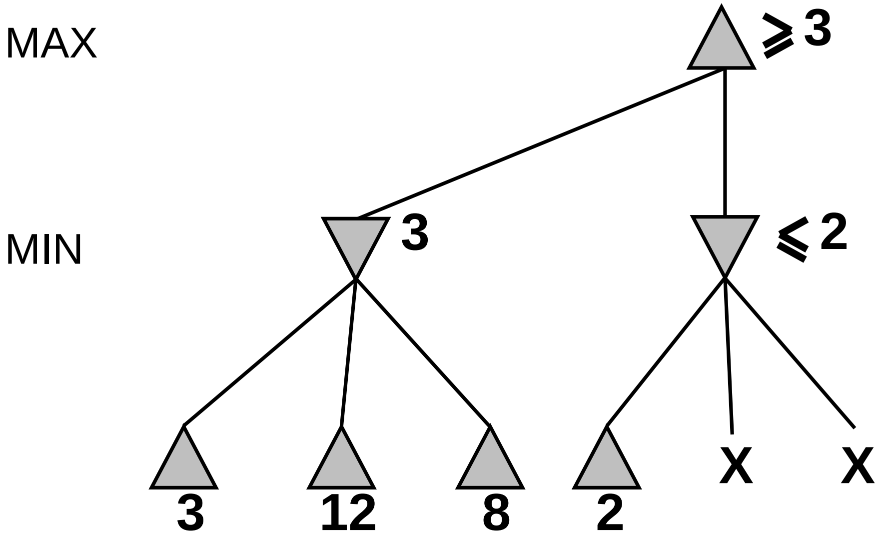
For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
$\Rightarrow$ exact solution completely infeasible

But do we need to explore every path?

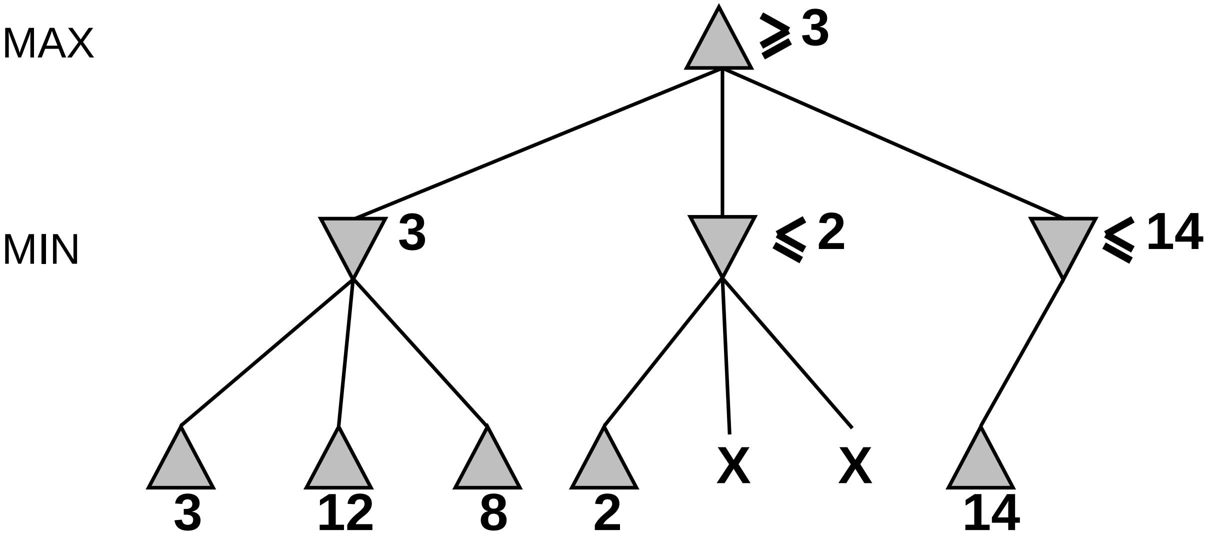
α–β pruning example



α–β pruning example



α–β pruning example



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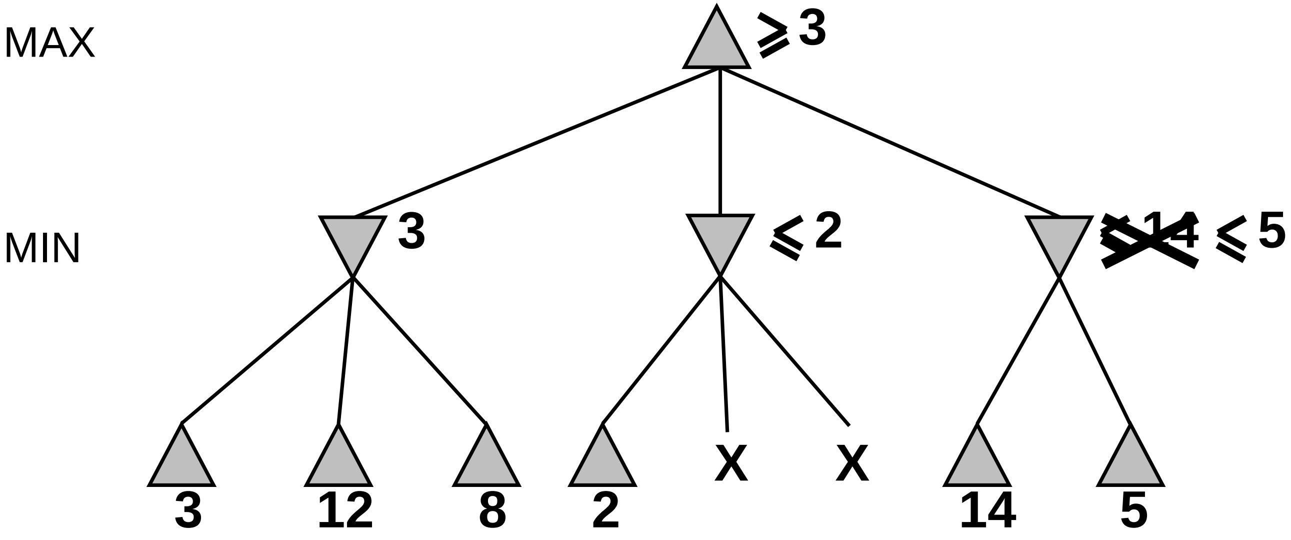
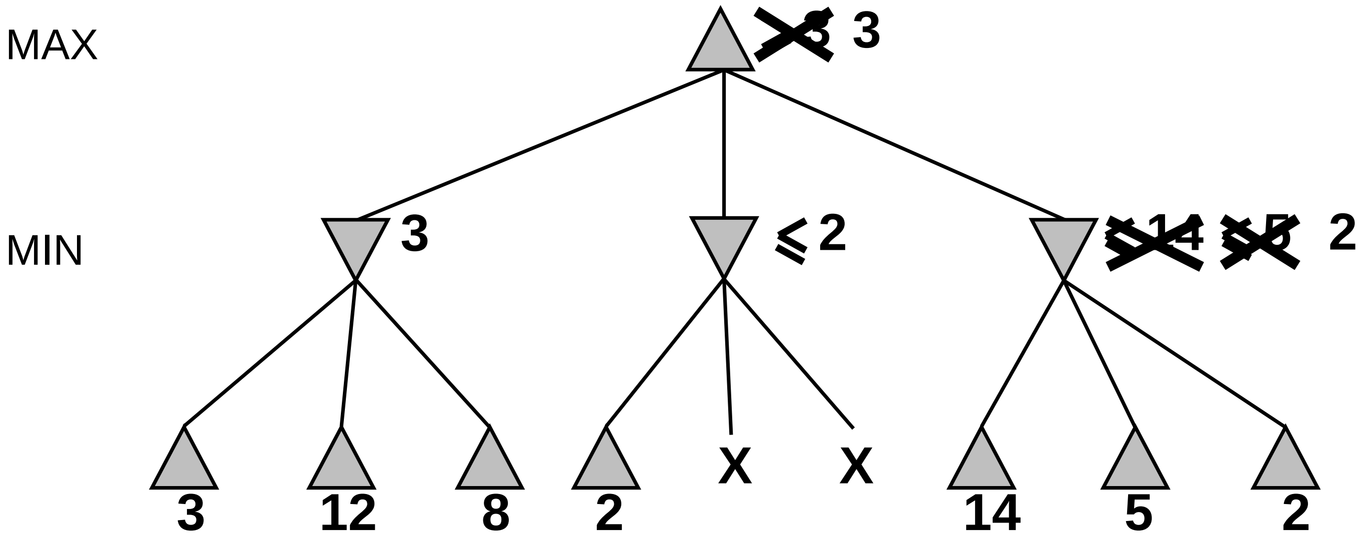
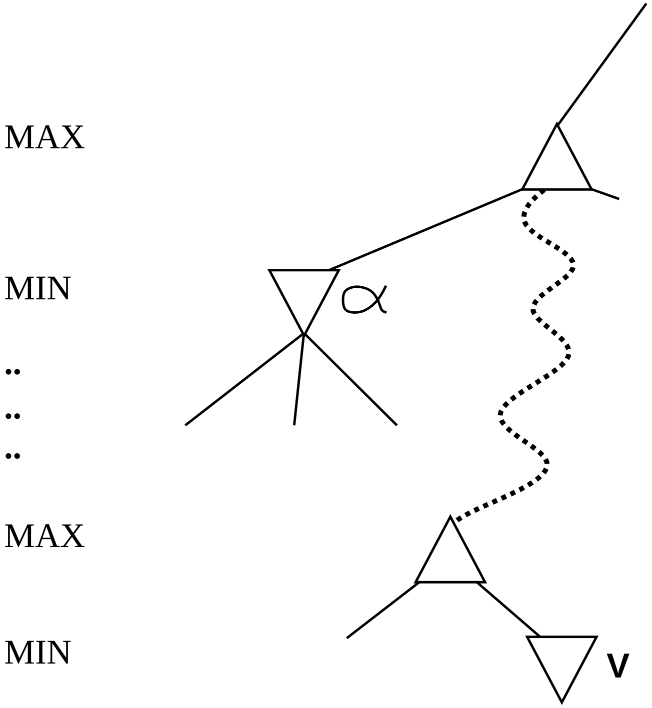


α–β pruning example



Why is it called α–β?



α is the best value (to MAX) found so far off the current path

If V is worse than α, MAX will avoid it ⇒ prune that branch

Define β similarly for MIN

The α–β algorithm

```
function Alpha-Beta-Search(state, game) returns an action
   action, state ← the a, s in Successors[game](state)
            such that Min-Value(s, game, $-\infty$, $+\infty$) is maximized
   return action
```

```
function Max-Value(state, game, $\alpha$, $\beta$) returns the minimax value of state
   if Cutoff-Test(state) then return Eval(state)
   for each s in Successors(state) do
      $\alpha \leftarrow \max(\alpha$, Min-Value(s, game, $\alpha$, $\beta$))
      if $\alpha \geq \beta$ then return $\beta$
   return $\alpha$
```

```
function Min-Value(state, game, $\alpha$, $\beta$) returns the minimax value of state
   if Cutoff-Test(state) then return Eval(state)
   for each s in Successors(state) do
      $\beta \leftarrow \min(\beta$, Max-Value(s, game, $\alpha$, $\beta$))
      if $\beta \leq \alpha$ then return $\alpha$
   return $\beta$
```

Properties of α–β

Pruning **does not** affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity $= O(b^{m/2})$
$\Rightarrow$ **doubles** solvable depth

A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Unfortunately, 35^{50} is still impossible!

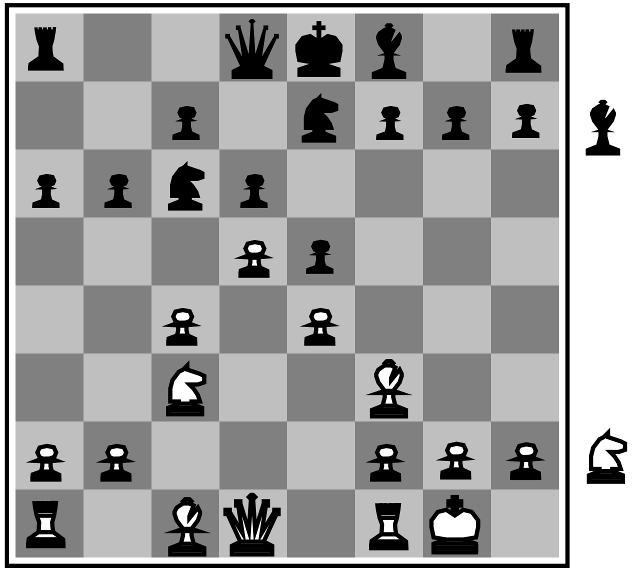
Resource limits

Standard approach:

- Use CUTOFF-TEST instead of TERMINAL-TEST
  e.g., depth limit (perhaps add quiescence search)
- Use EVAL instead of UTILITY
  i.e., evaluation function that estimates desirability of position

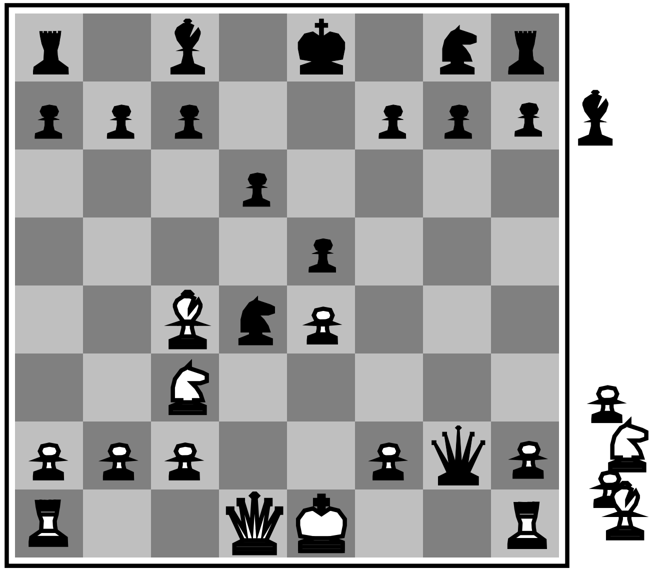
Suppose we have 100 seconds, explore 10^4 nodes/second
$\Rightarrow$ 10^6 nodes per move $\approx 35^{8/2}$
$\Rightarrow$ α–β reaches depth 8 $\Rightarrow$ pretty good chess program

Evaluation functions



Black to move

White slightly better



White to move

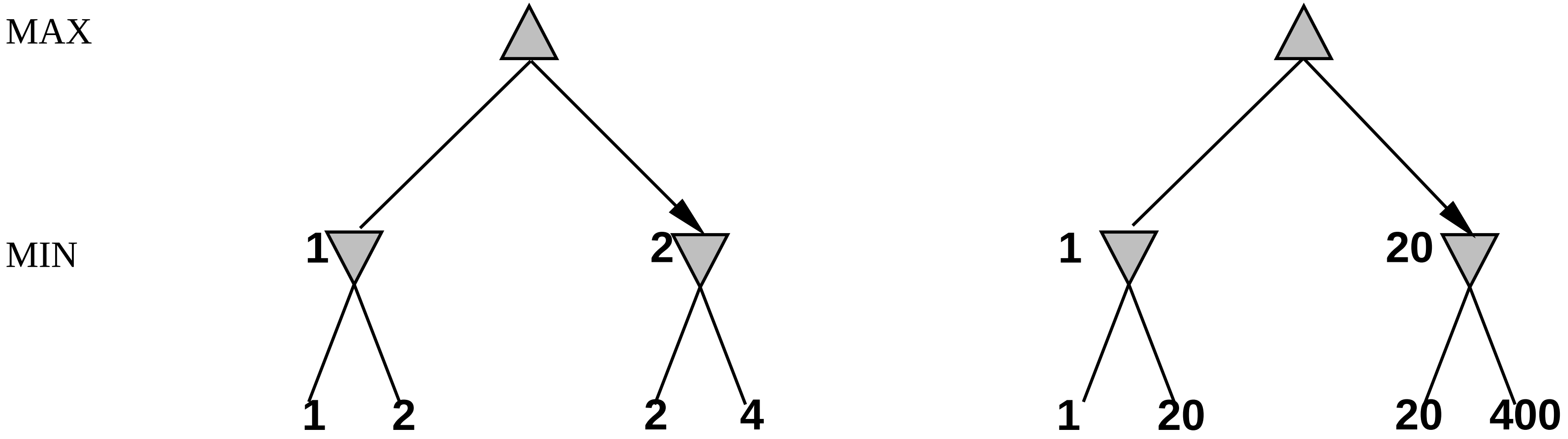
Black winning

For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

Digression: Exact values don't matter



Behaviour is preserved under any **monotonic** transformation of EVAL

Only the order matters:

payoff in deterministic games acts as an ordinal utility function

Deterministic games in practice

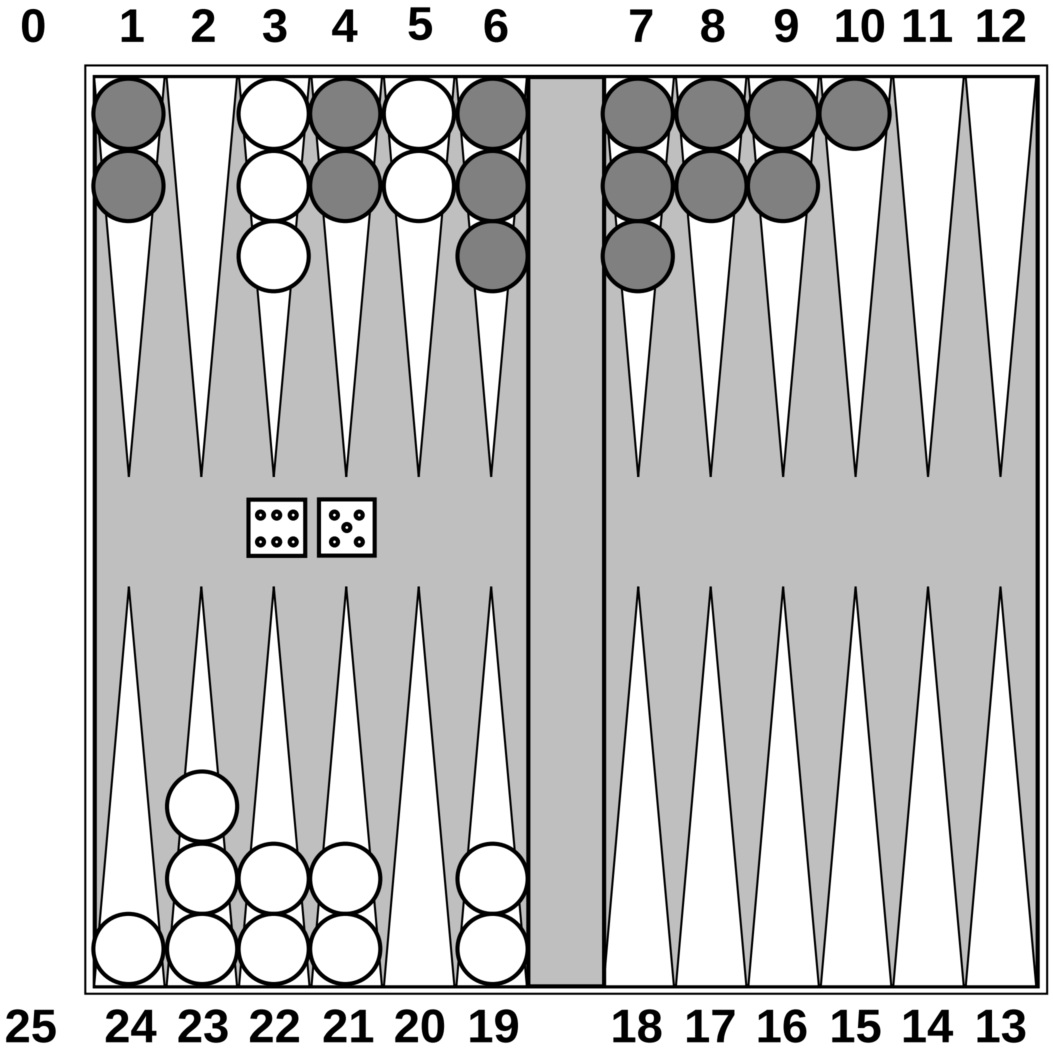
Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

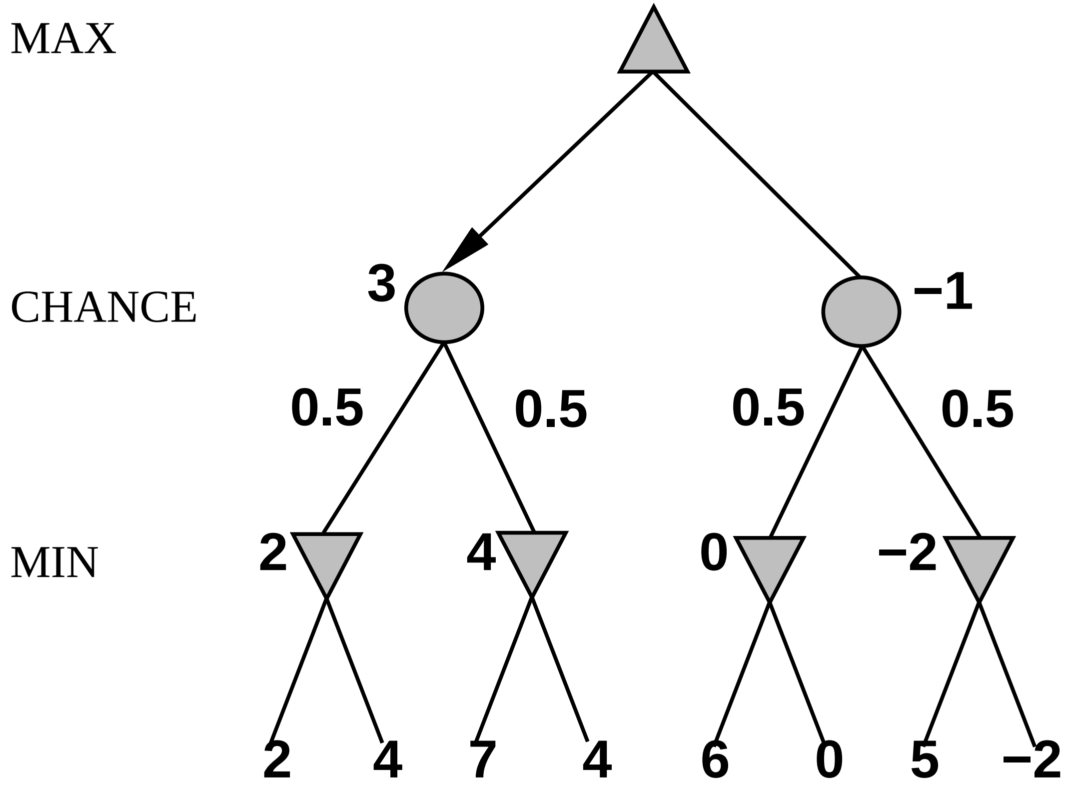
Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

```
. . .
if state is a MAX node then
        return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
        return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
        return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
. . .
```

Nondeterministic games in practice

Dice rolls increase b: 21 possible rolls with 2 dice
Backgammon ≈ 20 legal moves (can be 6,000 with 1-1 roll)

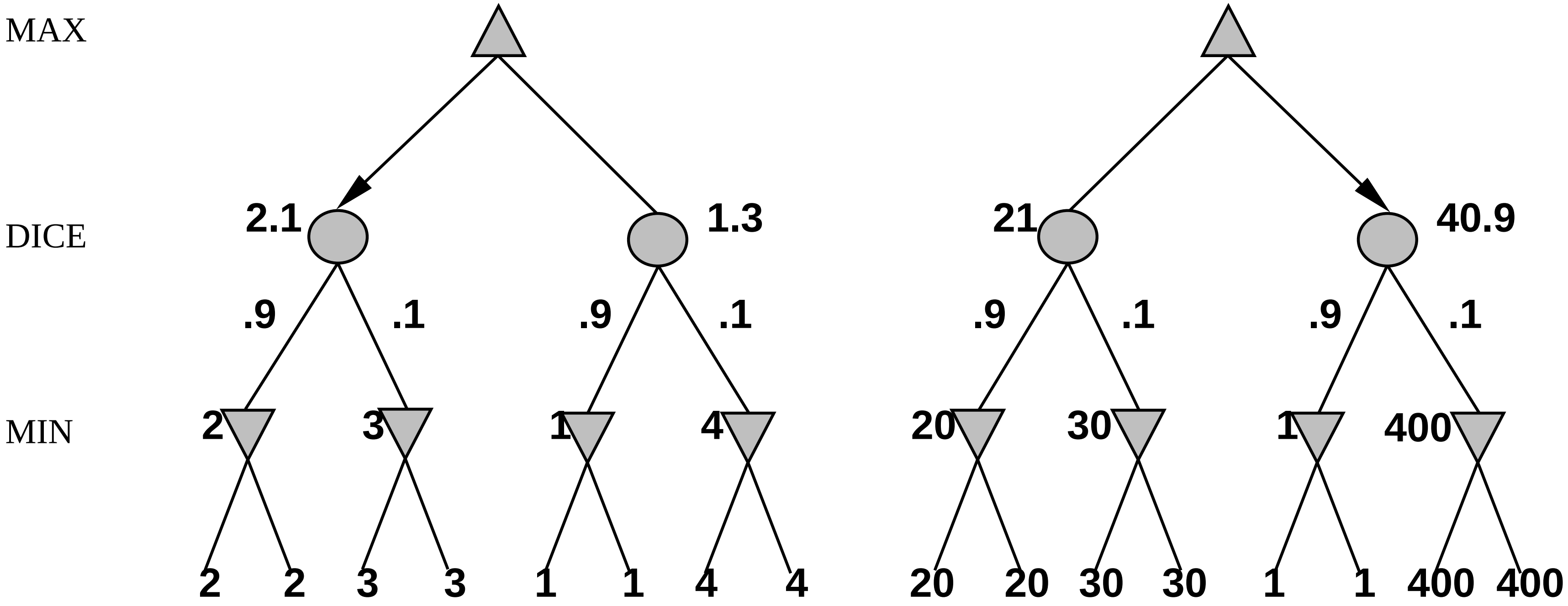
$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks
$\Rightarrow$ value of lookahead is diminished

α–β pruning is much less effective

TDGAMMON uses depth-2 search + very good EVAL
≈ world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by positive linear transformation of EVAL

Hence EVAL should be proportional to the expected payoff

Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game*

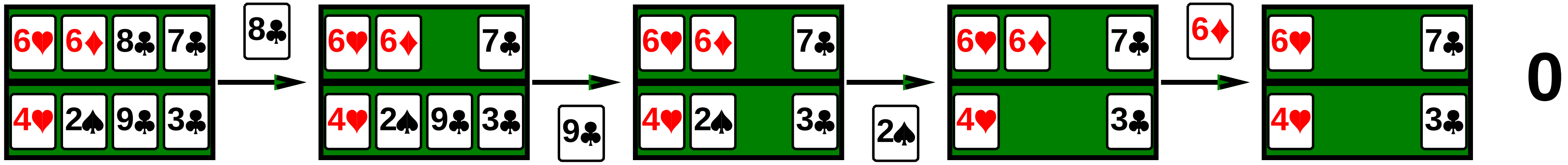
Idea: compute the minimax value of each action in each deal,
then choose the action with highest expected value over all deals*

Special case: if an action is optimal for all deals, it's optimal.*

GIB, current best bridge program, approximates this idea by
1) generating 100 deals consistent with bidding information
2) picking the action that wins most tricks on average

Example

Four-card bridge/whist/hearts hand, MAX to play first



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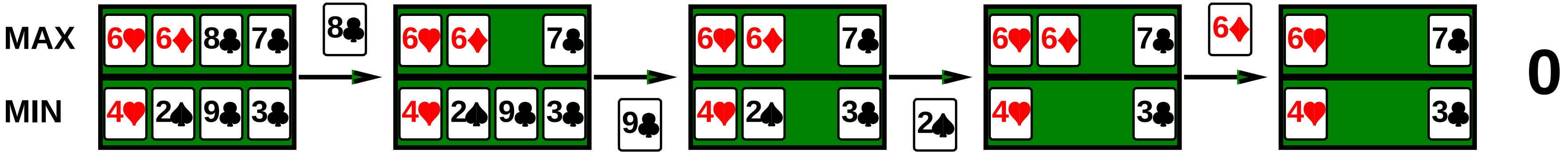

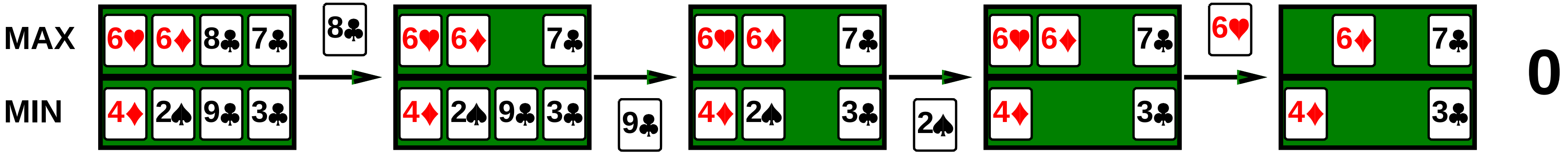

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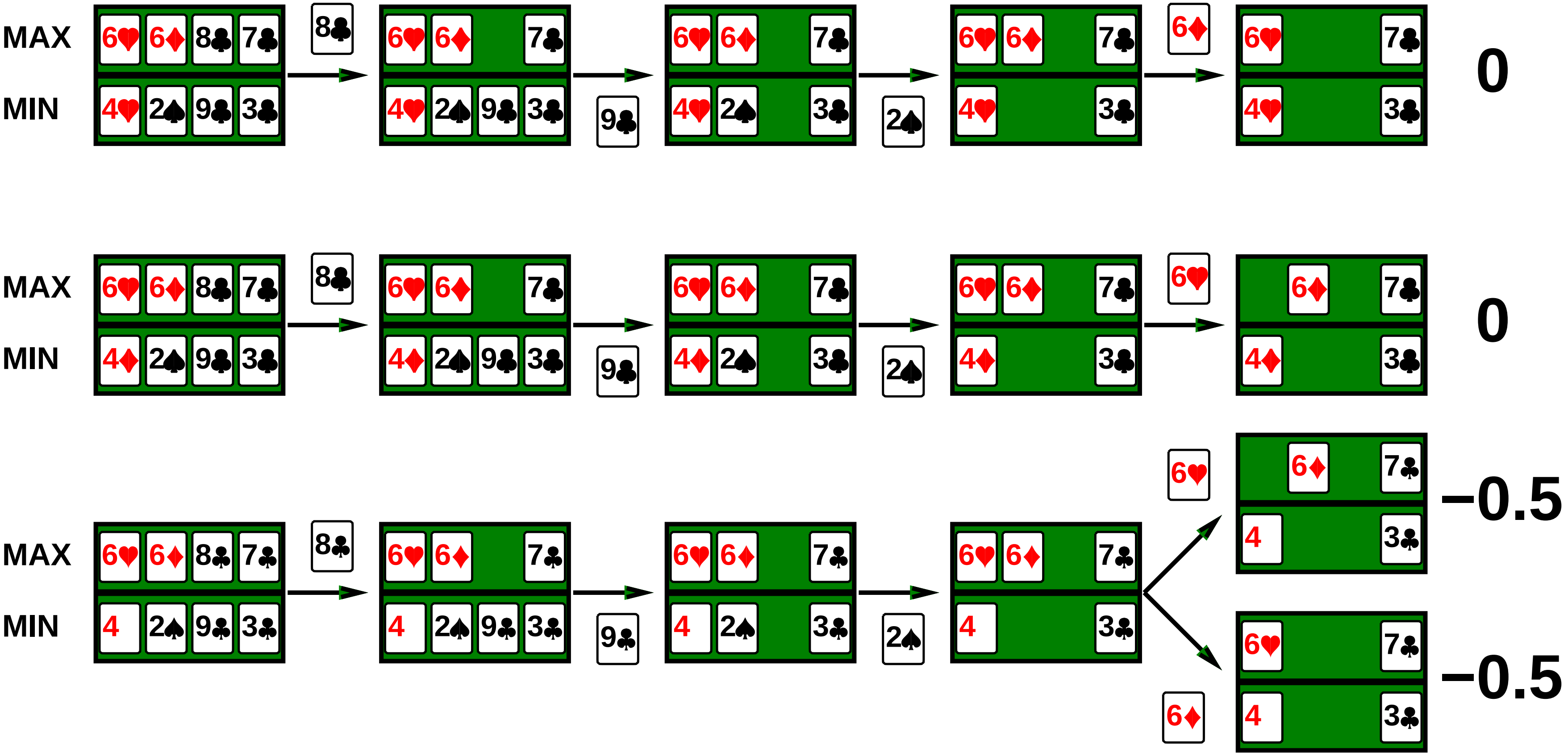

Commonsense example

Road A leads to a small heap of gold pieces
Road B leads to a fork:
    take the left fork and you'll find a mound of jewels;
    take the right fork and you'll be run over by a bus.

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    take the left fork and you'll be run over by a bus;
    take the right fork and you'll find a mound of jewels.

Road A leads to a small heap of gold pieces
Road B leads to a fork:
    guess correctly and you'll find a mound of jewels;
    guess incorrectly and you'll be run over by a bus.

Proper analysis

* Intuition that the value of an action is the average of its values in all actual states is **WRONG**

With partial observability, value of an action depends on the information state or belief state the agent is in

Can generate and search a tree of information states

Leads to rational behaviors such as

- ♢ Acting to obtain information
- ♢ Signalling to one's partner
- ♢ Acting randomly to minimize information disclosure

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ♢ perfection is unattainable ⇒ must approximate
- ♢ good idea to think about what to think about
- ♢ uncertainty constrains the assignment of values to states
- ♢ optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design